## TRANSMISSION PRINCIPLES

NOISSIWSNVYI
Sヨ7dITNIMd


WESTERN ELECTRIC COMPANY

TRANSMISSION PRINCIPLES


#### Abstract

This material is prepared for training purposes only and is the property of Western Electric Company, Incorporated, and is loaned on condition that it be returned to the Company upon termination of employment or upon request by the Company prior thereto. No reproduction of this material, in whole or in part, and no disclosure of any part of this material to other than fellow employees, as may be necessary in connection with work assignments, may be made without prior permission in writing from the Company.


Appreciation is expressed to the publishers:
D. Van Nostrand Co., Inc.

McGraw-Hill Book Co.
Prentice-Hall, Inc. and also to the
American Telephone and Telegraph Co. and others, for permission to use quotations and illustrations, A list of references is provided at the end of each chapter listing copyrights of the publishers to the quotations indicated by superscripts in the text.

Printed in U.S. A.

The material in this text has been compiled to give the Western Electric engineer a comprehensive survey of the transmission principles encountered in the Bell System.

We would like to express our sincere appreciation to past and present members of Graduate Engineering Education Staff and to the many other individuals within the Bell System who have given their time and effort to the organization of this book.

We wish to dedicate this book to the engineers of Western Electric who, in order to stay abreast of ever advancing technologies, must judiciously select reference materials that provide the greatest return for the time involved.
E. G. WALTERS

Manager, Graduate Engineering Education and Technical Training
Programs

## TRANSMISSION PRINCIPLES

## TABLE OF CONTENTS

Page
Chapter $1 \quad$ Characteristics of Sound and Wave Motion ..... 1. 1
Chapter 2 Networks ..... 2. 1
Chapter 3 Repeating Coils and Transformers ..... 3.1
Chapter 4 Transmission Lines ..... 4. 1
Chapter 5 Transmission Improvement ..... 5, 1
Chapter 6 Transmission Design ..... 6. 1

## CHAPTER I

## CHARACTERISTICS OF SOUND AND WAVE MOTION

## 1. 1 INTRODUCTION

The field of communications deals with the interchange of thoughts, or data signals to answer the needs of culture, entertainment or business. It follows that in order to convey the maximum information these thoughts or signals should take the form of something visual or audible and be capable of reaching, within spatial limitations, any person or apparatus to whom the thought is directed. For instance, speech is a thought expressed in audible form and may be transmitted to any person or apparatus. Pictures and printed words are thoughts expressed by visual means which may be interchanged between persons or suitable apparatusses. Each of these examples are an expression of some thought. It is the purpose of the communications industry to carry this expression to its destination by means of electromagnetic energy flowing over wires or radiated through space. The principles underlying the transmission of a thought expressed in the form of speech, are, with a few exceptions, identical to those of a thought expressed in all other forms. Since speech is the more common form, this text will deal with its transmission primarily.

The telephone transmits speech by employing the mechanical energy of the speaker's voice to produce electrical energy, of suitable characteristics, which is later converted into sound waves at the listeners station. To better understand the problems encountered we may well consider the nature of sound and hearing.

## 1. 2 BRIEF HISTORY OF COMMUNICATION TRANSMISSION

1835 Samuel Morse privately demonstrated his electromagnetic telegraph. Patents were granted to him in 1848.
1844 First telegraph line was constructed between Washington and Baltimore.
1851 First successful submarine telegraph cable was laid between England and France.
1866 First successful submarine telegraph cable was laid across the Atlantic between North America and Europe.
1876 Alexander Graham Bell invented the electric telephone and transmitted the famous sentence:
"Mr. Watson, come here, I want you."

1879 Phably the first cables used for telephone service were laid across the Brooklyn Bridge.
1870 Conellys and McTighe invented the dial telephone system and applied for patent.
1885 American Telephone and Telegraph Com any was incorporated.
1887 Strowger invented the step-by-step dial ystem.
1899 Loading coils were applied for the first ime. Theory of them was developed by Heaviside, Pupin and Jampbell. Prior to the use of loading, commercial service could be given over no greater distance than from New York to Chicago.
1911 Due to loading telephone service was established between New York and Denver.
1913 Vacuum-tube amplifier was adapted to telephone line repeaters.
1915 First transcontinental telephone circuits were opened for commercial service.
1918 The carrier telephone system was developed by Bell System engineers and was first used.

## 1. 3 SOUND

Sound in the scientific sense has two distinct meanings. To the psychologist it means a sensation. To the physicist it means an atmospheric disturbance or a stimulus whereby a sensation is produced in the human ear. More precisely it is a wave mot produced by some vibrating body such as a bell, tuning fork, tue human vocal chords, or similar objects capable of producing rapid to-andfro or vibratory motion of air particles.

Everyone is familiar with the series of waves that emanate from a stone cast upon the still water of a lake or a pond. This is one of many forms of wave motion. In a manner similar to that in which the stone coming in contact with the water establishes radiating rings formed by circular wave crests alternating with wave troughs, there emanate from a source of sound alternate condensations and raxefactions of the air. Instead of being rings on a single plane or surface, however, they are a series of concentric spheres expanding at a definite rate of travel. This velocity is approximately l, 075 feet per second but varies to some extent with? altitude, pressure, as well as with the temperatue and relative humidity of air. Compared to light or heat, which are also forms of wave motion, sound travels very slomily. Thus we see a flash of lightning before we hear the thunder, and vesee the smoke dispelled from the muzzle of a gun before we hear the, 's report.
(It is suggested that a special magnetic tane on sound theory be deint rated to class here. 23 minuted are needed for such demonstration.)

CH. 1-CHARACTERISTICS OF SOUND AND WAYE MOTION

## 1. 31 PURE SOUND

Pure sound is defined as a disturbance in the atmosphere whereby a sine wave motion is propagated from some source. An accurately constructed tuning fork produces the simplest kind of sound consisting of displacements following a single sine function. That is, the amount of displacement would vary with time as the amplitude of a sine curve varies along its time axis. To understand more clearly how this is so, consider a pendulum, which, though it vibrates so slowly that its motion is not audible as sound, none the less conforms to the laws of all vibratory motion. Once started swinging, the pendulum would continue, at the same rate, until it finally stops. Starting at the peak of its swing in one direction, the pendulum would complete the arc to the opposite peak in ( $t$ ) units of time. Each successive arc would $\exists 1$ so be completed in exactly ( $t$ ) units of time and the length of the arc would become smaller as the pendulum expended its energy. A pendulum starting from rest and returning to rest after having swung to both extremes of its arc, is said to have completed one cycle. The number of cycles it completes in one second is called its frequency, and the time it takes to complete one cycle is called a period.

If the horizontal motion of the pendulum were plotted as a function of time, the resultant curve would take the form of the familiar sine wave. Now since each cycle of the pendulum produces a disturbance of the atmosphere it follows that these disturbances or displacements will also vary as a sine curve, and if the frequency of the pendulum were increased enough, these displacements would become audible as a pure tone. A tuning fork is similar to a pendulum swinging at a very rapid rate, or frequency, producing atmospheric displacements following a single sine function, which we perceive as a pure sound.

It would seem that the atmosphere is necessary for the transmission of sound. This is true. Heat, light, and radio frequency waves, which are also examples of wave motion, may be transmitted through space where there is no atmosphere. Sound, however, cannot be perceived in a vacuum and so we assume that the air is necessary for its transmission.

## 1. 32 DEEMTTIONS AND UNITS OF SOUND

Pressure Units:
1 microbar $=\frac{1 \text { dyne }}{\mathrm{sq} \mathrm{cm}}$
or
1 bar $\quad=\frac{10^{6} \text { dynes }}{\mathrm{sq} \mathrm{cm}}$

## Sound Pressure Level:

Is expressed in decibels above the reference level and is calculated from the formula

$$
\mathrm{db}=20 \log _{10} \frac{\text { Sound Pressure in microbars }}{\text { Reference Pressure in microbars }}
$$

Two reference pressures are used -
a) $2 \times 10^{-4}$ microbar $=0.0002$ microbar

This reference pressure is equivalent to $10^{-16}$ watt/sq cm and is used for measurements of hearing and sound level in the air and in the liquids.
b) 1 microbar $=\frac{\text { dyne }}{s q \mathrm{~cm}}$

This reference pressure is used for calibrations of acoustic instruments and for measurements in certain laquids. 1000 microbars equal 1 millibar.

## Loudness Level:

Is often measured in "phons." It is numerically equal to the sound pressure level expressed in db relative to 0.0002 microbar of a pure 1000 cps (cycles por second) tone, which is judged by the listeners to be equivalein $n$ loudness to the investigated tone.

## Pitch:

Is that subjective quality of a tone that permits the listener to locate its position on a musical scale. Pitch changes with frequency but depends also upon the intensity of sound and the wave form of sound stimulus. A tone of fixed frequency of a few hundred cycles per second would decrease in pitch if its intensity is increased. For frequencies higher than about 2000 cps the pitch increases with increasing loudness. The lowest audible pitch is between 16 and 32 cps , highest between 16,000 and $32,000 \mathrm{cps}$. Pitch and frequency are not the same.

## 1. 33 COMPLEX SOUNDS AND WAVES

If the sound's source is a vibrating mechanism in simple form, such as the to-and-fro motion of a tuning fork, and it is sustained for a definite interval of time, the wave motion is said to be simple harmonic motion and may be represented by a sine curve. On the other hand, if the source performs a complex mechanical motion or is an object vibrating in parts as well as in its entirety, the wave motion is called complex.

In order to see how complex waves are produced, it is necessary to review some of the relationships between sine waves of various frequencies. Suppose a sine wave of frequency $X$ and amplitude 4 A is added to a second sine wave of frequency 2 X and amplitude lA. This is shown graphically in Figure 1. Notice that the resultant wave shape is obviously not a sine wave. Furthermore, it can be seen by inspection of Figure l that an amplitude of 2 A for the wave of frequency 2 X would cause the resultant wave to have a different and more irregular shape.

In Figure 1 the two component waves passed through 0 amplitude at the same time. Suppose now that the wave of frequency 2X were advanced in time 90 degrees as measured on its own scale. Then the wave of frequency 2 X would have a crest value of +1 at the time the wave of frequency X passed through zero. These two waves would then combine to give a resultant wave, as shown in Figure 2. This resultant wave is seen to be of different shape entirely from the resultant of Figure 1.


FIGURE I. RESULTANT COMPLEX WAVE AND TWO COMPONENT SINE WAVES WHICH ARE IN PHASE AT "O" TIME

From these illustrations it can be seen that an irregular wave may be produced by adding pure sine aves, that the shape of the irregular wave is influenced by the amplitudes of the component sire waves, and that the shape of the resultant wave is also influenced by the relative time (phase) displacement of the component sine waves, and their relative frequencies


FIGURE 2. ANOTHER COMPLEX WAVE AND THE SAME 2 SINUSOIDAL COMPONENTS. HERE THE $2 X$ COMPONENT IS LEADING BY ITS HALF CYCLE THE $X$ COMPONENT.

This leads to the statement that complex waves (or sounds) may be produced by a combination of sine waves (or simple sounds). Also, because the resultant wave shape is a function of so many variables, (number of sine waves, frequency of each, amplitude of
each, and phase relations) it is evident that the number of complex sounds is vely great in comparison to the number of simple sounds.

Waves may be periodic and nonperiodic.
When succeeding waves in a wave train 111 have the same shape and magnitude we call them periodic; w] on this is not the case, we call them nonperiodic. The simples periodic wave form is a sine wave, sometimes called a pure wave.

Periodic waveforms found in many transmission applications are usually irregular. By Fourier - series analysis we may analyze any periodic waveform into a number of individual sinusoidal components whose sum is identical to the original waveform. The frequencies of these component waves will be integral multiples of the number of times per second $f$ the original nonsinusoidal wave recurs. Such integral multiples are called 'harmonics." Frequency $f$ is called the fundamental, frequency $2 f$ the second harmonic, $3 f$ the third harmonic et c. By means of a laboratory instrument, called wave analyzer, these harmonics can be identified and measured in the laboratory.

The nonperiodic (non recurrent) waveforms can be analyzed into a band of frequencies in such a manner that a continuo curve may be plotted of relative amplitude of sinusoidal componeris versus their frequency. One of the methods which can be used for this is called the Fourier integral.

Suppose now that a given complex periodic wave is to be transmitted through some medium. The complex wave may be difficult to measure and therefore the quality of transmission through the medium may be doubtful. However, if the complex wave can be separated into its sine wave components, and such component sine waves are relatively easy to produce, and the quality of transmission through the medium can be readily measured or evaluated for these components, it is possible to say whether or not the medium will be satisfactory for the complex wave because of its effect on each of the component waves.

## 1. 34 VECTOR REPRESENTATION OF SINE WAVES

It may be well, at this time, to discass a concept which does not find much application in the study of sounc wat which is almost ef ential to an analysis of sine waves and elecoraco energy in reneral. This is the notion of vector representation of a sine wave. A.var has magnitude and direction. .... elation to sine wa sis developed in the following paragraphs.

First, a sine wave may be actually plotted by the method shown in Figure 3 where the horizontal lines are drawn through points $A, B, C$, etc. and the vertical lines $A^{3}, B^{\prime}, C^{3}$, etc. are equally spaced and indicate angular degrees of rotation. The intersections of lines $A$ and $A^{2}, B$ and $B^{j}, C$ and $C^{9}$, et $c$. indicate points on the sine curve. In this figure the horizontal acale represents time and the vertical scale eepresents instantaneous values of amplitude. The complete curve then shows the values of the amplitude for all instants. In is convenient and customary to divide the time scale into units of "degrees" wher then seconds, considering one complete cycle as being completed always in 360 degrees or units of time (regardless of the actual time taken in seconds). The reason for this convention is obvious from the method of constructing the sine curve since we take points around the circumference of the circle through 360 angular degrees. It needs to be kept in mind that in the sense now used, the "degæee" is a measure of time in terms of the frequency, and not of an angle.


FIGURE 3. VECTOR REPRESENTATION OF SINE WAVE

Having adopted this convention, it is not necessary to draw the complete sine curve figure whenever we wish to represent the amplitude of the curve at a particular instant, for example, that amplitude at the instant $t_{1}$, represented by the point $P_{c}$. If we know the frequency, and the length and the position of the single radius $R$ corresponding to the point $P_{C}$, we have all the information we need to define the curve. Here we have what we call a vector, which we can imagine as a radius of the circle, having a length equal to the peak value of the sine wave in question. The angle this vector makes
with the horizontal gives the position of point $P_{c}$. If we as sume a direction of rotation for the vector, (the accepted convention is counterclockwise) the instantaneous value of the curve at that time is measured by the vertical distance $P_{c} C^{\prime}$ which is equal to the length $R_{\text {max }}$ of the vector times the sine of the angle $X$ which it makes with the horizontal. Furthermore, we can determine by the position of the vector whether the value of th, curve is increasing or decreasing, and its direction.

Suppose now it is desired to study two sine waves, buch as electrical current and voltage or electromotive force, whose peak points are out of step, or out of phase, because the maximum value of current I will not have been established until some time after the emf E has reached its peak value. Figure 4 represents the relation of voltage and current that are out of phase due to the circuit having inductance. Here the vectorial representation must show the extent to which the voltage and current are out of phase. This is accomplished by having the voltage vector ahead of the current vector


FIGURE 4. SINE WAVES OUT OF PHASE
in its rotation (in the conventional counterclockwise direction) by an angle $\Theta$ which is a measure of the time by wion the current "lags" behind the voltage, and whose value is obvic ... rom the relative pesitions of the radii of the two circles.

## 1. 35 RESONANCE

Sounds produced by most musical instruments, as well as some other sources of sound, are caused by two separate actions. The vibrating element produces an original sound wave of a nature depending on its physical characteristics and on the method of causing it to vibrate. This original wave is further modified by the acoustic selectivity of the resonant parts of the instrument, which may strengthen some frequencies, and weaken others or suppress them almost entirely. The two parts of the instrument are called the "generator" and the "resonator" respectively. The resonator cannot add any sound which the generator does not supply, but it can exercise great selectivity by boosting or suppression. The resonating part of the instrument has its own natural frequency regions of vibration, and it amplifies those frequencies produced by the generator which fall in these regions. Since. however, a resonator may be put into "forced" vibration by frequencies outside its resonant ranges, those frequencies are not necessarily absent from the resulting sound, but they will be suppressed to some extent in relation to the resonant frequencies. The strings of a violin, for example, supply the original vibrations and some of their harmonics which are selected and strengthenef or subdued by the wooden body of the instrument. The resonator of any instrument is very important in determining the quality of the sound produced. Some famous violins made by masters from Cremona (Italy) such as Stradivari, Amati or Guarneri cost up to $\$ 60,000$. The excellency of the resonating characteristics of the wooden body is the main reason of fame and price. Other forms of wave motion, for example, electrical energy, exhibit this phenomenon of resonance when they pass through components which "select" or "amplify" certain frequencies.

## 1. 4 SPEECH



Fig. 5 - Resonant cavities provided by the throat, mouth and nasal cavity.

Speech sounds (see Fig. 5) are produced by the combined action of the lungs, the vocal cords, the throat, and the mouth and nose passages. The lungs supply the power, the vocal cords in the majority of sounds are the generators, and the throat, nose, mouth passages, head and chest bones function as resonators. The vocal cords, two in number, are under control of muscles, and hence may vary the pitch of the sound emitted. The sound produced by the vocal cords is very ric. in harmonics, and is greatly modified by the resonant action of the mouth and nasal passages; these air passages are also subject to great control and by their shape produce the typical sounds which, merged rapidly together, constitute speech. control of the air passages is produced by the action of the soft the spening through the nasal passages; the orgue, which regulates
: shape and size of the mouth cavity, a" mesition of the luer jaw ind 12 s which provide articulation alring with the tongue. Speech sounds can be also produced by passage of air through small openings
between teeth, for example "s", "sh" and "ts." Some sounds do not make use of the vibration of the vocal cords, but depend on the vibration of the air column in the mouth, in a manner similar to that of an organ pipe.

The frequency characteristics of speech sound are of particular importance in telephony, in order that the circuits used may transmit such frequency ranges as are necessary for the identification of the sounds. Although the fundamental pitch of vowel sounds is subject to rather wide variation by changes of the vocal cords, the sounds are characterized by resonent regions which are somewhat independent of the fundamental, and which determine the particular vowel being sounded. The characteristic frequency for sound " u " is about 350 cycles per second, and progressively increases for other vowels to the region $900-1100 \mathrm{cps}$ for sound a. The vowels "a" through "e", "i", and "e"' (long e) exhibit two resonance regions. In passing from "a" to "e"', the lower resonance goes to lower frequencies, while the upper resonance moves to higher frequencies. The sound è has two resonances at about 350 and 2800 cps . The characteristic frequencies of semi-vowels (1, r) and nasal consonants ( $\mathrm{m}, \mathrm{n}$ ) are principally below 3000 cycles, being in this respect in the class with the vowels. The other consonants have their important frequencies in the upper ranges. The wave forms of various sounds are shown in Figure 6.

The fundamental frequencies of the voice in singing cover an approximate range of 60 for the lowest note of a bass voice to 1300 cps for a high note of a soprano voice, while the overtones go as high as 10,000 cycles.

The pitch of voice fundamentals in speaking the vowels varies for different individuals, it is as low as about 90 cps for a deepvoiced man, and as high as about 300 cps for a shrill-voiced woman. The harmonics, or overtones, go as high as about $6,000 \mathrm{cps}$.

It is of interest to note that the power of speech is very small. The average speech power, including the short silent intervals, is about 10 microwatts; for a weak voice without whispering it is about 0.1 microwatt, and for very soft whisper 0.001 microwatt. The vowels are the most powerful speech sounds, giving an average of about 100 microwatts, with peak values of about 2000 microwatts. Those speakers having loud voices may reach peak values considerabiy higher. The range for the loudest sounds likely to be encountered in conversation to the weakest sounds is about 50 or 60 db . For any one individual, however, the range is usually not greater than about 35 to 70 db .
-


FIGURE 6. VARIOUS SOUND WIVEFORMS

## 1. 5 MUSIC

Music differs mainly from other combinations of sound by being sustained at definite pitches for comparatively long periods and by having the changes in pitch take place in definite steps known as musical intervals. Music is produced by three main classes of instruments besides the voice: vibrating strings, vibrating air column instruments (winds and brasses) and percussion instruments, whose sounds are often less definitely musical than those of the other two classes.

The range of the fundamentals used in instrumental music is wider than for the singing voice, extending beyond the voice range both in the high and low frequencies.

The frequency ranges for music, speart and noise are shown in Fig. 7.


Fig. 7 - Fundamental Frequency Ranges for Music, Speech and Noise.

The musical range is from about 16 to over $4: 600 \mathrm{cps}$ (fundamentals), a range encompassed by some pipe organs. Among the brasses, the tuba goes as low as 32 cycles, while the woodwinds reach to the upper ranges used in music, the flute having its highest fundamental at 4140 cps . Percussion instruments are confined to a relatively natrow range in the lower register, approximately from 85 to 200 cps. These frequencies are only the fundamentals; musical instruments are, for the most part, very rich in harmonics, which go to much higher values than the fundamental frequencies at least to $15,000 \mathrm{cps}$. For some instruments, the harmonics are higher in energy than the fundamental. For example, the second, third and fourth harmonics of the cello organ pipe are all at higher energy level than the fundamental.

In addition to the wide frequency range of music, it has great variaticins in intensity. An orchestra may show variations of power as high as 100,000 to 1 ( 50 db change); in this respect, rendering the proper transmission of such music is rather difficult.

For two reasons, the transmission of $m$ sic demands a much wider band of frequencies than does the voice. in the first place a good transmission of music requires inherentl: a much wider band, whereas in speech a narrower band is satisfactory, as was mentioned before (see Fig. 7). Secondly, in music naturalness is of great importance whereas in speech transmission naturalness is not as important but intelligibility is the prime consideration. This requires lower and upper limits of about 60 to 10,000 cycles (sometimes more) for the best transmission of music and only about 250 to 3,500 cycles for good transmission of speech. Problems of intelligibility and naturalness will be discussed later.

## 1. 6 HEARING

The ear consists (Fig. 8) basically of three parts:
The Outer Ear. The external portion, for diverting sound energy into the ear, and the auditory canal, for conducting this energy into the eardrum, comprise this part of the ear.

The Middle Ear. The hammer is attached to the eardrum T and communicates to the anvil motions imparted by the sound waves that strike the eardrum. These motions are then transferred by the anvil to the stirrup and through the foot plate to the oval window $O$, and thus to the inner ear.

The Inner Ear. There are three principal parts to the inner ear: namely, the semicircular canals $B$, which serve only in maintaining the person ${ }^{\text {s }}$ s equilibrium; the vestibule or space just behind the oval window; and the cochlea $S$, which serves as a terminating system for the mechanical sound vibrations, and in which these vibrations are converted. into nerve impulses.


Fig. 8 - Section Through The Human Ear. (From Fletcher ${ }^{3}$ s Speech and Hearing in Communication, Copyright 1953, D. Van Nostrand Co., Inc., Princeton, N.J.)

The cochlea is filled with fluid and is divided lengthwise into three parts by the basilar membrane and the membrane of Reissner. There are thus three parallel spiral canals, a cross section of which is shown in Fig. 9. The scala vestibuli $V_{t}$ terminates at one end on the oval window $O$, and the scala tympani $P_{t}$ terminates on the round window r. These two canals are connected at the extreme right inner end of the spiral.

The membrane of Reissner (Fig. 9) is very thin, and any impul ses transmitted to the fluid in the scala vestibuli by the foot of the stirrup at the oval window are readily transmitted through this membrane to the fluid of the canal of cochlea. The flexible basilar membrane extends toward the upper end of the cochlea, dividing it as shown. An impulse transmitted to the fluid of the scala vestibuli will readily pass to the canal of cochlea and will set the basilar membrane in motion.

The organ of Corti containing the nerve terminals in the form of small hairs extending into the canal of cochlea is along one side of the basilar membrane. Lying over these small hairs is a soft, loose membrane called the tectorial membrane.

The process of hearing is as follows. £ a low-frequency note, below about 20 cycles per second, impir, es on the eardrum, this sound variation is transferred as a mect nical impulse to the fluid of the scala vestibuli $\mathrm{V}_{\mathrm{t}}$. This fluid is in direct contact (at the far end) with that of the scala tympani $P_{t}$, as previously mentioned. At this low frequency, the liquid offers little reactance, owing to its mass, and therefore the liquids in the two canals move bodily back and forth. The basilar membrane is not affected, and accordingly no sound sensation is produced.

If a 1000-cycle tone is impressed on the ear, however, the mass reactance of the fluid is great enough so that the liquid does not move back and forth, and the impulse is transmitted through the membrane of Reissner to the canal of cochlea. The basilar membrane is caused to vibrate, and at some one spot the vibration will be greatest. The relative motion between the tectorial membrane and the basilar membrane then causes the small hairs to stimulate the nerve endings at their base, and this sends a sound sensation to the brain.


Fig. 9-Cochlea in Transve se Section. (From Fletcher ${ }^{\text {s }}$ Speech and Hearing in Communication, Copyright 1953, D. Van Nostrand Co., Inc., Princeton, N. J.)

If a tone of a different frequency than 1000 cps is used, a different part of the basilar membrane will vibrate with greatest amplitude, and thus different nerve endings will respond. When the frequency reaches and exceeds about 20,000 cycles per second, the hammer, anvil, stirrup, and associated parts absorb most of the sound energy, and little is transmitted to the inner ear; thus, the upper limit of audibility results. It can be shown that these three small bones together with the membranes on which they terminate act as a transformer to match the low impedance of the air to the high impedance of the fluid of the inner ear.

Hearing, the perception by our ears of sound waves, may be more easily studied by first considering the effects of simple sounds. It has been seen that any such simple sound is defined if its frequency and its amplitude are known, or in terms of sense perception, its pitch and its loudness.

Consider a tone of some frequency, say 1000 cps , and at a very low amplitude. This sound will not be perceived by the ear. As the amplitude is gradually increased, however, a point will be reached at which the sound is just perceptible. The amplitude at this point marks the so-called "Threshold of Audibility" for the 1000 cps tone. As the amplitude is further increased, the sound becomes louder and louder until the feeling of sound becomes uncomfortable. This level is called "Threshold of Feeling." A further increase of amplitude by some 20 db causes pain and this level is called "Threshold of Fain" $t$ " particular tone. Other frequencies will give like resule ..... in the amplitude values of the two thresholds vary with the $\mathrm{f}_{2}$. $u$ uency.

Fig. 10 shows the results of such tests for various frequencies.
The upper limit curve is actually the threshold of feeling above which tones become uncomfortable or even painful. The sound is audible if the frequency and intensity values lie within the area between the highest and the lowest curves. This area is known as auditory sensation area. We can see that for adequate hearing of speech this area is much smaller than for music.

As can be seen from this graph the important hearing range for the average normal ear lies between 30 cps and $20,000 \mathrm{cps}$, while the effective pressure varies at $1 \mathrm{kc}=1000 \mathrm{cps}$ from about 0.0002 dynes/sq cm to some 2, 000 dynes/sq cm, if threshold of pain is assumed as 140 db , or to 200 dynes/sq cm for the threshold of feeling assumed as 120 db .


The following formula is used for calculations of sound intensity for pressure in decibels, when the sound pressure level $P_{X}$ is given in regular units:

Sound pressure level db $=20 \log _{10} \frac{P_{X}}{\text { reference level }}$
The reference level frequently used here is 0.0002 dynes / sq cm equal to 0.0002 microbar equivalent in turn to $10^{-16}$ watts $/ \mathrm{sq} \mathrm{cm}$. Reference level and $P_{x}$ must be expressed in the same units.

## Example

At $1 \mathrm{kc} / \mathrm{s}$ the threshold of pain is 140 db . What is the respective sound intensity $\mathrm{P}_{\mathrm{x}}$ ?
$140 \mathrm{db}=20 \log _{10} \frac{P_{\mathrm{X}}}{0.0002 \text { dynes } / \text { sq cm }}=20 \log \frac{10^{4} P_{\mathrm{X}}}{2}$
$10^{7}=\frac{10^{4} P_{X}}{2} \quad \therefore P_{X}=10^{3} \times 2=2,000$ dynes $/ \mathrm{sq} \mathrm{cm}$

## The Loudness of Sounds

The magnitude of sensation produced in the brain is termed the loudness of a sound. Although the loudness of a sound is related to the intensity, the two are not the same. A sound that is loud for one person may not be loud for another. Also, two different sounds which produce equal intensities at the ear may not sound equally loud to the second observer.

These phenomena are illustrated by the curves of variations in the loudness-frequency response of the human ear in Fig. 11, the so called Fletcher-Munson curves of equal loudness. These curves are the result of statistics based on measurements performed on


Fig. 11 - Variations in the Frequency Response of the Human Ear with Sound Intensity.
many thousands of individuals under many different conditions. They are the chayacteristics of the average human ear and show its sensitiveness to sounds of different frequencies and intensities. Thus, at 100 cps , a sound must be about 36 db above the reference intensity value at 1000 cps to be just audible, but, at $10,000 \mathrm{cps}$, it needs to be only 8 db above the reference value to be audible.

Studies of the ear have shown that the increments of either energy or frequency that are necessary for the perception of an equal difference in either loudness or pitch are not constant but increase logarithmically when higher values of energy and of frequency are applied. The ear approximately follows the Weber-Fechner law of psychology, which states that sense perception varies as the

CH. 1-CHARACTERISTICS OF SOUND AND WAVE MOTION
logarithm of the stimulus. If the intensity level of the sound is changed so that it is about the smallest change that can be detected by ear we call this change of intensity one decibel. The fact that equal perceptible changes in frequency are logarithmic is illustrated by the musical scale. The octaves axe sensed as equal frequency changes although the note of one octave is twice the frequency of the corresponding note of the next lower octave, ad four times the frequency of the corresponding note two octaves lower.

## 1. 7 SOME ASPECTS OF SOUND TRANSMISSION

Having analyzed to a certain extent the sound as energy with reference to its characteristics, generation and reception, the transmission of sound can now be considered as a means of passing information from point to point. Several aspects of such transmission are discussed below.

## 1. 71 SPEECH ENERGY AND ARTICULATION

As previously stated, most of the power of speech is to be found in the vowels, which are ordinarily in the lower frequency range. Their average energy is 100 microwatts ( 2,000 peak) and the frequency ra ge of fundamentals is from 90 to 300 cps . The consonants, on the other hand, while carrying less energy, are very important to articulation. "Articulation" is the term used to designate the correctness with which speech sounds are perceived over a transmission system. In the "Syllable Articulation Test" lists of discrete monosyllables, usually without meaning, are read into the transmission system, and are recorded by listeners at the receiving end. The percentage of syllables correctly recorded is called the articulation of the circuit. Tests of this sort are more severe than is conversation test, since the lists of monosyllables have no connection, and are themselves meaningless, so that the recognizing of a sound by the context, as is possible in conversation, is impossible in the syllable tests. This accounts for the fact that a circuit may have an articulation rating of only 50 or 60 per cent and still afford satisfactory conversation.


FIGURE I2. FREQUENCY CHARACTERISTICS OF SPEECH
"Intelligibility" test measures the comparative perfection in the reception of sounds conveying ideas.
'Naturalness'". It is the correct reproduction at the receiving end of the relationships between the fundamental and its harmonics existing in a sound at the transmitting end.

Curves showing the importance of the various speech frequencies for intelligibility and articulation are shown on Figure 12. These general effects of speech indicate what results may be expected from a suppression of various frequency ranges. If the lower frequencies are suppressed by means of high-pass filters, the energy will be considerably reduced, causing a loss of naturalness, while the articulation will be only slightly affected. Suppression of the higher frequencies will considerably reduce the articulation and intelligibility
but the energy will be little reduced. It is evident, therefore, that both the low and the high ranges are necessary; the lower for naturalness and the higher for articulation and intelligibility.

From what has been stated above, consideration may hereafter be given to the requirements for satisfactory telephone transmission in texms of the received acoustic energy. The general adequacy of the received sounds is dependent on three factors, as follows:

1. Loudness
2. Distortion
3. Extraneous interference

These three charateristics of a transmission system affect both the intelligibility of the received sounds and their naturalness.

## 1. 72 MASKING

It is a common experience that when any sound is impressed upon the ear it reduces the ability of the ear to sense other sounds.

An experiment can be set up so that a sound A called "maskee tone" is impressed on the ear at a constant intensity. At the same time another sound B, called "masker tone" is also impressed on the ear but its intensity is increased from zero level until sound A can no longer be heard. The sound A is said to be then masked by the sound B.

It was observed at first that low-pitched sounds had a masking effect different from high-pitched sounds, namely that, in general, a tone of low pitch would completely mask one of higher pitch, but that a tone of high pitch would not mask a tone of lower pitch. These phenomena were observed by A. A. Mayer and were further investigated by the Bell Telephone Laboratories. Their experiments were conducted as follows.

The masker tone B was kept at a constant level while the maskee tone A of other pitch was gradually increased in intensity from zero level (when it was completely masked) until it was just perceptible in the presence of the masker tone $B$.
A. A. Mayer's conclusions were found correct but only under certain circumstances. The B.T. L. found that:

1. A low tone will not obliterate to any degree a high tone far removed in frequency, except when the low tone is raised to a very high intensity.
2. A tone of higher frequency can easily obliterate a tone of lower frequency if the two frequencies are near to each other.

The interfering effect of a masker tone (or complex sound) is also somewhat greater if it is interrupted than if it is continuous. This is doubtless due to the fact the beginning of an interfering sound engages the interest because of a desire to identify it, while the latter stages of a continuous interfering sound claim a smaller share of attention. With an inter rupted tone, this focusing of attention on the interference occurs periodically at the beginning of each spurt of sound, with consequent detriment to the sounds or conversation which it is desired to hear.

Because of its non-linear characteristic the ear acts similarly to a modulating vacuum tube, and supplies the harmonics, the difference frequencies, and the summation frequencies of any two frequencies contained in the impressed sound wave. This makes it possible for the ear to supply the fundamental of a sound, of which the overtones are present, even though the fundamental is suppressed in some way. The difference frequency of the second and third harmonic would be the fundamental, as would also be the difference frequency of the third and fourth harmonics, and so on. As an example, a voice singing "ah" at a fundamental frequency of 145 cps had the range from 0 to 1250 cps suppressed by a filter. This change caused no change in the pitch, although the naturalness and intelligibility were entirely changed. Such supplying of frequencies lost by suppression is probably of material advantage to the adequacy of transmission over systems in which distortion occurs. On the other hand, the non-linear characteristic of the ear imposes a limit on the increase in energy output of a transmission system; such limit is considerably below that set by the threshold of feeling. Thus ear starts distorting sounds of higher intensity, below the 120 db threshold of feeling.

## 1. 73 NOISE

Noise may be defined, broadly, as unwanted sound. Some noises are entirely sounds or their harmonics, different from, while in the case of other noises, the difference is slight.

When speech is transmitted, either directly or over an electrical system, there is always an interference to the proper reception of speech because of the presence of other sounds. It may be caused by induced effects between telephone lines and power lines.

The main cause is powerful harmonics of 60 cps , some of which may have energies of the order of 10,000 watts. Such levels should be compared with the power of speech currents in the telephone lines which may be as low as 10 microwatts. The ratio of the 2 powers is of the order of 90 decibels. One of the ways to reduce such interference is to avoid running telephone lines pa rallel and in a close distance to power lines.

Another way is to have the power supply company either rearrange connections of their power transformers or insert so called "harmonic filters" in their power lines which would divert the most troublesome harmonics from the sections adjacent to the disturbed telephone lines.

Another main cause of noises is the disturbances originating within the telephone plant such as key clicks or crosswtalk between adjacent circuits. Still another main cause is ambient noises at the transmitting or receiving location.

In any case, the presence of noise tends to reduce the ability of the ear to detect the signal. In effect, the threshold of hearing is raised by an amount which depends on both the volume and the frequency of the components of the interference. In other words more signal energy is needed than the minimum at that frequency to convey the intelligence.

Noises are usually classified as of 2 types:

1. Random
2. Impulse

Random noise is the common type which occurs for instance in ventilating systems, jets, blowers, combustion chambers etc. It is also present in communication circuits in form of "electrical noise" due to cross-talk, induction voltages from power lines and similar sources. This type of noise does not have a well defined pitch and has energy distributed non-uniformly over a band of frequencies. Sometimes random noise is called room noise, background noise or thermal noise.

One of the types of random noise is the so called "white noise." In this type of noise the energy of noise per cycle is distributed uniformly over a wide band of frequencies. White noise is sometimes injected to improve privacy by notifying the speaker that somebody is eavesdropping.

CH. 1-CHARACTERISTICS OF SOUND AND WAVE MOTION

When the broadband noise has only little energy at low frequencies it sounds more like a hissing sound. When most energy is concentrated in a narrow band of low frequencies, the noise sounds like a "roar".

Impulse (or impact) noise. Typical examples are clicks or statics. This type of noise is usually ten times stronger than the white noise.

To obtain satisfactory transmission quality signal power should be higher by 10 db than the noise for data transmission and by 20 db for speech.

The disturbing effect of noise to a listener depends first upon its volume. It also depends upon the frequency of noise currents. Disturbing effect peaks rather sharply around 1100 cps .

In telephone practice the noise interference problem may be attacked in two ways. Either the volume of the signals must be raised to compensate for the shift in the hearing threshold, or attention must be directed toward eliminating the noise at its source or suppressing it at the point where it enters the telephone plant. The solution is often an economic compromise between the two methods.

## 1. 8 FREQUENCY BANDS FOR OTHER SERVICES

The preceding discussion has been directed principally toward establishing the wire trans ission requirements for normal speech telephone service. In ack. a, the wire facilities may be used for the transmission of ringing, signaling and telegraph impulses below 250 cycles per second; special services such as radio program circuits may require an extension of the upper frequency limit to 10,000 or 15,000 cycles per second; and with sufficient amplification the same facilities may be used in future for carrier circuits requiring an upper frequency of around 260 kilocycles per second. In addition to the extension of the frequency ranges required for such services, it is frequently necessary to handle greater volume ranges than those encountered in normal speech, and a corresponding improvement in noise conditions is often required.

## 1. 9 UNITS USED IN TRANSMISSION ENGINEERING

1. 91 DECIBEL is used in electrical and acoustic work. It is derived from the formerly used unit "bel," which is defined as:

$$
\begin{array}{r}
\text { bel }=\log _{10} \frac{P_{2}}{P_{1}} \quad \text { where } P_{1} \text { is the input power } \\
\text { and } P_{2} \text { the output power }
\end{array}
$$

This unit was found to be too large and was replaced by another unit decibel ( db ) which is 10 times smaller and may be calculated from the formula:

$$
\mathrm{db}=10 \log _{10} \frac{\mathrm{P}_{2}}{\mathrm{P}_{1}}
$$

It is very important to keep in mind that decibel expresses the ratio of powers expressed on a logarithmic scale and not the power levels. Following examples illustrate the use of db.

## Example 1

The input power going into a device is 50 mic rowatts and the output power is 100 mic rowatts. What is the gain, or loss of the device?

$$
\begin{aligned}
\mathrm{db}=10 \log \frac{100 \text { microwatts }}{50 \text { microwatts }} & =10 \log 2=10 \times 0.3= \\
& = \pm 3 \text { decibels }
\end{aligned}
$$

Since the answer is positive, we have a gain, if the input and output powers were interchanged, we would have $10 \log \frac{1}{2}$ and the answer would have been -3 db , or a loss. In the first case we had an amplifying device in the second an attenuator.

## Example 2

a. Calculate "system gain" for the system shown in the following block diagram Fig. 13. Its components are explained by notes.


FIGURE 13.

An input power $P_{1}=10$ microwatt is injected and an output power $P_{2}=10$ watt is obtained. Thus the system gain is

$$
\mathrm{db}=10 \log \frac{10 \mathrm{watt}}{10 \times 10^{-6} \mathrm{watt}}=10 \log 10^{6}=+60 \mathrm{db}
$$

b. Calculate gain X of the I -amplifier $A$ to produce system gain of 60 db . We can write an expression in which the system gain is equated to the algebraic sum of losses and gains of all system components.
$60 \mathrm{db}=-3 \mathrm{db}+\mathrm{X}-10 \mathrm{db}+60 \mathrm{db}$ or

$$
X=+13 \mathrm{db}=\text { the gain of preamplifier } \mathrm{A} .
$$

Many errors in calculations of db occur from improper use of voltage or current ratios in the formulas for calculation of db's. These formulas are as follows:

$$
\begin{aligned}
\mathrm{db} & =20 \log _{10} \frac{\mathrm{~V}_{2}}{\mathrm{~V}_{1}} \text { for voltages, and } \\
\mathrm{db} & =20 \log _{10} \frac{\mathrm{I}_{2}}{\mathrm{I}_{1}} \text { for currents. }
\end{aligned}
$$

Both formulas can be used only when the input and output impedances $R_{1}$ and $R_{2}$ of the device are equal. If they are not equal following correct formulas must be used:

$$
10 \log _{10} \frac{\mathrm{P}_{2}}{\mathrm{P}_{1}}=10 \log \frac{\mathrm{R}_{2}}{\mathrm{R}_{1}} \frac{\left(\mathrm{I}_{2}\right)^{2}}{\left(\mathrm{I}_{1}\right)^{2}}=20 \log _{10} \frac{\mathrm{I}_{2}}{\mathrm{I}_{1}}+10 \log _{10} \frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}
$$

or

$$
10 \log _{10} \frac{\mathrm{P}_{2}}{\mathrm{P}_{1}}=10 \log \frac{\mathrm{R}_{1}}{\mathrm{R}_{2}} \frac{\left(\mathrm{~V}_{2}\right)^{2}}{\left(\mathrm{~V}_{1}\right)^{2}}=20 \log \frac{\mathrm{~V}_{2}}{\mathrm{~V}_{1}}+10 \log \frac{\mathrm{R}_{1}}{\mathrm{R}_{2}}
$$

## 1. 92 dbm

These units express power levels relative to one milliwatt
level. Thus $\mathrm{O} \mathrm{dbm}=1$ mwatt. dbm are calculated from the formula

$$
\mathrm{dbm}=10 \log \frac{\mathrm{P}_{\mathrm{x}}}{1 \mathrm{~m} \text { watt }}
$$

## Example 1

where $P_{X}$ is the power level which is to be expressed in dbm.

What is the power level of 10 microwatt expressed in dbm?

$$
\mathrm{dbm}=10 \log \frac{10 \mathrm{w} \times 10^{-6}}{1 \mathrm{w} \times 10^{-3}}=10 \log 10^{-2}=-20 \mathrm{dbm}
$$

## Example 2

The part of a transmission system is given as shown on the sketch below.

Find dbm levels on the input and output side as well as the db gain.

a. Level on the input side

$$
\begin{aligned}
\mathrm{dbm} & =10 \log \frac{(0.5 \mathrm{v})^{2}}{6000 \times 10^{-3} \mathrm{w}}=10 \log \frac{0.25 \times 10^{3}}{6 \times 10^{2}}= \\
& =-10 \log \frac{6}{2.5}=-3.8 \mathrm{dbm}
\end{aligned}
$$

b. level on the output side

$$
\mathrm{dbm} \quad 10 \log \frac{(94.8 \mathrm{v})^{2}}{900 \Omega \times 10^{-3} \mathrm{w}}=+40 \mathrm{dbm}
$$

c. gain in db (and not in dbm)

$$
10 \log _{10} \frac{(94.8 \mathrm{v})^{2}}{900 \Omega} \times \frac{600 \Omega}{(0.5 \mathrm{v})^{2}}=43.8 \mathrm{db}
$$

This gain may be calculated also from the difference of dbm levels between the output and the input:

$$
\text { gain }=[40 \mathrm{dbm}-(-3.8 \mathrm{dbm})]=43.8 \mathrm{db}
$$

1.93 dbw

These units express power levels relative to one watt level, and the standard load is 600 ohms.

## Example

What is the power level of 20 watts expressed in dbw?

1. 94 dbv

$$
\mathrm{dbw}=10 \log \frac{20 \mathrm{w}}{1 \mathrm{w}}=10 \times 1.3=13 \mathrm{dbw}
$$

These units are used frequently for general measurement of video signal strength. dbv's express the voltage level relative to 1 volt. Both peak-to-peak or RMS voltages may be used in various contexts. Therefore, the reference level must be used accordingly as 1 volt peak-to-peak or 1 volt RMS.

The following formula shall be used.

$$
\mathrm{dbv}=20 \log _{10} \frac{\mathrm{~V}}{1 \text { volt }}
$$

1.95 VU

These units were standardized by the industry about 1940.
Speech and music ${ }^{4}$ waves vary with time in a complex manner and it is not possible to measure their precise values in terms of watts or decibels, except on an instantaneous basis. 2. . $z$ frequently desirable, however, to know the overall average strength of transmitted speech or music. A device known as volume indicator may be used for this purpose. This is essentially a high impedance voltmeter made up of an attenuator, a copper-oxide rectifier, and a d-c milliammeter having specified dynamic characteristics. The meter, which may be bridged across a line without appreciable effect on the transmission line characteristics, is calibrated in terms of VU.

VU is a logarithmic unit that measures strength or volume above or below a specified reference level. In general, this reference level, designated O VU, indicates no precise electrical quantity, but the volume indicator is calibrated to read O VU on 1 milliwatt of l, 000 cycle sine-wave power dissipated in a 600 ohm resistance.

The VU represents the same power ratio as the $d b$ and the volume indicator may therefore be used sometimes to measure transmission losses or gains in db when the current being measured is a 1,000 cycle sine-wave, although its basic purpose is to measure the volume of complex waves.

## 1. 95 DBRN

These units express in decibels the electrical noise power in the telephone circuits related to reference noise.

Reference noise level " $O$ " is the magnitude of current no: that will produce a circuit noise meter reading equal to that proc ed by $10^{-12}$ watt of electric power of a $1,000 \mathrm{cps}$ sine-wave.

## Example

Calculate the DBRN for a noise of 10 microwatt.

$$
\begin{aligned}
\text { DBRN }=10 \log _{10} \frac{10 \mathrm{w} \times 10^{-6}}{10^{-12} \mathrm{w}} & =10 \log _{10} \frac{10^{-5}}{10^{-12}}=10 \log 10^{7}= \\
& =70 \text { DBRN }
\end{aligned}
$$


#### Abstract

1. 97 dba

Noise is evaluated ${ }^{4}$ here in the same numerical way as for DBRN. However, different weighting networks may be used with differing receiving devices. In practice an adjusted unit dba (where "a" stands for adjusted) is used. It measures the acoustic interfering effect of the frequency-weighted energy. Weighting network used must be specified always. Equal values of dba measured across any receiving device, with proper weighting used, should indicate approximately equal interfering effects.


## Reference for Chapter 1

4. (C) American Telephone and Telegraph Company, 1961.

## NETWORKS

## 2. 1 ELECTRICAL NETWORKS

The transmission of any message to a remote point by electrical means involves three essential processes:
a. Converting the original message to the electrical signal which varies in a manner similar to the message.
b. Passing this signal through a series of connected electrical networks until the receiving point is reached.
c. Reconverting the electrical signal to the form of original message.

The original message may have many forms, the common characteristic being that some varying element is present. Thus, for sounds, variations in air pressure are involved; for pictures, variations in light intensity; for remote gauging, mechanical variations such as the height of a float. These variations in the original messages when translated into electrical signals, may have a wave form ranging from the simple to the highly complex. As previously shown, any repetitive (recurrent) electrical wave may be resolved using Fourrier Series techniques into a series of a single frequency waves each having the form of a sine wave. Similarly, any complex non repetitive voltage may be expressed using Fourrier-Integral techniques as a continuous band of frequencies and the response of an electrical network to this impressed voltage is a current which can also be expressed as a continuous band of frequencies. Furthermore, the ratio of the voltage to the current for any particular frequency component is given by the steady-state impedance of the network at that frequency. While the actual analysis is only rarely attempted, the knowledge that it is theoretically possible governs the whole philosophy of transmission analysis and leads to the classical method of attacking the problem by means of single frequency sine wave computations.

Obviously, no single frequency is completely representative of a changing complex electrical wave. However, for many purposes in the telephone transmission analysis, the performance of a circuit or piece of equipment over the voice range of frequencies may be roughly judged from a knowledge of its performance at a frequency of around 1000 cycles per second. This procedure has the merit of simplicity,
with the acco..panying disadvantage that it ignores the noise and the quality elements of the transmission process. Where comparisons are made between circuits and equipment having essentially the same frequency response and noise characteristics, the differences in volume at 1000 cycles per second are indicat ve of the relative merits of the various arrangements. The resu $s$ of computations at. 1000 cycle frequency are designated as "volur 'losses or gains to distingus? them from ratings including the effects of noise and distortion waich are called "effective" or "subjective" losses or gains. This chapter is largely devoted to volume considerations.

The actual transmission of the electrical wave from transmitter to receiver is accomplished by transferring energy from one electrical network to the next, until the terminal circuit is reached. Before any quantitative analysis of the telephone transmission process can be attempted, therefore, it is necessary to review some general principles of alternating current networks.

An electrical network is an assembly of resistors, inductors and capacitors and, in certain cases, control devices (controlled sources) such as transistors and electron tubes. The usual resistors and capacitors are linear (i. e., the current is directly proportional to the voltage) and bilateral (i.e., they are capable of transfering energy equally well in either direction). This is also true oi air-core inductors. Iron-core inductors, while bilateral, are not usually linear but may frequently be considered so over the limited ranges of current used in telephony. Control devices (controlled sources) are generally unilateral and often non linear and involve a local source of power which is controlled by the input signal.

In the analysis of the usual electrical networks making up communication circuits, Ohm ${ }^{\mathfrak{q}}$ s and Kirchoffis laws are the primary tools. By the aid of these, certain other principles have been derived which are of considerable assistance in minimizing the efforts required for retwork analysis. Although this material is familiar to some students, it is presented here for the sake of completeness.

## 2. $2 \mathrm{OHM}^{1} \mathrm{~S}$ LAW

The current I which will flow throigh an impedance $Z$ is equal to the voltage divided by the impedance, or $=\underline{E}$. Two examples of the solution are shown in Figures 14 and 15.


FIGURE 14. SIAPLE SERIES CIRCUIT CONTAINING AN IAPEDANCE $Z$.

$$
I=\frac{E}{Z}
$$



FIGURE 15. SIMPLE SERIES CIRCUIT CONTAINING TWO IMPEDANCES $z_{1}$ AND $z_{2}$

$$
I=\frac{E}{Z_{1}+Z_{2}}
$$

## 2. 3 KIRCHOFF'S LAWS

Kirchoff's Laws are a means of solving for unknown circuit parameters in electric circuits no matter how complex.

Law I: At any point in a circuit there is as much current flowing into the point as there is current flowing away from it; in other words the sum of currents in a point (a node) is zero.

Law II: In any closed electrical loop the algebraic (or vectorial) sum of the voltages generated by the sources and the potential drops across loop components is equal to zero, when going around the loop in one relected direction.

Applying Law I to figure 16, this equation can be written for point "a"

$$
I_{1}=I_{2}+I_{3}
$$

Law I for figure 16 will net the following three equations
For loop 1:

$$
\begin{equation*}
\mathrm{E}=\mathrm{I}_{1} \mathrm{Z}_{1}+\mathrm{I}_{3} \mathrm{Z}_{3} \tag{1}
\end{equation*}
$$

For loop consisting of $\mathrm{E}, \mathrm{Z}_{1} \& \mathrm{Z}_{2}$ :

$$
\begin{equation*}
F=I_{1} Z_{1}+I_{2} Z_{2} \tag{2}
\end{equation*}
$$

For loop 2:

$$
\begin{equation*}
I_{2} Z_{2}-I_{3}^{r}:=0 \tag{3}
\end{equation*}
$$

E, $Z_{1}, Z_{2}$ and $Z_{3}$ in these three equations are given and therefore they can be solved algebraically (or vectorially) to obtain the actual values for the three unknowns $I_{1}, I_{2}$ and $I_{3}$.


FIGURE 16. SIMPLE SERIES - PARALLEL CIRCUIT

The application of these laws to more complicated circuits involves setting up simultaneous linear equations for solution. This can be very laborious in a practical case, and several theorems, known as network theorems, have been developed to facilitate this process. Two important types of networks are called, from their configurations, the T and $\pi_{i}$ network. $\left(\pi^{i}\right.$ i. the sixteenth letter in the Greek alphabet; it is pronounced the same $\quad \checkmark$ as the English wo. "pie".)

## $\therefore$ - TUIVALENT NETWORKS

In any linear, bilateral, passive network at a single frequen y, a three element $T$ network can be interchanged with a three-element $\pi$ network, provided certain relations exist between the elements of these two structures.


Figure 17 shows a $T$ network on the left side and an equivalent $\Pi$ network on the right side. We will develop ${ }^{3}$ the relationships between $\mathrm{Z}_{1}, \mathrm{Z}_{2}$ or $\mathrm{Z}_{3}$ and $\mathrm{Z}_{\mathrm{A}}, \mathrm{Z}_{\mathrm{B}}$ and $\mathrm{Z}_{\mathrm{C}}$ and vice versa. This will permit to convert any $T$ into a $\pi$ network and vice versa.

The designations used are as follows:
$\mathrm{Z}_{1 \mathrm{sc}}-$ is the impedance looking inside into each network from terminals l-l with terminals 2-2 short circuited (strapped).
$\mathrm{Z}_{10 c}$ - is the impedance looking inside into each network from 1-1 with terminals 2-2 open.
$Z_{2 o c}$ - is the impedance looking inside into the networks from terminals 2-2 with terminals $1-1$ open.

Following equations can be written for the $T$ network.

$$
\begin{aligned}
& \qquad \begin{array}{l}
Z_{1 o c}=Z_{1}+Z_{3} \quad Z_{2 o c}=Z_{2}+Z_{3} \\
Z_{1 \mathrm{sc}}= \\
Z_{1}+\frac{Z_{2} Z_{3}}{Z_{2}+Z_{3}}
\end{array} \\
& \text { and for the f: network. }
\end{aligned}
$$

$$
\begin{gathered}
Z_{1 o c}=\frac{Z_{A}\left(Z_{B}+Z_{C}\right)}{Z_{A}+Z_{B}+Z_{C}}=\frac{Z_{A}\left(Z_{B}+Z_{C}\right)}{Z_{T}} \\
\text { where } \quad Z_{T}=Z_{A}+Z_{B}+Z_{C} \text { and } \\
Z_{2 o c}=\frac{Z_{C}\left(Z_{A}+Z_{B}\right)}{Z_{T}} \\
Z_{1 s c}=\frac{Z_{A} Z_{B}}{Z_{A}+Z_{B}}
\end{gathered}
$$

Since the two circuits have to be equivalent impedances for T and Pi with identical subscripts must be equal.
$\underline{\text { Part A. Find } \mathrm{Z}_{1} ; \mathrm{Z}_{2} \text { and } \mathrm{Z}_{3} \text { in terms of } \mathrm{Z}_{\mathrm{A}} ; \mathrm{Z}_{\mathrm{B}} \text { and } \mathrm{Z}_{\mathrm{C}} \text {. }}$

$$
\begin{array}{rlr}
\mathrm{Z}_{1}+\mathrm{Z}_{3} & =\frac{\mathrm{Z}_{\mathrm{A}}\left(\mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{C}}\right)}{\mathrm{Z}_{\mathrm{T}}} & \text { (1) from } \mathrm{Z}_{1 o c} \text { formula } \\
\mathrm{Z}_{2}+\mathrm{Z}_{3} & =\frac{\mathrm{Z}_{\mathrm{C}}\left(\mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{B}}\right)}{\mathrm{Z}_{\mathrm{T}}} & \text { (2) from } \mathrm{Z}_{2 \mathrm{oc}} \text { formula } \\
\mathrm{Z}_{1}+\frac{\mathrm{Z}_{2} \mathrm{Z}_{3}}{\mathrm{Z}_{2}+\mathrm{Z}_{3}} & =\frac{\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{B}}}{\mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{B}}} & \text { (3) from } \mathrm{Z}_{1 \mathrm{sc}} \text { formula }
\end{array}
$$

By subtracting (3) from (1) we obtain

$$
\mathrm{Z}_{3}-\frac{\mathrm{Z}_{2} \mathrm{Z}_{3}}{\mathrm{Z}_{2}+\mathrm{Z}_{3}}=\frac{\mathrm{Z}_{\mathrm{A}}\left(\mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{C}}\right)}{\mathrm{Z}_{\mathrm{T}}}-\frac{\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{B}}}{\mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{B}}}
$$

By taking the right side to common denominator and developing the bracketed terms:

$$
\frac{\mathrm{Z}_{3}^{2}}{\mathrm{Z}_{2}+\mathrm{Z}_{3}}=\frac{\mathrm{Z}_{\mathrm{A}}^{2} \mathrm{Z}_{\mathrm{C}}}{\mathrm{Z}_{\mathrm{T}}\left(\mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{D}}\right)} \cdot \text { Equation (2) is now }
$$

substituted in the denominator of the left side:

$$
\begin{align*}
\frac{\dot{Z}_{3}^{2} \mathrm{Z}_{\mathrm{T}}}{\mathrm{Z}_{\mathrm{C}}\left(\mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{B}}\right)} & =\frac{\mathrm{Z}_{\mathrm{A}}^{2} \mathrm{Z}_{\mathrm{C}}}{\mathrm{Z}_{\mathrm{T}}\left(\mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{B}}\right)} \\
\mathrm{Z}_{3}^{2} & =\frac{\left(\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{C}}\right)^{2}}{\left(\mathrm{Z}_{\mathrm{T}}\right)^{2}} \text { and } \mathrm{Z}_{3}=\frac{\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{C}}}{\mathrm{Z}_{\mathrm{T}}} \tag{4}
\end{align*}
$$

If we substitute formula (4) into equation (1), the following equation is obtained

$$
\begin{align*}
\mathrm{Z}_{1}+\frac{\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{C}}}{\mathrm{Z}_{\mathrm{T}}} & =\frac{\mathrm{Z}_{\mathrm{A}}\left(\mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{C}}\right)}{\mathrm{Z}_{\mathrm{T}}} \\
\mathrm{Z}_{1} & =\frac{\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{B}}}{\mathrm{Z}_{\mathrm{T}}} \tag{5}
\end{align*}
$$

If we substitute now (4) into equation (2) the following equation is obtained:

$$
\mathrm{Z}_{2}+\frac{\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{C}}}{\mathrm{Z}_{\mathrm{T}}}=\frac{\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{C}}+\mathrm{Z}_{\mathrm{B}} \mathrm{Z}_{\mathrm{C}}}{\mathrm{Z}_{\mathrm{T}}} \quad \text { and }
$$

simplifying it we obtain:

$$
\begin{equation*}
\mathrm{Z}_{2}=\frac{\mathrm{Z}_{\mathrm{B}} \mathrm{Z}_{\mathrm{C}}}{\mathrm{Z}_{\mathrm{T}}} \tag{6}
\end{equation*}
$$

Part B. Find $\mathrm{Z}_{\mathrm{A}} ; \mathrm{Z}_{\mathrm{B}}$ and $\mathrm{Z}_{\mathrm{C}}$ in terms of $\mathrm{Z}_{1} ; \mathrm{Z}_{2}$ and $\mathrm{Z}_{3}$.
Taking equation (3) to a common denominator, we obtain

$$
\begin{array}{r}
Z_{1}+\frac{Z_{2} Z_{\overline{3}}}{Z_{2}+Z_{3}}=\frac{z_{A} Z_{B}}{Z_{A}+Z_{B}}  \tag{3}\\
\frac{Z_{1}\left(Z_{2}+Z_{3}\right)+Z_{2} Z_{3}}{Z_{2}+Z_{3}}=\frac{Z_{A} Z_{B}}{Z_{A}+Z_{B}}
\end{array}
$$

and multiplying

This equation by equation (2), namely

$$
\mathrm{Z}_{2}+\mathrm{Z}_{3}=\frac{\mathrm{Z}_{\mathrm{C}}\left(\mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{B}}\right)}{\mathrm{Z}_{\mathrm{T}}} \quad \text { we obtain }
$$

$$
2.7
$$

$$
\begin{align*}
\mathrm{Z}_{1} \mathrm{Z}_{2}+\mathrm{Z}_{1} \mathrm{Z}_{3}+\mathrm{Z}_{2} \mathrm{Z}_{3} & =\frac{\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{B}}}{\mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{B}}} \times \frac{\mathrm{Z}_{\mathrm{C}}\left(\mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{B}}\right)}{\mathrm{Z}_{\mathrm{T}}}= \\
& =\frac{\left(\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{C}}\right)}{\left(\mathrm{Z}_{\mathrm{T}}\right)} \mathrm{Z}_{\mathrm{B}} \tag{7}
\end{align*}
$$

We will now use a new designation $Z_{123}$ samely: $Z_{1} Z_{2}+Z_{1} Z_{3}+Z_{2} Z_{3}=Z_{123}$. The term inside the brackets of (7) is equal $Z_{3}$. [See formula (4)].

Thus $Z_{123}=Z_{3} Z_{B}$, and therefore

$$
\mathrm{Z}_{\mathrm{B}}=\frac{\mathrm{Z}_{123}}{\mathrm{Z}_{3}}=\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}+\mathrm{Z}_{1} \mathrm{Z}_{3}+\mathrm{Z}_{2} \mathrm{Z}_{3}}{\mathrm{Z}_{3}}
$$

From (7) and (5) we obtain:

$$
Z_{C}=\frac{Z_{123}}{Z_{1}}=\frac{Z_{1} Z_{2}+Z_{1} Z_{3}+Z_{2} Z_{3}}{Z_{1}}
$$

and from (7) and (6) we obtain:

$$
\mathrm{Z}_{\mathrm{A}}=\frac{\mathrm{Z}_{123}}{\mathrm{Z}_{2}}=\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}+\mathrm{Z}_{1} \mathrm{Z}_{3}+\mathrm{Z}_{2} \mathrm{Z}_{3}}{\mathrm{Z}_{2}}
$$

All the necessary equations were thus obtained for converting any $T$ into a $\pi$ network or vice versa. Due to this any passive 3 terminal network, no matter how complex, can be converted into a simple T (or $\pi$ ) network.

For example, the use of $\pi$ and $T$ transformations shows how the circuit of Figure 18 may be simplified into the $T$ circuit of Figure 22. The sequency of sim ifications is shown in Figures 18 to 22.


FIGURE 18. NETWORK


FIG. 20. STEP 2 OF REDUCING THE NETWORK


FIGURE 19. STEP 1 OF REDUCING THE NETWORK


FIG. 21. STEP 3


FIG. 22. STEP 4 FINAL
2.9

These relationships apply only to networks having three terminals. Similar relations can be developed for 4 -terminal networks (frequently called two-port networks). Figure 23A is a typical 4-terminal network. If only voltages mea sured between terminals $a$ and $b$ and $c$ and $d$ are significant, the five impedances of Figure 23A can be replaced with the $T$ netwo $k$ shown in Figure 23B.


FIGURE 23. EQUIVALENT 4-TERMINAL NETWORKS

## 2. 41 PADS - COMMON TYPES

In the operation of various telephone circuits, it is frequently necessary to reduce currents and voltages within the circuits. To accomplish this result, attenuating networks are inserted at the required points. To attenuate all currents of the different frequencies by the same amount, the attenuating network obviously must be made up of resistances. By arranging appropriate resistances in a network consisting of series and shunt paths, any specific value of attenuation may be obtained without introducing impedance mismatch in the circuit in which the network is connected. Such resistance networks are usually called pads or attenuators and the most common of these are the "T" and "Pi" types illustrated in Figure 24.

The "T" and "Pi" pads may be made up in "H" and "Square" networks, In the latter the series resistances in Wires 1-3 and 2-4 are equal to $R_{1} / 2$. This balances the two sides of the circuit without chancing the electrical characteristics of the pad. Thus, in Figure 24 the "T" and "H" pads are electrically identical from both an impedance (resistance) and attenuation standpoint. The same applies to the "Pi" and "Square" pads illustrated.


FIGURE 24. PADS - COMMON TYPES

In most cases pads are symmetrical; that is, their impedances, as seen from either terminals 1-2 or 3-4, are the same and they can be designed to have any desired loss value. It is possible, however, for a "T, " "H, " "Pi" or "Square" type pad to be designed so as to have a different impedance as seen from either side. Under these conditions the pad may be used to match two unequal impedances and at the same time to produce any desired loss value which is equal to or greater than a minimum value depending upon the ration of the two impedances to be matched which are connected to two sides of the pad.

## 2. 42 T-PAD CALCUI_ATIONS

Problem 1) Given are two impedances $R_{1}$ and $R_{2}$ and voltages $E_{1}$ and $E_{2}$ across them, see Fig. 25. Calculate values of resistors $L, M$ and $N$ which would form a $T$ pad providing a good match between $R_{1}$ and $R_{2}$ and also calculate the attenuation $Y$ in decibels produced by this pad.
2) Instead of given $R_{1}, R_{2}, E_{1}$ and $E_{2}$, the input and output powers may be given, namely $P_{1}$ and $P_{2}$.
3) Another alternative may be a problem where $R_{1}$ and $R_{2}$ have to be matched by a pad which would provide $Y$ decibels of attenuation.
The calculations which follow show steps to obtain the solution.


1. Calculate parameter " X "

$$
X=\log _{10} \sqrt{\frac{P_{2}}{P_{1}}} \text { or } \quad X=\log _{10} \sqrt{\left(\frac{E_{2}}{E_{1}}\right)^{2} \times \frac{R_{1}}{R_{2}}}
$$

2. Calculate attenuation Y in decibels when X is known, or calculate X when Y is given, from the formula:

$$
Y=20 X
$$

3. Calculate " M "

$$
M=\frac{2 \times 10^{x}}{10^{2 x}-1} \sqrt{R_{1} R_{2}}
$$

4. Calculate "L"

$$
L=R_{1} \frac{\left(10^{2 x}+1\right)}{\left(10^{2 x}-1\right)}-M
$$

5. Calculate "N"

$$
N=R_{2} \frac{\left(10^{2 x}+1\right)}{\left(10^{2 x}-1\right)}-M
$$

6. To check the values obtained calculate the impedance of the pad looking into the terminals 1 and 2 with $R_{2}$ connected across 3 and 4. Then check the impedance of the pad looking into the terminals 3 and 4 with $R_{1}$ connected across 1 and 2. These impedances should be equal to $R_{1}$ and $R_{2}$ xespectively.
7. Assume $E_{1}=10$ volts, for instance, injected across terminals 1 and 2 and find from the calculated values of $L, M \& N$ the voltage $E_{2}$. Check whether the voltage ratio $E_{2}$ substituted into formula:

$$
\mathrm{X}=\log _{10} \frac{\mathrm{E}_{2}}{\mathrm{E}_{1}} \sqrt{\frac{\mathrm{R}_{1}}{R_{2}}}
$$

produces $X$ equal to $\frac{Y}{20^{\circ}}$. If so, the pad was designed correctly.

## 2. 43 PRACTICAL DETERMINATION OF EQUIVALENT NETWORKS

The problem may arise of some unknown passive, complex network, usually called "black box", of which only the input terminals 1-2 and output terminals 3-4 are accessible. Such "black box" can be replaced always, at any frequency, by an equivalent T or Pi network by making any three of the four measurements described hereafter.;

INPUT
OUTPUT


FIG. 26. TYPICAL BLACK BOX

In Fig. 26 the designations used have the following meaning:
$Z_{\text {ao }}$ - inputimpedance with output terminals $(3-4)$ open
$\mathrm{Zas}_{\mathrm{as}}$ - 11 " 11 " shorted
$\mathrm{Z}_{\mathrm{bo}}-$ output impedance $_{11}$ with input terminals (1-2) open
$\mathrm{Z}_{\mathrm{bs}}$ - " " " " " shorted
The "black box" is shown by dashed lines.
From the selected $T$ configuration (we could as well start from $a \mathrm{P}_{\mathrm{i}}$ ) we can write a set of following equations:

$$
\begin{align*}
& Z_{\mathrm{as}}=Z_{1}+Z_{3}  \tag{1}\\
& Z_{\mathrm{bo}}=Z_{2}+Z_{3} \tag{2}
\end{align*}
$$

$$
\begin{equation*}
\mathrm{Z}_{\mathrm{as}}=\mathrm{Z}_{1}+\frac{\mathrm{Z}_{2} \mathrm{Z}_{3}}{\mathrm{Z}_{2}+\mathrm{Z}_{3}} \tag{3}
\end{equation*}
$$

From (1) we obtain:

$$
\begin{align*}
& \mathrm{z}_{1}=\mathrm{Z}_{\mathrm{ao}}-\mathrm{Z}_{3}  \tag{4}\\
& \mathrm{z}_{2}=\mathrm{Z}_{\mathrm{bo}}-\mathrm{Z}_{3} \tag{5}
\end{align*}
$$

and from (2)

By subtracting (3) from (1) and substituting (5) thereafter, we obtain:

$$
\begin{aligned}
\mathrm{Z}_{\mathrm{ao}}-\mathrm{Z}_{\mathrm{as}} & =\mathrm{Z}_{3}-\frac{\mathrm{Z}_{2} \mathrm{Z}_{3}}{\mathrm{Z}_{2}+\mathrm{Z}_{3}}=\mathrm{Z}_{3}-\frac{\left(\mathrm{Z}_{\mathrm{bo}}-\mathrm{Z}_{3}\right) \mathrm{Z}_{3}}{\mathrm{Z}_{\mathrm{bo}}}= \\
& =\frac{\mathrm{Z}_{3} \mathrm{Z}_{\mathrm{bo}}-\mathrm{Z}_{3} Z_{\mathrm{bo}}+\mathrm{Z}_{3}^{2}}{\mathrm{Z}_{\mathrm{bo}}} \\
\mathrm{Z}_{3}^{2} & =\mathrm{Z}_{\mathrm{bo}}\left(\mathrm{Z}_{\mathrm{ao}}-\mathrm{Z}_{\mathrm{as}}\right) \quad \text { or } \\
\mathrm{Z}_{3} & =\sqrt{\mathrm{Z}_{\mathrm{bo}}\left(\mathrm{Z}_{\mathrm{ao}}-\mathrm{Z}_{\mathrm{as}}\right)} ; \text { by substitu } \mathrm{g} \text { this formula }
\end{aligned}
$$

into (4) and (5) we obtain the remaining two answers:

$$
\begin{gathered}
\mathrm{Z}_{1}=\mathrm{Z}_{\mathrm{ao}}-\mathrm{Z}_{3}=\mathrm{Z}_{\mathrm{ao}}-\sqrt{\left(\mathrm{Z}_{\mathrm{bo}}\left(\mathrm{Z}_{\mathrm{ao}}-\mathrm{Z}_{\mathrm{as}}\right)\right.} \text { and } \\
\mathrm{Z}_{2}=\mathrm{Z}_{\mathrm{bo}}-\mathrm{Z}_{3}=\mathrm{Z}_{\mathrm{bo}}-\sqrt{\left(\mathrm{Z}_{\mathrm{bo}}\left(\mathrm{Z}_{\mathrm{ao}}-\mathrm{Z}_{\mathrm{as}}\right)\right.} \\
\mathrm{q} .14
\end{gathered}
$$

We could have started by selecting any other three impedances instead of $\mathrm{Z}_{\mathrm{ao}} ; \mathrm{Z}_{\text {as }}$ and $\mathrm{Z}_{\text {bo }}$, and would have obtained similar formulas.

Summarizing: any passive network, no matter how complex, as long as it is composed of linear, bilateral impedances and has 2 input and 2 output terminals (such a network is often called 4terminal network or a 2-port network) can be represented at any selected single frequency by a simple $T$ (or $\mathrm{Pi}_{\text {i }}$ ) section consisting of resistors $Z_{1} ; Z_{2}$ and $Z_{3}$ which may be calculated from any three out of four input and output impedances measured in the laboratory.

In one of the preceding paragraphs 2.4 we have seen an example how a simple $T$ or $\pi$ network may be obtained from a fairly complex passive network by successive simple steps consisting of T and $\pi$ transformations.

## 2. 5 SUPERPOSITION THEOREM

If a linear network has more than one generator, the resulting current through any impedance component of this network is identical to the sum of partial currents obtained by considering each source of current alone with all other sources removed and replaced by their respective internal impedances, and then summing partial currents. By the use of the principle of duality the superposition theorem can be also applied to voltages. All that has to be done is to replace the words "current" by "voltage", since the latter is the dual of current.

It is important to remember that the superposition theorem holds only for linear networks.

Multi-generator networks can be solved by Kirchoffis Laws (sometimes this method is called loop and node analysis). However, the solution by using the superposition theorem often requires much less mathematical computations. Perhaps of even greater importance is the fact that this theorem provides a useful insight into the operation of the circuit.

It may be useful, at this stage, to review the concept of "the internal impedance of a generator. "


#### Abstract

T ? wrell known that if we use a high impedance VTVM (Vacuum Iube Voltmeter) and measure with it the so called opencircuit voltage across terminals of a battery we will find that this voltage is higher than the voltage when that battery is supplying current to a load. We call the open-circuit vollage the "EMF" or Electro Motive Force; it is determined by rysical conditions of the battery and also by the electro-chemical py perties of the matexials of which the battery is made. The dif rence between these two volta ${ }_{6}$ es is called the internal voltage drop V , and it is caused by the battery's internal resistance $R$ and by the current I flowing from it. This voltage drop equals IR; $R$ can be calculated as a quotient $\frac{V}{I}$.


- It is customary to represent such physical battery by a circuit model consisting of an ideal DC voltage generator, which has an internal resistance zero, and by a resistor $R$ connected in series with the generator. If we had some actual (physical) current generator (instead of a voltage generator) we could represent it by an ideal current generator with an infinitely high internal impedance and by a resistor $R$ across it; the $R$ value would depend on the internal impedance of the physical current generator.

Returning to Superposition Theorem, it can be illustrated by working out a simple problem.

## PROBLEM

In the circuit shown in Fig. 27 what is the direction and magnitude of the current flow in the center 10 ohm resistor?


FIGURE 27. NETWORK WITH TWO SOURCES

Our theorem instructs to determine the currents caused by each battery in turn, with all other batteries replaced by their internal resistances. Ohm ${ }^{\text { }}$ s law gives us the currents indicated in Figures 28, a and b.


FIGURE 28. THO SOURCE NETWORK REDRAWN

The currents flowing in the circuit with two batteries will be the sum of these component currents. Of course, sum means algebraic sum (or vector sum if the problem is AC ) and thus currents flowing in opposite directions subtract.


FIGURE 29. TWO SOURCE NETWORK WITH RESULTANT CURRENTS

Tho resultant currents are shown in Figure 29, and we see that the center 10 -ohm resistor carries lampere upward, which is the difference $(2 \mathrm{~A}-1 \mathrm{~A})=1 \mathrm{~A}$. We could have estimated the direction of the current in the $10-0 h m$ resistor by inspection, since the resistances are symmetrical with respect to two batteries and the 60 -volt battery will produce a twice larger c mponent of current. Going through the arithmetic illustrates an exar le of the application of the theorem.

## 2. 6 THE THEVENINS THEOREM

Thevenin's theorem states that, in general, any electric network including impedances and voltage sources, such as Fig. 30, may be replaced, insofar as an external load is concerned, by and equivalent circuit containing one constant-voltage source and one series resistance as shown on Fig. 31. The Thevenin ${ }^{8}$ s Theorem reads in full as follows:

Any network containing voltage sources (generators) and impedances may be replaced, insofar as terminal characteristics are concerned, by a constant-voltage source and a series impedance, at any particular frequency, providing this generated voltage is equal to the opencircuit voltage between the output terminals of the original network, and the series impedance is the impedance between the output terminals of the original network calculated with all voltages generated by the sources reduced to zero (short circuited) without opening any branch in which these voltages occur, or removing any impedance.



FIGYEE 31. THEVENINS EQUIVALERI NETTWORK

An example of the application of Thevenin ${ }^{\text {s }}$ s theorem is shown in Figs. 30 and 31.

A simple circuit of Fig. 30 has to be transformed. This circuit is included in the dashed rectangle, called a "black box", to indicate that little or nothing is known of what is inside it. The circuit has two external terminals 1 and 2, to which a load may be connected.

The purpose of the Thevenin ${ }^{3}$ s transformation is to obtain another circuit, consisting of a constantwvoltage source $e_{o c}$ and a series resistor $R_{0}$. The latter circuit, called the Thevenin's Equivalent Network, when connected between the terminals 1 and 2, would produce the same voltage and current in the external circuit (load) as the original "black box" would do.

If the circuit Fig. 30 is to be equivalent to one shown in Fig. 31, the open circuit voltages between terminals 1 and 2, must be equal. When circuit of Fig. 30 is solved for open-circuit voltage $e_{o c}$ the following is obtained:

$$
\begin{equation*}
\mathrm{e}_{\mathrm{oc}}=\mathrm{e}_{1} \frac{\mathrm{R}_{2}}{\mathrm{R}_{1}+\mathrm{R}_{2}} \tag{1}
\end{equation*}
$$

This mustalso be the open-circuit voltage of the generator of Fig. 31.

As a second condition for equivalent circuit, the shortcircuit currents obtained when shorting 1 and 2 must be the same in both circuits. The short-circuit current in Fig. 30 circuit is:

$$
\begin{equation*}
\mathrm{i}_{\mathrm{sc}}=\frac{\mathrm{e}_{1}}{\mathrm{R}_{1}} \tag{2}
\end{equation*}
$$

and the short circuit current in Fig. 31 is:

$$
\begin{equation*}
\mathrm{i}_{\mathrm{sc}}^{\prime}=\frac{\mathrm{e}_{\mathrm{oc}}}{\mathrm{R}_{\mathrm{o}}} \tag{3}
\end{equation*}
$$

Equating (2) to (3), thus assuming ${ }^{i_{S C}}=i_{\text {SC }}^{\prime}$

$$
\begin{equation*}
\frac{e_{1}}{R_{1}}=\frac{e_{o c}}{R_{o}}, \text { thus } \quad R_{o}=\frac{e_{o c}}{e_{1}} R_{1} \tag{4}
\end{equation*}
$$

and from (1)

$$
\frac{e_{\mathrm{oc}}}{e_{1}}=\frac{R_{2}}{R_{1}+R_{2}} ; \text { substituting this into (4) we obtain }
$$

$$
\begin{equation*}
R_{o}=\frac{R_{1} R_{2}}{R_{1}+R_{2}} \tag{5}
\end{equation*}
$$

As is can be easily seen the value of $R_{0} ;$ often equal to the computed value of impedance between the same o points 1 and 2 as Weve used in computing the open-circuit voltage oc but with all values of ers reduced to zero.

It should be noted that the internal characteristics of both networks (power, efficiency etc.) are not identical; only the characteristics at the terminals 1 and 2 are identical.

Care shall be exercised when Thevenin ${ }^{3}$ s theorem is applied to circuits containing controlled sources such as transistors or tubes. The correct Thevenin equivalents can be determined in a variety of ways. The procedure that is probably the simplest and most convenient is to evaluate the internal resistance $R_{0}$ from the ratio of open - circuit voltage to short - circuit current, or

$$
R_{o}=\frac{e_{o c}}{i_{s c}}
$$

## 2. 7 THE NORTON${ }^{2}$ THEOREM

This theorem is the dual of Thevenin's theorem.
Norton ${ }^{2}$ s theorem states that, in general, any electric network including impedances and voltage sources, for instance a.s shown in Fig. 32 may be replaced $d^{2}$ far as terminal conditions axe concerned, by constant-current generator shunted by a conductance as shown on Fig. 33. The Norton's theorem reads in full as follows:

Any circuit consisting of voltage sources and impedances may be replaced, insofar as terminal conditions are concerned at any particul frequency, by a constant current source and a shunt conductance, providing the magnitude of the constant current is equal to the current obtained by short circuiting the texi. 21 s she original network, and the shunt conductance is 1 to the reciprocal of the impedance between the outp: ..ftrals of the original network calculated with all - Mages generated by f sources reduced to zero wit. pening any branct n which these voltages occur or romoving any impedances.


FIG. 32


FIG. 33

An example of the application of Norton ${ }^{8}$ s theorem to the same circuit of Fig. 32 as of Fig. 30 is shown in Fig. 32.

The shoxt - circuit current $i_{\text {Sc }}$, obtained by strapping terminals 1 and 2 in Fig. 32, is obtained as follows:

$$
\mathbf{i}_{\mathrm{sc}}=\frac{\mathrm{e}_{1}}{\mathrm{R}_{1}}
$$

and the shunt conductance $g_{o}$

$$
R_{o}=\frac{1}{g_{o}}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}, \text { thus } g_{o}=\frac{R_{1}+R_{2}}{R_{1} R_{2}}
$$

It is obvious from the method of calculation of values for the circuit of Fig. 33 that $i_{S C}, e_{o c}$ and $R_{o}$ are identical with the same values obtained for the Thevenin ${ }^{2}$ s equivalent circuit of Fig. 31.

The simplest and most convenient procedure, especially if applied to circuits containing controlled sources, is to find first the equivalent Thevenin's circuit, applying the rule

$$
R_{o}=\frac{e_{o c}}{i s c}
$$

After finding $\mathrm{R}_{\mathrm{o}}$

$$
g_{o}=\frac{1}{R_{o}}=\frac{i_{s c}}{e_{o c}}
$$

Now the constant - current generator producing the current $i_{s c}$ is shunted by $g_{o}$ and the necessary values for the circuit of Fig. 33 have been found.

## 2. 8 POWER TRANSFER

In Figure 34, E and $\mathrm{Z}_{1}$ together represent a source of power. This source may be a telephone instrument, a repeater amplifier, or the sending side of any point in a telephone connection. $\mathrm{Z}_{2}$ is the load which receives the power transmitted. It may be another telephone instrument or a radio transmitting antenna or the receiving side of any point in a connection. The amount of power transferred from the source to the load will be determined by the relative values of $Z_{1}$ and $Z_{2}$.


FIGURE 34. POWER TRANSFER CIRCUIT

Three cases should be considered here:
a. If $z_{\text {. }}$ is an impedance and there is no restriction on the selection of $Z_{2}$, the power transferred will be a maximum when $Z_{2}$ and $Z_{1}$ have equal components of resistance, but the reactive components are equal and opposite, one inductive, the other capacitive $\left(R_{2}=R_{1}\right.$ and $\left.X_{2}=-X_{1}\right)$; such $Z_{1}$ and $Z_{2}$ are called "complex conjugates".
b. If $Z_{1}$ is an impedance and the magntude of $Z_{2}$ can be selected, but not its angle, the power transferred will be a maximum when the absolute values of $Z_{2}$ and $Z_{2}$ are equal: $Z_{2}=Z_{1}$. That is, the values of their magnitudes are equal and their phases are disregarded.
c. If both $Z_{1}$ and $Z_{2}$ are pure resistances, the power transferred will be a maximum when the source and load resistance are equal $\left(R_{2}=R_{1}\right)$.

The pure resistance case "c" will be investigated hereafter and three curves illustrating the circuit behavior are plotted in Fig. 35.


FIGURE 35. POWER TRANSFER AND EFFICIENCY WHEN SOURCE AND LOAD ARE RESISTIV

On the abscissa axis ratios $\frac{R_{2}}{R_{1}}$ are plotted for the range of values from 0 to 2 . On the ordinate ${ }^{R_{1}}$ axis are plotted: power $P_{2}$ developed in load $R_{2}$, curve (A); total power $P_{T}$ delivered by the source $E$ to the total load ( $R_{1}+R_{2}$ ) shown by curve (B); and the percent efficiency of the circuit, shown by curve (C).
$\mathrm{P}_{\mathrm{T}}$ can be calculated from formula

$$
P_{T}=\frac{E^{2}}{R_{1}+R_{2}}
$$

Since $R_{1}$ is fixed and $R_{2}$ is variable, the graph of $P_{T}$ is a hyperbola ( $B$ ); its maximum value occurs when load $R_{2}=0$, that is when $R_{2}$ is shorted. Then

$$
\mathrm{P}_{\mathrm{Tmax}}=\frac{\mathrm{E}^{2}}{\mathrm{R}_{1}}
$$

Current I equals in the original circuit

$$
I=\frac{E}{R_{1}+R_{2}} \text {, therefore power } P_{2} \text { in the load } R_{2} \text { may be }
$$

calculated as follows

$$
P_{2}=I^{2} R_{2}=\frac{E^{2}}{\left(R_{1}+R_{2}\right)} \times R_{2}
$$

It may be found by standard methods of calculus that maximum value of $P_{2}$ occurs when $R_{1}=R_{2}$. Then

$$
P_{2 \max }=\frac{\mathrm{E}^{2}}{\left(2 \mathrm{R}_{1}\right)} 2 \times \mathrm{R}_{1}=\frac{\mathrm{E}^{2}}{4 \mathrm{R}_{1}}
$$

or, using the formula for $\mathrm{P}_{\mathrm{T} \text { max }}$, we can write

$$
P_{2 \text { max }}=\frac{P_{T \text { max }}}{4}
$$

Therefore the maximum power which can be delivered to a matched load equals $25 \%$ of the maximum power $P_{T m a x}$ available from the source.

The relationship between per cent efficiency and the ratio $\frac{R_{2}}{R_{1}}$ will be investigated now.

Efficiency at any value of $\frac{\mathbf{R}_{2}}{\mathbf{R}_{1}}$ is defined as the ratio of power $P_{2}$ to the total power $P_{T}$ delivered by the source at that particular value of $\frac{R_{2}}{R_{1}}$.

$$
\begin{aligned}
& \text { Efficiency }=\eta=\frac{\mathrm{P}_{2}}{\mathrm{P}_{\mathrm{T}}}=\frac{\mathrm{E}^{2}}{\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)^{2}} \mathrm{R}_{2} \times \frac{\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)}{\mathrm{E}^{2}}=\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}+\mathrm{R}_{2}}=\frac{1}{\frac{\mathrm{R}_{1}}{\mathrm{R}_{2}}+1} \\
& \text { a) When } \mathrm{R}_{2}=0 \quad \frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}=0 \quad \text { and } \eta=0 \\
& \text { b) When } \mathrm{R}_{2}=\mathrm{R}_{1} \frac{\mathrm{R}_{2}}{1 \mathrm{R}_{1}}=1 \quad \text { and } \eta=50 \% \\
& \text { c) When } \frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}=2 \frac{\mathrm{R}_{1}}{\mathrm{R}_{2}}=\frac{1}{2} \quad \text { and } \quad \eta=662 / 3 \% .
\end{aligned}
$$

When $R_{1}=R_{2}, P_{2}$ is maximum and the efficiency, as we have seen, is only 50 per cent; then half of the total power $P_{T}$ is dissipated in the internal resistance $R_{l}$ of the source and the other half in the load $R_{2}$. This approximates the desirable condition in telephony since in most telephone applications we are interested primarily in receiving all possible signal power $P_{2}$ without too much regard to efficiency. Thus very often we try to operate at $50 \%$ efficiency; as a result we will deliver to the load $\mathrm{R}_{2}$ only $25 \%$ of the maximum power which the source is capable of supplying. The latter is designated as $P_{T \text { max }}$ and occurs when load $R_{2}=0$.

Actually, most telephone circuits contain some reactance so that condition "a" (where the load impedance is the complex conjugate of the source impedance) would appear optimum. However, it is well known from the theory of transmission lines that complex conjugate termination will cause reflections or echo. Therefore, we usually compromise on condition " b " and choose a load impedance of the same absolute magnitude as that of the source, disregarding the phase relationships.

## 2. 8 RESONANCE

In a circuit ${ }^{4}$ containing a given inductance $L$, the reactance, $X_{L}=L \omega$ (where $\omega=2 \pi f$ ), depends on frequency $f$. If $f$ is doubled, the reactance is doubled too. In the case of a given capacitance value $C$, on the other hand, the negative reactance,

$$
x_{c}=-\frac{1}{c \omega}
$$

is reduced when $f$ is increased. This is illustrated by Fig. 36B where the inductive reactance $X_{L}$ and the capacitive reactance $X_{C}$ are plotted against frequency. In a series resonant circuit containing both inductance and capacitance, as shown in Figure 36A, there is therefore some frequency at which the negative
reactonce $X_{C}$ becomes equal but opposite in value to $X_{L}$. Where the dolluine (a hyperbola) crosses the abscissa axis in Figure 36 B , the combined reactance

$$
X_{T}=X_{L}+X_{c}=L \omega-\frac{1}{\mathrm{c} \ell} .
$$

is equal to zero. The frequency at the inter ction is called resonance


FIGURE 36

SERIES RESONANT CIRCUIT

frequency $F_{r}$. Thus, since $\omega \quad 2 \pi f$

$$
\begin{aligned}
2 \pi f_{\mathrm{r}} \mathrm{~L}-\frac{1}{2 \pi f_{\mathrm{r}} \mathrm{C}} & =0 \text { and } \\
\mathrm{f}_{\mathrm{r}} & =\frac{1}{2 \pi \sqrt{L \mathrm{C}}}
\end{aligned}
$$

The value of the resonant frequency, $f_{r}$, therefore can be determined in terms of the inductance $L$ and capacitance $C$ from the above equations. Here $L$ is expressed in Henry ${ }^{1}$ s and $C$ in Farads. Since every coil has a resistance $R$ and at resonance the total reactance of the circuit is equal zero, therefore the total impedance which is the sum of resistance and reactance equals $R$. Under these conditions the circuit current is determined solely by R.

Figure 37 illustrates the behavior of a series resonant circuit similar to that shown in Figure 36A, but including some resistance $R$, when the applied voltage is varied through a band of frequencies. The curves were plotted by assuming a constant impressed voltage from the generator $E$ of 1 volt for each frequency of the band, and three different values of resistance R. As will be noted, the peak current values depend entirely upon the values of resistance $R$, for at the peak the positive and negative reactances cancel each other and therefore current is determined solely by the resistance $R$. The increase of resistance $R$ in the series resonant circuit reduces the selectivity (or sharpness) of the resonance peak.

FIGURE 37. CURVES OF CURRENT VALUES IN A SERIES RESONANT CIRCUIT WHICH INCLUDES R.

That is, The ratio of the current at the resonant frequency $f_{r}$ to the current at L -quencies near this resonant frequency is reduced.
Below the resonant frequency, the capacitance $C$ in the circuit will have the major effect in limiting the current, and the circuit will tend to look like capacitance with the current leading the voltage. Above resonance the inductance $L$ will limit th current and the circuit will look like inductance with the curre t lagging behind the vo tage. At resonance the circuit current and roltage are in phase and the carrent may be relatively very large. The actual voltages, across the inductor and capacitor may therefore be many times as great as the voltage E applied to the circuit, but since they are equal and opposite in sign they partially cancel.

Example: To what frequency is the circuit shown by Figure 37 resonant if C is 0.254 , L is 0.10 H ; what current $\mathrm{I}_{\mathrm{r}}$ will flow at resonance when $R$ is 4 ohms and $E$ is 1.0 volt, and what is the voltage $\mathrm{drop} \mathrm{E}_{\mathrm{L}}$ across the inductance?
Solution:

1) Resonant frequency $f_{r}=\frac{1}{6.28 \sqrt{0.10 \mathrm{H} \times 0.254 \mathrm{~F} \times 10^{-6}}}$

$$
\begin{aligned}
& =\frac{-1}{6.28 \times 10^{-3} \sqrt{0.0254}} \\
& =\frac{1.000}{6.28 \times 0.159}=\frac{1,000 \text { cycles per }}{\mathrm{sec}}
\end{aligned}
$$

2) What current flows at resonance?

$$
\mathrm{I}_{\mathrm{r}}=\frac{\mathrm{E}}{\mathrm{R}}=\frac{1.0 \mathrm{v}}{4 \mathrm{ohms}}=0.25 \mathrm{amp} .
$$

3) What is the voltage drop across L ?

$$
\begin{aligned}
& \mathrm{E}_{\mathrm{L}}=\mathrm{IX}_{\mathrm{L}} \\
& \mathrm{X}_{\mathrm{L}}=2 \pi \mathrm{fL}=6.28 \times \mathrm{cns} \times 0.10 \mathrm{H}=628 \text { ohms } \\
& \mathrm{E}_{\mathrm{L}}=0.25 \mathrm{~A} \times 626 \text { has }=157 \text { volts. }
\end{aligned}
$$

This voltage should now be compared with the
$E=1 \mathrm{v}$ injected into the circuit.

CH. 2 NETWORKS

The resonance principle has numerous and interesting uses in connection with communication circuits. One application is the use of a capacitor $C$ of proper value in series with a telephone receiver winding, repeating coil winding, or other winding having inductance, where it is desired to increase the current. The 1 capacitor $C$ decreases the total reactance by reducing $X_{I}$ by $\overline{c \omega}$ Thus the total impedance is reduced and current I is increased.

A much more common use of the series resonance principle is the so-called "tuned" circuit which is extensively employed in radio and other high frequency applications. It is an arrangement whereby the circuit has a much lower impedance to some particular frequency than to any other frequency; if a band of frequencies is impressed, it selects, so to speak, a high current (and therefore a high voltage) for the particular frequency but permits only a small current (voltage) for any other frequency. Figure 37 illustrates this principle.

In many tuned circuits, capacitance and inductance are connected to form a parallel-resonant circuit as shown in Figure 38A. For this connection, when the positive reactanceL wis equal and opposite to the negative reactance $-\frac{1}{c}$ and the resistance $R$ of the inductor is low, the combined impedance $\mathrm{X}_{\mathrm{t}}$ presented to the generator is extremely great and there is a minimum current flowing from the generator. In other words, the generator circuit is practically open. Figure 38B shows the resultant reactance, $X_{t}$, presented to the generator by this circuit. It can be seen that at the resonant frequency the two parallel reactances combine to give a resultant of extremely high value. At the same time, at resonance there must be a current through the inductance L, determined by dividing the voltage E of the generator by the impedance of this branch. Similarly, there must be a current through the capacitor C which can be determined in the same way. These two currents are equal in value, but are flowing in opposite directions, thereby canceling each other in the lead to the generator. Effectively, this gives an open-circuit insofar as the generator circuit is concerned, but gives a circuit equal to either the inductance or capacitance alone connected to the generator insofar as either of the branches is concerned. The physical explanation of the phenomena here is that a current is oscillating around through the inductor and capacitor, with the emf of the generator merely sustaining and triggering this oscillation. Of course, since the inductance $L$ must have some resistance $R$, there will be an $I^{2} R$ loss in the inductance, and it would never be possible to have the theoretical case where the generator current is actually zero, or the load is actually an open circuit.

Figure 39 illustrates the selectivity of a parallel-resonant circuit made up of the same units as were used in the series resonant circuit. It will be noted that the selectivity of the parallelresonant circuit is also decreased as the resistance $R$ is increased. Indeed, there is a value of resistance $R$ beyon which the circuit loses its resonant characteristics altogether. Moreover, in this case, the resistance may be seen to have sor effect on the value of the resonant frequency. When $R$ is increa $\quad \mathrm{dd} \mathrm{f}_{\mathrm{r}}$ decreases.

In radio and other high frequency work the parallelresonant circuit is often called a "tank circuit," because it acts as a storage reservoir for electric energy. Here it may be more helpful to think in terms of energy transferring back and forth between the electric field of the capacitor $C$ and the magnetic field of the inductor $L$ rather than merely of current oscillating back and forth in the parallel circuit.

It may be said, in general, that the series-resonant circuit steps up voltages injected in to it, and the parallel-resonant circuit steps up currents.

## 2. 9 FILTERS

An electrical network which consists of inductors, capacitors and resistors designed to permit the flow of curent at certain frequencies with little or no attenuation, and to present high attenuation at other frequencies is called an electric filter. Therefore, the purpose of a filter is to provide a circuit which will easily transmit certain frequencies and suppress others.

The action of a filter depends upon the fact that the higher the frequency the easier it is for the current to flow through a capacitor and the more difficult it is to flow through an inductor. In other words a capacitor passes high frequencies readily and offers a decreasing reactance with increase in frequency, while the reverse is true of inductance, which passes low frequencies readily.

The filter characteristics show usually either attenuation provided by the filter or current which it passes as functions of frequency.

$$
x_{T}=\frac{x_{1} x_{C}}{x_{L}-x_{C}}
$$


figure 38. parallel-resonant circuit


FIGURE 39. Curves of current values in parallel-resonant CIRCUIT WHICH TNCLUDES R.


FIGURE 40 . LOW-PASS FILTER


FIGURE 4 2. BAND-PASS FILTER


FIGURE 4I. HIGH-PASS FILTER


FREQUENCY
FIGURE 43. BAND-ELIMINATION FILTER

Four basic filter attenuation characteristics are shown. Figure 40 shows a low-pass filter, which passes readily lower fr zencies and attenuates higher ones; Figure ' a high-pass filt $\because$, whose action is reverse to low-pass. $L$. Figure 42 is bova the characteristic of a band-pass file; it passes only a definute band of frequencies while it attenuates frequencies outs de the pass-band. In Figure 43 is shown a band-elimination filter
whose action is opposite to band-pass; it attenuates (eliminates) a definite band of frequencies, while passing readily others outside the elimination band.

In Figure 44 are shown typical examples of the circuits of low-pass and high-pass filters as well as their current versus frequency characteristics.


The presence of resistance in the inductors used in filter sections introduces additional losses in the transmitting bands, and reduces the sharpness of cut-off (See Figure 44). In telephone and telegraph carrier systems, the number of channels which can be used in a given frequency range depends on the width of the pass band plus the "transition bands" on each side of it.

Ora of the most practical ways to obtain a high ratio of reactance to iesistance is to use mechanical vibrating systems, such as the piezo-electric crystals. In an electric circuit such as a filter, a crystal acts as an impedance. Crystal filters find wide application in "broad-band" ( $J, K$, and $L$ ) carrif r systems. In Figure 44 A is shown the equivalent electrical n :work and the reactance characteristics of a quartz crystal.


FIG. 44 A. A QUARTZ CRYSTAL ACTS, WHEN ASSOCIATED WITH A SUITABLE ELECTRIC CIRCUIT, AS THE. NETWORK SHOWN ABOVE. THE REACTANCE CHARACTERISTICS OF THIS CIRCUIT ARE SHOWN BELOW.

Electrical filters have a large number of applications in the telephone art. In addition carrier systems, composite sets and many central office power plants require their use. In fact, much of the present electronic art has been made possible by their development.
2. 10 QUALITY OF A COIL

Where resonant circuits are used ${ }^{4}$ for rposes of tuning or equency selection, it is important that timective resistance ithe circuit be held to a minimum. This: illustrated by 18 37 and 39 where it is shown that sharpness of tunir is greater the lower the resistance R. "Effective resistance" $R$ is no
the DC resistance but one measured at the actual operating frequency; its value takes care of all the high-frequency losses.

Since the resistance in the circuit is largely contained in the inductor, the objective is to have the ratio of the reactance of the inductor to its effective resistance as high as possible. This ratio is known as the Quality or $Q$ of an inductor and is usually expressed by the equation

$$
\mathrm{Q}=\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}=\frac{2 \pi f \mathrm{~L}}{\mathrm{R}}
$$

and by $Q=\frac{1}{2 \pi f R C}$
a capacitor, represented as a series model.
The $Q$ of resonant circuits used in practice varies from values in the order of a hundred in the case of iron-core coils to values as high as 20 thousand or more in the case of certain cavity resonators or quartz crystals.

In a series resonant circuit (Figure 37) at the resonant frequency, the voltage developed across the capacitor $C$ is $Q$ times the net voltage $E$ applied by the generator, or $E_{c}=Q E$. In a parallel-resonant circuit when $Q$ is large, the total impedance of the $L$ and $C$ combination at the resonant frequency is $Q$ times the coil or capacitor reactance, or $Z=X_{L} Q=-X_{C} Q$ (at resonance $X_{C}=X_{L}$ ).

Example: In a parallel-resonant circuit having an inductance $L$ of 50 microhenries and an effective resistance $R$ of 10 ohms, what is the $Q$ of the circuit and what is the impedance Z at a resonant frequency of 1000 kc ?


## Solution:

$$
\begin{aligned}
50 \text { microhenries } & =5 \times 10^{-5} \text { Henries; } \\
1000 \mathrm{kc} & =10^{6} \text { cycles. } \\
X_{\mathrm{L}} & =2 \pi \mathrm{fL}=6.28 \times 10^{6} \mathrm{cps} \times 5 \times 10^{-5} \mathrm{H} \\
& =314 \mathrm{ohms} \\
\mathrm{Q} & =\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}=\frac{314}{10}=31.4 \\
\mathrm{Z} & =\mathrm{X}_{\mathrm{L}} \mathrm{Q}=314 \times 31.4=9860 \text { ohms. }
\end{aligned}
$$

## References for Chapter 2

3. From John D. Ryder, NETWORKS, LINES AND FIELDS, 2nd Edition. 1955, by permission of Prentice-Hall, Inc., Englewood Cliffs, N. J.
4. (C) American Telephone and Telegraph Company, 1961.

## CHAPTER 3

## REPEATING COILS AND TRANSEORMERS

### 3.1 THEORY OF THE TRANSFORNIERS

The inductive effects ${ }^{4}$ of the magnetic interlinkages from one turn of a coil winding to the other turns of the same winding are defined as self-inductance. The current resulting from the induced emf is superposed upon the current resulting from electromotive force (emf) impressed across the coil.

In practice, we may experience inductive effects in circuits other than the first one in which the current is flowing due to the impressed emf. That is to say, two coils may be so related that the lines of magnetic induction established by a current in the first coil may cut the turns of the second coil (which may be connected to an entirely different circuit) in the same way that similar lines established by any one turn of a first single coil cut the other turns of the same coil. This effect is called mutual induction and the property of the electric circuit that is responsible for the effect is known as its mutual inductance.

### 3.2 THEORY OF THE TRANSFORMER

In the study of magnetism it is found that a wire through which there flows a current is always surrounded by a magnetic field. This field, when created by a current estishing itself in the conductor, extends concentrically from the wire and is proportional to the current magnitude at any instant of time. Figure 45 shows a group of lines of a magnetic field around a conductor (shown in cross-section) in which the current is flowing.


FIGURE 45
MAGNETIC FIELD AROUND A CURRENT CARRYING CONDUCTOR

If a secon conductor is in the vicinity, it will be cut by the se lines extending concentrically outward from the current-carrying conductor. This induces an emf in the second conductor, if the current in the first conductor is non-stationary. As illustrated in the Figure 45, this will establish a current in the oposite direction to that in the first conductor. The induced current will cease to flow, however, when the current in the first conducto reaches its maximum value, or at any other instant when it has .. steady, unchanging value because the magnetic field has become stationary and the lines of magnetic induction are also stationary.

If the current in the first conductor is decreased, we have the reverse condition, or that shown in Figure 46. Here the lines, instead of being concentric in the clockwise direction, are again concentric but counterclockwise. Now the current induced flows in the opposite direction. It is now in the same direction in the second conductor as in the first. The law for induced emf may be expressed as follows: For any two parallel conductors, a changing current in one induces an emf in the other, tending to establish a current that will counteract the mechanism that produces it.



FIG. 47. PRINCIPLE OF INDUCTION COIL
Instead of two single conductors shown in Figure 45 and 46 , let us consider two separate coils, one inside the other, as in Figure 47. If we call the one carrying the original current the primary, which in this case we may represent by the inside coil, and the other the secondary, we shall find that a magnetic field is established by a changing current in the primary. This will cut the entire group of conductors represeded by the turns of the secondary, thereby inducing a potential in the secondary. The ordinary telephone induction coil operates in this manner. The primary, when connected in series with the telephone transmitter, carries a current which decreases and increases in value in response to the varying resistance of the transmitter. Consequently, an alternating current is induced in the secondary of the coil.

If now the two separate coils of Figure 47 are wound on the same iron core the effect will in intensified. Because iron offers a path of low reluctance to the magnetic flux, the total number of flux lines will be greatly increased and almost all of the lines set up by the primary winding, $P$, will cut all of the secondary winding, $S$.

If the windings, $P$ and $S$ have the same number of turns, and both the coils and core are constructed so as to have negligible energy losses, we shall find that the reading is the same when a voltmeter is connected across the terminals of $S$ as when connected across the terminals of $P$. In other words, the induced emf of the secondary winding is equal to the impressed emf of the primary winding. Such a device is called an ideal transformer of unity ratio.

I:. N $N$, we should increase the number of turns $\mathrm{N}_{\mathrm{S}}$ of the secondary winding $S$, we would find that the voltmeter reading would be greater on the secondary than on the primary side of the transformer. If we should decrease the number $N_{S}$ of turns of the winding $S$, the effect would be reversed. We have here a means of controlling the voltage applied to a load; we rr .y effectively increase or decrease the generator voltage by a $p$ per choice of tranis, former. If a transformer has a greater n sber of turns on the secondairy $N_{S}$ than on the primary $N_{p}$ so that the voltage $i-i_{n}-$ creased, it is called a step-up transformer; if it has a lesser number of turns on the secondary than on the primary so that the voltage is decreased, it is called a step-down transformer. The voltage across the two windings is directly proportional to the number of turns. This relation is expressed by the equation:

$$
\begin{equation*}
\frac{\text { primary volts }}{\text { secondary volts }}=\frac{\mathrm{E}_{\mathrm{p}}}{\mathrm{E}_{\mathrm{s}}}=\frac{\mathrm{N}_{\mathrm{p}}}{N_{\mathrm{s}}}=\frac{\text { primary turns }}{\text { secondary turns }} \tag{1}
\end{equation*}
$$

We may explain this relation between the number of turns and voltage by our original law governing inductive effects, winch states that the induced voltage is proportional to the rate of ring lines of magnetic induction. Each time the alternating emf i, the primary completes a cycle, it establishes a magnetic flux in the iron core which collapses to be established in the opposite direction, to again collapse; etc. This flux will cut each and every turn about the iron core. In doing so, for the ideal case where there is no loss due the magnetic leakage, etc., the same voltage is induced in each individual turn of secondary. This voltage may be represented by the symbol V. Now, the voltage measured across the secondary (with no load connected)

is merely the sum of these individual turn voltages (See Fig. 48) or

$$
\begin{equation*}
E_{S}=N_{S} V \tag{2}
\end{equation*}
$$

where $N_{S}$ is the number of turns on the secondary.
In the primary the induced emf $V$ in one turn must be exactly equal and opposite to the originally impressed emf since the emf due to an IR drop is practically negligible. This could be expressed by an equation similar to equation (2), thus assuming that $V$ is the same for primary as for the secondary -

$$
\begin{equation*}
E_{p}=N_{p} V \tag{3}
\end{equation*}
$$

Since $V$ is the same in both equations (2) and (3), we may derive equation (1) by dividing (3) by (2).


FIGUPE 49

If a load $Z_{S}$ is connected across the secondary winding $S$, as shown in Figure 49 , the emf induced in the secondary $S$ causes Current $I_{s}$ to flow through the impedance $Z_{S}$. This current is:

$$
I_{S}=\frac{E_{S}}{Z_{s}}
$$

When current $I_{s}$ starts to flow through the secondary it will establish additional lines of magnetic induction in the transformer core, which lines will oppose those established by the primary current $I_{p}$. This phenomenon tends to neutralize partially the magnetic field in the iron core, thereby tending to counteract the effect of the inductance of the primary winding $P$.

As a result the primary winding $P$ will behave more nearly like a resistance than an inductance. Therefore, own induced emf of the primary, which opposes the voltage of the generator G, will be reduced and a larger current $I_{p}$ will flow from $G$ through the
primary $2 . \quad \therefore$ in turn will increase the flux in the iron core. Finally, there is produced the same induced emf in the secondary as in the case without load $Z_{s}$ when the secondary was open. Thus it may be said that the transformer adjusts itself to a load $Z_{s}$ which is connected to the secondary just as if an "equiv lent" load were connected directly to the generator $G$ (without th transformer), i. e. the current supplied by the generator $G$ to $t$ e primary increases with an increase of the current $I_{s}$ in the secondary of the transforme

The relation between current values is the inverse ratio of the number of turns. In other words, the winding having the greater number of turns has a proportionately smaller current. We know from the law of conservation of energy that the energy $P_{s}$ existing in the secondary circuit can never exceed, but for an ideal transformer will be just equal to, the energy of the primary circuit, where since

$$
P_{p}=P_{S} \text { and in general } P=E I \text {, }
$$

we have

$$
E_{s} I_{S}=E_{p} I_{p}
$$

from which

$$
\begin{equation*}
\frac{I_{S}}{I_{p}}=\frac{E_{p}}{E_{S}} \text { or } \frac{I_{S}}{I_{p}}=\frac{N_{p}}{N_{s}} \tag{4}
\end{equation*}
$$

The value of the current in the secondary circuit is of course dependent on the value of the load impedance $Z_{g}$ - that is, $I_{s}=E_{s} / Z_{s}$. From vhich

$$
\mathrm{Z}_{\mathrm{s}}=\frac{\mathrm{E}_{\mathrm{s}}}{\mathrm{I}_{\mathrm{s}}}
$$

Similarly the impedance presented to the generator by the primary of the transformer is

$$
z_{p}=\frac{E_{p}}{I_{p}}
$$

The "nonship between the impedance $\quad$ and the turns ratio in then determined, with the help of equatione (1) and (1), es

$$
\begin{align*}
& \frac{Z_{S}}{Z_{p}}=\frac{E_{S}}{I_{S}} \times \frac{I_{p}}{E_{p}}=\frac{E_{S}}{E_{p}} \times \frac{I_{p}}{I_{S}}=\frac{N_{S}}{N_{p}} \times \frac{N_{S}}{N_{p}} \\
& \quad \text { or } \quad \frac{Z_{S}}{Z_{p}}=\left(\frac{N_{S}}{N_{p}}\right)^{2} \quad \text { and } \quad \frac{N_{S}}{N_{p}}=\sqrt{\frac{Z_{S}}{Z_{p}}} \tag{5}
\end{align*}
$$

Transformers which have different numbers of turns of primary and secondary windings are called in telephone work "inequality ratio repeating coils." They may be rated either according to their voltage ratios, step-up cr step-down as the case may be or in accordance with their impedance ratios.

Before taking up specific uses of the transformer, let us review in general what its presence in Figure 49 has or may have accomplished.
a. The characteristics of the electric energy may have been changed, or we might say its state may have been "transformed," inasmuch as in the primary circuit we may have had high current and low voltage, while in the secondary circuit we may have had low current and high voltage, or vice versa, depending upon whether the transformer was step-up or step-down.
b. The electric energy was transferred from one circuit to another without any metall: connection being made between the two circuits; Liom a direct-current aspect the circuits are separate units. Thus the transformer separates D.C. components from A.C. components contained in the primary winding. Only A.C. components are transferred to the secondary winding.
c. The transformer in effect changed the nature of the con nected load, or in other words changed the impedance of the load to a different value unless the transformer was of unity ratio. This is called impedance transformation and formula (5) is used to calculate the transformer's turns ratio

$$
\frac{N_{\mathrm{S}}}{\mathrm{~N}_{\mathrm{p}}}=\sqrt{\frac{Z_{\mathrm{S}}}{\mathrm{Z}_{\mathrm{p}}}}
$$

or to find the value of "reflected impedance" $Z_{p}$ when $Z_{s}$ is connected across the secondary:

$$
\mathrm{Z}_{\mathrm{p}}=\mathrm{Z}_{\mathrm{s}}\left(\frac{\mathrm{~N}_{\mathrm{p}}}{\mathrm{~N}_{\mathrm{s}}}\right)^{2}
$$

### 3.3 TRANS TORMER APPLICATIONS TO TELEPHONE CIRCUITS

The applications of transformers ${ }^{4}$ to telephone circuits are numerous and varied. The reduction of energy losses in alternatingcurrent transmission due to stepped-up voltages 1 as an application to telephone transmission but is not so important as to other uses especially to power transmission. One very gen cal use is to ac= complish the result given as "b" above. In this wase, the primary function of the transformer is to transfer energy to another circuit (separating D.C.) rather than to change the voltage and current values. When so used in telephone work, the transformers are generally called repeating coils rather than transformers because their primary function is to "repeat" the variation of energy into a different circuit rather than to transform it into a different voltage or current. There are, however, inequality ratio repeating coils which perform both functions. On the other hand, in connection with telephone repeater circuits and certain other telephone apparatus, input and output inequality ratio coils are used primarily to match impedances to permit maximum energy transfer. Another very general use of repeating coils in the telephone plant is for deriving "phantom" circuits. Here the coils serve a unique purpose which has no counterpart in electric power work, and is not included in the classification of transformer functions given above.

### 3.4 THE PHANTOM CIRCUIT

Figure 50A shows a simplified diagram ${ }^{4}$ of two adjacent and similar telephone circuits 1-2 and 3-4 arranged for phantom operation. By means of repeating coils installed at the terminals of the wire circuits, a third circuit is obtained. This circuit is known as the phantom; the pair of conductors of each of the two metallic or side circuits is utilized as one conductor for the phantom circuit. The two side circuits and the phantom circuit are together known as a phantom group. The three circuits, employing only four line conductors, can be used simultaneously without interference from, or interfering with, any of the others of this combination. The four wires must have identical electrical characteristics and be properly transposed to prevent crosstalk, and can carry three different channels of communication.

Since the two wires of each side circuit are almost identical, any speech current set up in the phantom circuit in E will divide almest equally at the midpoint of the repeating $c$. line windings. As suown by the full line arrows in Figure 50, one part of the fom current will flow through one half of the line winding, and the othe_, rt of the current will flow in the opposite direction through the other half of the line winding. The speech currents in side 1-2 and side 3-4 are shown by dashed arrows. The inductive

effects of the coil windings will be almost neutralized, as shown in Figure 50B, and there will be a very small resultant current set up in the "drop" side of the repeating coil. As the phantom current i divides in approximately two equal parts, the halves of i current will flow in the same direction through the respective conductors 1 and 2 of one side circuit, and likewise return in the other side circuit via 3 and 4. At any one point along a side circuit, there will be ideally no potential difference between the two wires caused by currents $\mathrm{i} / 2$ in the phantom circuit. This can be seen easily from Figure 5lA. Here the two line conductors 1 and 2 are electrically identical. If a telephone receiver is installed at any place $F G$ along the line the impedances $(1-1+A)$ and $(2-1+B)$ are equal. The same is true for the right part of the line between $F$ and 2 and $G$ and 2, that is impedances, $(1.2+C)$ and $(2.2+D)$ are equal. Thus component currents of $i$ in the wires 1 and 2 must be equal and due to that there is no potential difference between points $G$ and $F$. Therefore, no conversation over the phantom circuit will be heard in side 1-2 circuit. The same reasoning is true for the side circuit 3-4.



A


B

FIGURE 5I. Principles of no interference within the phantom group

It will be equally true that the conversation over a side circuit cannot be heard in the phantom circuit. As can be seen from Figure 51B a signal from side l-2 is applied across two parallel impedances $(A+B)$ and (wire $1+C+D+$ wire 2). Here $A=B$ and $(1+C)=(D+2)$, as required by the balanced condition. Therefore, there will be no potential difference between points 1 and 2, and an A. C. voltmeter connected across these points will read zero voltage. This proves that no signal from the circuit l-2 will be heard in the phantom circuit. The same reasoning is true for the side 3-4 circuit.

In considering the theory of the phantom ${ }^{4}$ it should be kept in mind that the conductors are assumed to be electrically identical, or in other words, the conductors are perfectly "balanced." The phantom is very sensitive to the slightest upset of this balance, and circuits that are sufficiently balanced to prevent objectionable cros. talk or noise in physical circuit operation, may not be sufficiently balanced for successful phantom operation.

### 3.5 STANDARD REPEATING COILS

A number of general types of repeating coils ${ }^{4}$ are currently $\ldots . .2$ in the Bell System. One principal type, illustrated by tre 62 and 4 ies, has four windings, the terminals of which are designated by numbers as shown by Figure 52A. The other type, illustrated by the 173 series, has six windings which may be connected as shown in Figure 525 with four windings on the line side, or


FIGURE 52 STANDARD REPEATING COILS
with the 9-10, 11-12 windings not used, depending on the impedance ratio required. In all types, the windings which are used to form the line side are precision manufactured so as to be as nearly identical electrically as possible. This balance is required on the line side, as we have already seen, to avoid crosstalk where the coils are used in phantom operation. The drop windings (that is, 1-2 and 5-6) do not need to be as well balanced in normal use on both coils.

TABLE 1
STANDARD REPEATING COILS

| IMPEDANCE RATIO "LINE" TO <br> "DROP" 4-3 AND 8-7 TO 2-1 AND 6-5 <br> 3-8 AND 1-6 STRAPPED | 93-TYPE | 62-TYPE |
| :---: | :---: | :---: |
| 1:1 | 93-A | 62-A |
| 1:1.62 | $93-\mathrm{B}$ | 62-B |
| 1.62:1 | 93-F | 62-C |
| 2.66:1 | 93-G | 62-E |
| 1.24:1 | $93-\mathrm{H}$ |  |
| 2. 28.1 | $93-J$ |  |
| 1:1. 28 |  | 62-F |
| 1:2.34 |  | 62-G |

(. . . nd 93-type coils have toroidal cores made of many turns of fine-gay - silicon-steel wire sawed through at one point to introduce a gap in the magnetic circuit. In the 93 -type coil this gap is filled with compressed powdered iron which, while increasing the reluctance of the uncut core slightly gives it . high degree of magnetic stability due to preventing permanents agnetization under abnormal service conditions. In the 62 -type co the gap in the mag. tic circuit is an unfilled air gap which te as to make the coil even more c.ide. This coil is especially well adapted for use on circuits used for d-c telegraph operation. The same feature, however, tends to make the 62 series inefficient due to low inductance at low frequencies and they cannot be used on circuits employing 20 -cycle signaling, whereas the 93 series may be used for such purposes. Standard 173-type coils are built with permalloy cores of high permeability thus very efficient magnetically.

The types of repeating coils discussed above are manufactured with a number of different turn ratios to provide various impedance matching combinations. Table I gives the standard impedance ratios for 93 - and 62 -type coils. The 173 -type coils are likewise available in a wide range of impedance ratios. The impedance ratio obtained in their use depends on whether all four of the line windings are used and on how those used are connected. The m pedance ratios that can be obtained accordingly do not lend tanselves readily to tabular presentation, but various ratios line-tom drop ranging rom as low as $0.6: 1$ to as high as $2.52: 1$ may be obtained.

### 3.6 THE HYBRID COIL

In telephone repeater operation, as well as in duplex telegraphy, we mist receive incoming energy and direct it into a receiving circuit ( $n$ ut) which is separate and distinct from the sending (output) circuit. This is necessary because the amplifier used for voicc Frequency currents operates usually in one direction only. It woul not be possible for two such amplifiers to be connected at the wame point in a telephone circuit as shown in Figure 53, because part of energy amplified in one circuit, in say the output of amplifier \#1, would be delivered to the input of the other \#2, to be again amplified in it. Part of this returning energy would again reach the input of the \#l amplifier and the rocle would be repeated, with energy thus circulating through the two an fiers and in$\mathrm{c} x$ sing in level until a condition of "howl" $\mathrm{o}_{\mathrm{i}}$ ing" was reached. The epeater would then continue in this condid on rexdering the 1. $n$ circuit inoperative.

To eliminate such possibility of repeater singing, the two circuits must be connected to the same 2 -wire line so that any current flowing in one amplikier must not in any way affect the other.


AN INCORRECT AMPLIFIER CONNECTION SINCE SINGING IS POSSIBLE
FIGURE 53

We can obtain this desired result by applying the balance principle of the A. C. Wheatstone bridge. To illustrate this, we have a repeating coil connected in an alternating current Wheatstone bridge in Figure 54A. Here the source of voltage is a telephone transmitter instead of a generator. An emf is then impressed across a and b by mutual induction instead of by direct connection. In place of a galvanometer, we have substituted a telephone receiver or in practice an amplifier.


FIGURE 54. BALANCED BRIDGE

A: we clear and equivalent representation of the same circuit is shown in Figure 54B; here the telephone transmitter is replaced by an AC generator and an amplifier is substituted for a telephone receiver. Figure 54B clearly shows a Wheatstone bridge. With the $R$ and $L$ components of the variable arm of the bridge adjusted to give a good balance, any voice curre coming from the telephone transmitter (AC generator) cannot be ard in the receiver ' ${ }^{\text {fier }}$ ) circuit for the same reason that a $g$ : vanometer needle is stationery in any balanced DC bridge. If we apply two bridoe circuits, we have double-tracked, so to speak, the ordinary 2 -way telephone circuit as shown in Figure 56; the operation of such circuit will be explained later.

The coil that takes the place of the bridge mechanism in Figure 54A and 54B is known as a "hybrid coil" or "three winding transformer." In the actual coil, there are a few additional details of design that do not permit the identity of the simple AC bridge circuit to be so readily recognized. The line coils are divided and connected on both sides of the two wire line as shown by Figure 55, in order that symmetry in the wiring of the talking circuit may be maintained. Each hybrid coil has six windings, and all of them are wound on a single magnetic core. Both sets of windings are inductively coupled to the third winding.


The circuit arrangement also includes ${ }^{4}$ wo balanoing net. works, $Z_{2}$ and $Z_{5}$,each desipued and adjuted to have exectly the same impedance as the line section $Z_{1}$ or $Z_{6}$ which is connected to the same coil. The function of the overall circuit arrangement is to permit energy shown by dashed line arrows entering the coil from the East line to pass to the E-W amplifier: $Z_{3}$ and thence out to the West line through the West Hybrid coil (1). Similarly energy coming from the West line shown by full line arrows passes through the W-E amplifier $Z_{4}$ and therefrom to the line East. At the same time, the circuit must prevent the output energy of either amplifier from crossing a coil to enter the input of the other amplifier, since this would set up a local circulation that would cause the repeater to "sing" or "howl."

How the hybrid coil acts to meet those requirements may be understood by an analysis of the circuit of Figures 56, and simplified schematics of Figures 57 and 58.

For the proper operation of the hybrid circuit $Z_{2}$ is adjusted to equal $\mathrm{Z}_{1}$ and $\mathrm{Z}_{5}$ to equal $\mathrm{Z}_{6}$.

A. Operation of the hybrid coil (l) for EAST to WEST transmission Energy flowing from EAST to WEST shown by dashed line arrows in Figure 56 enters from the 2 -wire line $Z_{6}$. It is delivered via coil (2) to the input of amplifier $Z_{3}$. How this energy passes coil (2) will be explained in the following part B. The operation


## CH. 3 - REPEATING COILS AND TRANSFORMERS

of the coil (1) is explained here by means of the simplified circuit of Figure 57. Two of the windings Ll and L2 are omitted for simplicity since their presence would have no bearing on the analysis.

The output of the amplifier $\mathrm{Z}_{3}$ may be represented as a generator in series with the impedance $\mathbb{Z}_{3}$. This establishes ${ }^{4}$ a current I through $\mathrm{Z}_{3}$ and the identical coil windings $L_{5}$ and $L_{6}$, as indicated by the arrows. This induces voltages across $\mathrm{L}_{3}$ and $\mathrm{L}_{4}$ which are exactly equal in value because these two windings are also identical. The resultant current i will produce equal voltage drops across $Z_{1}$ and $Z_{2}$, because these two impedances have equal values. The potential at a is accordingly the same as at $b$ and therefore no current will flow in $\mathrm{Z}_{4}$. In other words, there is no transmission from $Z_{3}$ the output of one amplifier to $Z_{4}$, the input of the other amplifier. It may be noted, however, that only half of the energy delivered from $Z_{3}$ goes into the line $Z_{1}$, the other half being dissipated and wasted in the balancing network $\mathrm{Z}_{2}$. The amplifier must therefore be adjusted to supply twice as much energy as it is desired to feed to the line.
B. Operation of the Hybrid coil (l) for WEST to EAST Transmission Energy flowing from WEST to EAST, shown by full line arrows in Figure 56, enters from the 2 -wire line $Z_{1}$. This part of the operation is explained by means of the simplified circuit of Figure 58.


Here the signal from WEST may be represented as a generator in series with the impedance $\mathrm{Z}_{1}$. The behavior ${ }^{4}$ in this case is best followed by first assuming that $\mathrm{Z}_{2}$ is disconnected, leaving the terminals cb open. The voltage source then sets up a
currc.. $u$ gh $Z_{1}, Z_{4}$ and the winding $L_{3}$, as shown by the arrows. Th current through $\mathrm{L}_{3}$ induces a current in winding $\mathrm{L}_{5}$ which also flow ethroagh $\mathrm{Z}_{3}$ and winding L6. This in turn induces a voltage across a c which is the same in value as that across a since these two windinge are iden cal. The turn ratio of the transformer is such that the mance $Z_{3}$, which is the reflected impedance of $Z_{3}$ as seen thr gh the winding $L_{3}$, हैua? to $\mathrm{Z}_{4}$. The potential difference be. .een d and a is then equai 1 drop between $a$ and $b$ across the impedance 7 is also equal to the potential difference between a and $c$, as noted above. But the voltage across a c is opposite in direction to the drop across $Z_{4}$ so that points $c$ and $b$ are at the same potential. Therefore, no current will flow in $Z_{2}$ if it is now reconnected. This means that energy coming from the line $Z_{1}$ divides equally between $Z_{3}^{1}$ (or $Z_{3}$ ) and $Z_{4}$ and none reaches the balancing network $Z_{2}$. Again, however, only half of the incoming energy reaches the input of the amplifier $Z_{4}$ where it is useful, the other half being dissipated and wasted in the output of the other amplifier $Z_{3}$. The amplifier $Z_{4}$ must be designed to take into account this loss also.
C. Overall Operation of the Complete 2 -wire Repeater with Two Hybrid Coils
The complete WEST-EAST flow of energy will now be ey ined. It is show by full line arrows in Figure 56.

Energy of the WEST signal coming from a 2 -wire line $Z_{1}$ is split in half in the hybrid coil (1). Only one half of it reaches the input of $Z_{4}$ amplifier, the other half is dissipated and wasted in the output of $Z_{3}$ amplifier as was explained before in part B. Therefore, dis regarding the amplification required by other considerations, the $\mathrm{Z}_{4}$ amplifis: must supply at leasi twice the energy delivered to it.

From Atput of the $Z_{4}$ amplifier the signal energy is delivered to the gi coil (2). The operation of coil (2) was explained in part A fo: coil (1). Again, disregarding other consideratic: $:$ $\mathrm{Z}_{4}$ must supply wice more of the signal energy, whose flow is shown, as before, by full line arrows. One half of $Z_{4}$ output flows to EAST line $Z_{6}$ and the other half is dissipated and wasted in the balancing network $\mathrm{Z}_{5}$.

[^0]
### 3.7 FOUR-WIRE TERMINATING SETS

In Figure 59, are shown two basic methods of telephone operation, the two-wire and the four-wire. In the upper part, typical repeaters are shown using hybrid coils which are discussed in the preceding part 3.6. LPF - designates low-pass filters which are used to ease the out-of-band balancing problem, namely to provide high loss above the voice-frequency transmission band.

For the four-wire circuits, one of which is shown in the bottom part of Figure 59, a hybrid coil is often replaced by a different transformer arrangement known as, four-wire terminating set. It is shown in Figure 60. The set consists of two repeating coils \#1 and \#2 connected with one winding B reversed. A few comments regarding comparison of the two methods will be given later.

Suppose a signal is to be transmitted from line 1 EAST to the 2 -wire line WEST. The energy flow is shown by full line arrows in Figure 60. Energy from amplifier \#l is supplied to winding C of the coil \#l. Due to transformer action this energy is sent into 2 -wire WEST line and also induces a voltage in coil A. The same energy from amplifier \#l is supplied to coil D as well. It also produces, due to transformer action of coil \#2 terminated in the balancing network N , an induced voltage in coil B . If the balancing is done properly voltages (or currents) in windings $A$ and $B$ are equal and opposite and therefore cancel each other. Therefore, there will be no energy transfer between the output of amplifier \#l and the input to amplifier \#2.

Now suppose that the signal is to be transmitted from the 2 -wire line WEST to line 2 EAST. The energy flow is shown by dashed arrows in Figure 60. Energy from 2-wire line WEST produces a current in winding $A$ which also flows through $B$ winding and the input circuit of amplifier \#2. After amplification signal energy flows into line 2 EAST. Due to transformer action coil \#l produces a certain voltage in coil $C$. On the other hand current flowing through winding $B$ produces, also by transformer action of coil \#2, a voltage in winding $D$. This voltage is equal and opposite to the voltage in winding C. If the balancing is done properly voltages (or currents) in windings $C$ and $D$ will cancel. Therefore, there will be no energy transfer between 2 -wire line WEST and the output side of amplifier \#1.

As shown in the bottom part of Figure 59 at the EAST end of the 4 -wire line there is another four-wire terminating set which operates in the same way.


FIG. 59. TWO-WIRE AND FOUR-WIRE METHODS OF OPERATION.


When the operation of the two-wire circuit is compared with that of a four $\sim$ wire circuit ${ }^{2}$ it may be said that, in general, the advantage of four-wire circuit lies in the higher gain which can be obtained with one-way repeaters, both because of absence of loss in balancing circuits and the impossibility of singing in a single, properly designed, one-way repeater. Also, as will be described later, possibilities for echoes are greatly reduced. Thus because of higher gain the number of repeater points may be reduced, or the size of the conductor may be reduced, or both. Most long-cable circuits operate on the principle of four-wire circuit.

The four wire terminating sets work up to about $4,000 \mathrm{cps}$ whereas the hybrid coils are used for much higher frequencies. The hybrid principle has been successfully used for operation in the thousand megacycle range. Thus it can be said that hybrid coils may be used for broad-band operation, they have to be manufactured in a much more precise manner and therefore they are more expensive than the four-wire terminating sets.

## References for Chapter 3

2. From Everitt \& Anner's COMMUNICATION ENGINEERING, 3 ed. Copyright 1956 McGraw-Hill, Inc. Used by permission of McGraw-Hill,Book Company.
3. (c) American Telephone and Telegraph Company, 1961.
4. Standards for noise measurements. American Standards,Assn., publication Z24.2, February, 1936.

## 4. 1 GENERAL

A transmission line is used to conduct or guide electrical energy from one point to another. In long distance telephony the telephone transmitter is the source of energy; the line from one end to another, including its parts such as conductors, coils and connections, may be regarded as the medium of transmission; and the telephone receiver at the other end may be considered to be the third part of the transmission system, or the device which converts electric currents into audible vibrations of air, called sound waves.

There are several types of transmission lines in use. One of them is the open-wire line. The telephone line strung on crossarms on poles is an open-wire line.

Another type is the cable. ${ }^{3}$ For telephone use it consists of many individually insulated conductors, twisted in pairs, and combined inside a lead or plastic sheath.

Still other forms of construction are employed in the coaxial line in which one conductor is a holler tube, the other being located inside and coaxial with the tube. $\mathrm{I}_{\mathrm{c}}$. dielectric may be solid or gaseous, but if the cable is gas-filled, occasional solid dielectric disks are employed to maintain accurate spacing and location of the central wire. Coaxial cables in present practice ${ }^{4}$ may be manufactured to include 2, 4, 6 or 8 tubes and in all cases may also include a number of ordinary wire pairs or quads.

Our study here will concern itself with an analysis of the characteristics of a transmission line consisting of simple openwire conductors at relatively low frequencies up to 150 kc .

A transmission line has certain characteristics which combine to form a "line impedance." If a uniform line of this type is made infinitely long, all of the energy applied to the sending end will be transmitted along the line and progressively absorbed. Such a uniform line has constant impedance along its length, called $\mathrm{Z}_{0}$. In other words, the impedance determined by dividing the voltage by the current at any point in the line will be the same. If such a line is cut (that is made finite in length) and then terminated
in this $Z_{0}$ or "characteristic" impedance, it will behave in the same fashion as a ine of infinite length. This is the type of line which is normally used in telephone plant for the transmission of voice frequency currents.

## 4. 2 EQUIVALENT CIRCUIT

In our discussion of networks in paragr h 2.43 it was found that any 2 -port passive network composed of linear, bilateral impedances can be represented by a T (or $\mathrm{Pi}_{\mathrm{i}}$ network. Since a transmission line is a passive network and, as will be seen later, corresponds to the above definition its equivalent T-network can and will be developed hereafter.

Basically, a transmission line is a pair of conductors uniformly spaced and extending for a considerable distance. These conductors have a total resistance R. And since the two conductors are terminated at both ends they form a one turn coil and, therefore, have an inductance L. If we wish to construct a lumped constant network that is electrically equivalent to such line, we must start with a resistor ( $R$ ) and an inductor (L) connected in series. But there are other factors to consider. The insulation between the line wires is never perfect; there is some leakage between them. The leakage resistance may be very large, as in a cable, or it may be fairly small in the case of a wet open-w. e lead. This leakage resistance is denoted by a conductance ( $G$ ) measured in mhos per unit length of line. (G) is the reciprocal of the leakage resistance (bit not of $R$ ). The quantity ( $R$ ) obviously represents the imperfection of the conductor, while (G) represents the imperfection of the insulation. Thus our equivalent circuit must contain a resistor having a conductance (G) in shunt between the line conductors. Any two conductors separated by an insulator across which a potential difference exists have the properties of a capacitor. So our circuit must have a capacitor (C) in shunt. Finally, if as usual, a line appears the same (electrically) when viewed from either end, our equivalent circuit must also be symmetrical.

On the basis of preceeding discussion we can represent the equivalent circuit for a transmission line as the $T$-network shown in Figure 61. R, L, C, and G are called the "primary constants" of a transmission line. For convenience, a can lump the series constants $R$ and $L$ into an impedance $Z_{1}$ and th ohunt constants $C$ anc $\mathfrak{x}$ into another impedance $Z_{2}$ as has been ac.e in Figure 61B.


It is obvious from Figure 61 that $Z_{1}=R+j \omega L$ and

$$
z_{2}=\frac{1}{G+j \omega C}
$$

The equivalent circuit in Figue 6lB is a poor approximation of a real physical line, for we have lumped all of the distributed constants at one point namely the center of the line. All of the leakage, for instance, does not occur at the center of the line, and the same is true for other primary constants. We would improve the approximation by having two T-sections in series, each containing a half of the line constants, but the best ideal approximation would be to have an infinite number of T -sections, each having the constants of an infinitely short section of the real, physical line.

## 4. 3 CHARACTERISTIC IMPEDANCE

Let us simulate an infinitely long line with an infinite number of identical recurrent $T$-networks as suggested by Figure 62. If we measure the impedance looking into the terminals $a-b$, we shall obtain a value $Z_{0}$. Suppose now that we open the line at $\mathrm{c}-\mathrm{d}$ and again measure the impedance of the line looking to the right. The effect of removing one section from an infinite number of sections is negligible, so we would still measure $Z_{0}$. This


FIGURE 62.UNIFORM INFINITE LINE SIMULATED BY A LARGE NUMBER OF IDENTICAL NETMORKS
impedance, $Z_{0}$, is called the characteristic impedance of the network. It may be thought of as the impedance of a line if that line were of infinite length. A "long line," while not of infinite length, does for all practical purposes exhibit the characte tics of an infinite line and an impedance $Z_{0}$.

The impedance $a t a-b$ was $Z_{0}$ when the first section was connected to an infinite line presenting an impedance $Z_{o}$ at $c-d$. Therefore, if we terminate the first section at c-d with a lumped impedance having a value of $Z_{0}$, we shall still measure $Z_{o}$ looking to the right at $\mathrm{a}-\mathrm{b} . \mathrm{Z}_{\mathrm{O}}$ depends only ${ }^{4}$ on the impedance values of the unit sections of the network; this is obvious from the fact that those were the on: impedance values used in determining it. The first section between $\mathrm{a}-\mathrm{b}$ and $\mathrm{c}-\mathrm{d}$, terminated in $\mathrm{Z}_{\mathrm{O}}$ is shown in Figure 63.


FIGURE 63. LINE TERMINATED IN CHARACTERISTIC IMPEDANCE

$$
\begin{align*}
& z_{o}=\frac{z_{1}}{2}+\frac{z_{2}\left(\frac{z_{1}}{2}+z_{0}\right)}{\frac{z_{1}}{2}+Z_{2}+z_{0}} \\
& \frac{\mathrm{Z}_{\mathrm{o}} \mathrm{Z}_{1}}{2}+\mathrm{Z}_{\mathrm{o}} \mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{o}}^{2}=\frac{\mathrm{Z}_{1}^{2}}{4}+\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}}{2}+\frac{\mathrm{Z}_{\mathrm{o}} \mathrm{Z}_{1}}{2}+\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}}{2}+\mathrm{Z}_{\mathrm{o}} \mathrm{Z}_{2} \\
& \mathrm{Z}_{\mathrm{o}}^{2}=\frac{\mathrm{Z}_{1}^{2}}{4}+\mathrm{Z}_{1} \mathrm{Z}_{2} \\
& z_{o}=\sqrt{\frac{z_{1}^{2}}{4}}+Z_{1} z_{2} \tag{1}
\end{align*}
$$

As has already been pointed out, the more identical Tsections that are used in the multisection network to represent a given length of transmission line, the more closely does the network approximate the line. If we take a transmission line having a series impedance. $Z_{1}$ per mile and a shunt (parallel) impedance $Z_{2}$ per mile and represent each mile by $n T$-network sections, the impedance of each series $T-a r m$ will be $\mathrm{Z}_{1} / 2 n$ and of each shunt
arm will be $\mathrm{nZ}_{2}$. The characteristic impedance of the network may then britten from the last equation as

$$
\mathrm{z}_{\mathrm{o}}=\sqrt{\frac{\mathrm{z}_{1}^{2}}{4 \mathrm{n}^{2}}+\frac{\mathrm{z}_{1}}{\mathrm{n}} \mathrm{n} \mathrm{Z}_{2}}
$$

From this it may be seen that as the number $r$ f sections is increased to a very large number, the first texm under the radical approaches zero and the equation reduces to

$$
\begin{equation*}
z_{o}=\sqrt{Z_{1} Z_{2}} \tag{2}
\end{equation*}
$$

Actually the use of higher level mathematics will show that this is the exact expression for the characteristic impedance of a uniform transmission line.

The impedance $Z_{1}$ contains an inductance and $Z_{2}$ a capacitance. Since the reactances of inductor and capacitor change with frequency, the magnitude and the phase angle of $Z_{0}$ of a transmission line also change with frequency. This property must be recognized when selecting a network which is to terminate a line in its characteristic impedance $Z_{o}$ over a band of frequencies.

It is sometimes necessary to determine the characteristic impedance $Z_{o}$ by performing measurements on an actual physical transmission line. This can be done easily if we have a telephone circuit terminating at the same end points as the transmission line to be tested. Now testing can be accomplished at both the near and far end of the transmission line based on instructions send over the second or parallel line. First impedance $Z_{s c}$ is measured at the sending end with the conductors strapped (shorted) at the receiving end. Thereafter a second measurement is made of the impedance $\mathrm{Z}_{\mathrm{oc}}$ at the sending end; during this the two conduators at the receiving unt are kept open-circuited.

A formula will be developed now for the calculation of $Z_{0}$ after the two impedances $\mathrm{Z}_{\mathrm{SC}}$ and $\mathrm{Z}_{\mathrm{Oc}}$ have been measured. Figu: 63 is used for this purpose.

If the right side of the circuit sho - in that Figure is open-circuited, instead of being terminated a ' $\quad$, the impedance $a \quad-B$ is

$$
\mathrm{Z}_{\mathrm{oc}}=\frac{\mathrm{Z}_{1}}{2}+\mathrm{Z}_{2}
$$

And if the right side is short-circuited, the impedance a.t $\mathrm{A}-\mathrm{B}$ is

$$
\mathrm{z}_{\mathrm{sc}}=\frac{\mathrm{Z}_{1}}{2}+\frac{\mathrm{Z}_{2} \cdot \frac{\mathrm{Z}_{1}}{2}}{\mathrm{Z}_{2}+\frac{\mathrm{Z}_{1}}{2}}
$$

The product of $\mathrm{Z}_{\mathrm{oc}}$ and $\mathrm{Z}_{\mathrm{Sc}}$ will now be developed:

$$
\begin{aligned}
\mathrm{Z}_{\mathrm{oc}} \mathrm{Z}_{\mathrm{Sc}} & =\left(\frac{\mathrm{Z}_{1}}{2}+\mathrm{Z}_{2}\right)\left(\frac{\mathrm{Z}_{1}}{2}+\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}}{2 \mathrm{Z}_{2}+\mathrm{Z}_{1}}\right)= \\
& =\frac{\mathrm{Z}_{1}^{2}}{4}+\frac{\mathrm{Z}_{1}^{2} \mathrm{Z}_{2}}{2\left(\mathrm{Z}_{1}+2 \mathrm{Z}_{2}\right)}+\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}}{2}+\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}^{2}}{\mathrm{Z}_{1}+2 \mathrm{Z}_{2}}= \\
& =\frac{\mathrm{Z}_{1}^{2}}{4}+\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}}{2}+\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}}{\mathrm{Z}_{1}+2 \mathrm{Z}_{2}}\left(\frac{\mathrm{Z}_{1}}{2}+\mathrm{Z}_{2}\right)= \\
& =\frac{\mathrm{Z}_{1}^{2}}{4}+\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}}{2}+\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}}{2}\left(\frac{\mathrm{Z}_{1}+2 \mathrm{Z}_{2}}{\mathrm{Z}_{1}+2 \mathrm{Z}_{2}}\right)=\frac{\mathrm{Z}_{1}^{2}}{4}+\mathrm{Z}_{1} \mathrm{Z}_{2}
\end{aligned}
$$

If now a square root of the final development of the equation is taken, the following is obtained

$$
\sqrt{\mathrm{Z}_{\mathrm{oc}} \mathrm{Z}_{\mathrm{sc}}}=\sqrt{\frac{\mathrm{Z}_{1}^{2}}{4}}+\mathrm{Z}_{1} \mathrm{Z}_{2}
$$

$$
\begin{equation*}
\mathrm{z}_{\mathrm{o}}=\sqrt{\mathrm{Z}_{\mathrm{oc}} \mathrm{Z}_{\mathrm{sc}}} \tag{3}
\end{equation*}
$$

Using two formulas $Z_{1}=R_{1}+j \omega L$ and $Z_{2}=\begin{gathered}1 \\ G+j \omega C\end{gathered}$

$$
\begin{equation*}
Z_{o}=\sqrt{Z_{1} Z_{2}}=\sqrt{\frac{R+j \omega L}{G+j \omega C}} \tag{4}
\end{equation*}
$$

Here $R$ is resistance in ohms ${ }^{4}$ per unit length of line; $L$ is inductance in henries; $G$ is conductance in mhos; $C$ is capacitance in farads.

As the above equation shows, the value of characteristic impedance is dependent solely on the primary constants of the line and the frequency. At any given frequency, therefore, this impedance has a fixed value for any given type of line regardless of the length of the line or what may be connected to the line terminal. One of its most useful applications in practical telephone work lies in the fact that receiving devices to be connected to a line may be designed with impedance equal to the characteristic impedance $Z_{o}$ of the line, thus permitting a maximum transfer of power out of the line.

## 4. 4 PROPAGA.TION CONSTANT

We have seen that a line of infinite length (or of finite length when terminated in $\mathrm{Z}_{0}$ ) has a special property; at every point along the line, its impedance is the characteristic impedance, $Z_{0}$. Now impedance is just another way of saying voltage divided by current ( $\mathrm{E} / \mathrm{I}$ ). Therefore, for Figure 62, we can write the equation:

$$
\frac{\mathrm{Eab}}{\mathrm{I}_{\mathrm{a}}}=\frac{\mathrm{Ecd}}{\mathrm{I}_{\mathrm{c}}}=\frac{\text { Eef }}{\mathrm{I}_{\mathrm{e}}}=\frac{\mathrm{Egh}}{\mathrm{I}_{\mathrm{g}}}=\ldots . .=\mathrm{Z}_{\mathrm{o}}
$$

Bear in mind that the voltage and current are not constant as we proceed down the line, only their ratio is constant. In fact, voltage and current decrease. This follows from Kirchoff²s Laws. The voltage $\mathrm{E}_{\mathrm{cd}}$ is less than $\mathrm{E}_{\mathrm{ab}}$ by the voltage drop across $\mathrm{Z}_{1}$, and the current $I_{c}$ is less than $I_{a}$ by the current shunted :" sugh $\mathrm{Z}_{2}$. These effects result in only a f . on of the signal en ering the line at $a-b$ being passed on to the next section of line com Suppose that the energy reachin. $\cdots$ is $1 / 2$ of that supplied $a t a-b$ Since the line constants are unitozmly distributed alc.g the line, the energy reaching e-f will be $1 / 2$ of that at $c-d$. So the
signal at e-f will be $1 / 2$ of $1 / 2$ or $1 / 4$, of the original signal sent at a-b. This decrease in energy, as a signal is propagated down a line, is called "attenuation." Our main interest in the line will be to know the attenuation. We will also be interested in how long it will take the applied voltage, current or energy to reach the receiving device.

Referring now again to Figure 62 let us assume, as before, that all the small T-sections are identical. Therefore, we can write the following equation.

$$
\frac{I_{c}}{I_{\mathrm{a}}}=\frac{\mathrm{I}_{\mathrm{e}}}{\mathrm{I}_{\mathrm{c}}}=\frac{\mathrm{I}_{\mathrm{g}}}{\mathrm{I}_{\mathrm{e}}}=\ldots .=\frac{\mathrm{I}_{\mathrm{n}}}{\mathrm{I}_{\mathrm{n}-1}}=\text { constant }
$$

We can also write another equation, obviously true

$$
\frac{I_{n}}{I_{a}}=\frac{I_{c}}{I_{a}} \times \frac{I_{e}}{I_{c}} \times \frac{I_{g}}{I_{e}} \times \ldots . x \frac{I_{n}}{I_{n-1}}=\text { constant }
$$

The last equation can be represented in another form

$$
\frac{I_{n}}{I_{a}}=\left(\frac{I_{n}}{I_{n-1}}\right)^{n} \quad \text { Taking the logarithms to the }
$$

base of "e" of both sides of the equation we obtain

$$
\ln \frac{I_{n}}{I_{a}}=n \ln \frac{I_{n}}{I_{n}-1}
$$

The propagation constant $\gamma$ is defined by the equation

$$
\begin{equation*}
\gamma=-\ln \frac{I_{n}}{I_{n-1}} \tag{5}
\end{equation*}
$$

Because $\mathrm{I}_{\mathrm{n}}<\mathrm{I}_{\mathrm{n}-1}$ the logarithm is negative and
thus $\gamma$ is positive. When we consider that $I_{a}$ is the input current and $I_{n}$ is the output current we can use a different pair of symbols namely $I_{a}=I_{\text {in }}$ and $I_{\mathrm{n}}=\mathrm{I}_{\text {out }}$.

Thus we may write the following equation

$$
\ln \frac{I_{\text {out }}}{I_{\text {in }}}=\ln \frac{I_{n}}{I_{a}}=n \ln \frac{I_{n}}{I_{n-1}}=-\gamma n
$$

Using the well known formula for division of total current into two pas... ' S hrunch currents we may write the following equation using circuit shown in Figure 63.

$$
\frac{I_{\text {out }}}{I_{\text {in }}}=\frac{I_{n}}{I_{\mathrm{a}}}=\frac{\mathrm{Z}_{2}}{\mathrm{Z}_{2}+\frac{\mathrm{Z}_{1}}{2}+Z_{o}} \text { from which } \gamma \text { can be }
$$

calculated as follows

$$
\gamma=-\frac{1}{\mathrm{n}} \ln \frac{\mathrm{I}_{\text {out }}}{\mathrm{I}_{\text {in }}}=+\frac{1}{\mathrm{n}} \ln \frac{\mathrm{I}_{\text {in }}}{\mathrm{I}_{\text {out }}} .
$$

By expansion as the power series and by means of binomial series* the following expressions may be obtained:

$$
\begin{equation*}
\gamma=\frac{1}{\mathrm{n}} \ln \frac{\mathrm{I}_{\text {in }}}{\mathrm{I}_{\text {out }}}=\sqrt{\frac{\mathrm{Z}_{1}}{\mathrm{Z}_{2}}}=\sqrt{(R+j \omega \mathrm{~L})(\mathrm{G}+\mathrm{j} \omega \mathrm{C})} \tag{6}
\end{equation*}
$$

From this formula it can be seen that $\gamma$ is a vector qu "ity because it is equal to $\sqrt{Z_{1} / Z_{2}}$, and both $Z_{1}$ and $Z_{2}$ are vector quantities and functions of fred ency $\omega$. Both $Z_{1}$ and $Z_{2}$ are composed of resistive elements $R$ and $G$ which dissipate power, and reactive elements $\omega L$ and $\omega C$, which cause a phase shift in time.

The propagation constant $\gamma$ may be separated into a "real" and "imaginary" components and therefore, represented as

$$
\begin{equation*}
\gamma=\alpha+\mathfrak{j} \beta \tag{7}
\end{equation*}
$$

$\alpha$ is called "attenuation constant" and is the real part of $\gamma$, and $\beta$ is called "phase constant" and is the imaginary component.
a. The "attenuation constant" 11 s what fraction of the current entering a ser ion of line is passed on to the next section. In other words, it is a
*See Everitt \& Anner, Communication Engineering, 3rd edition p. 301.
measure of the rate of decay of the current transmitted along the transmission line. Since the impedance is the same at all points along a uniform line, the attenuation constant $\alpha$ is also a measure of the difference in power at the two ends of each section of line. Attenuation is calculated in nepers per mile, but often for practical application the result is converted to "db per mile." One $d b=8.686$ nepers.
b. The "phase constant" $\beta$ gives directly in radians the phase shifi in current or voltage over a unit length of line such as one mile. A radian is $360 / 2 \pi$ or 57.3 degrees. For our purposes, we will consider later the phase constant in the more familiar form of "velocity." Velocity $V$ can be expressed in "miles per second." Theoretically the maximum possible velocity of propagation would be the speed of light or radio waves in free space, which is 186,000 miles per second.

Example. Consider a typical open-wire telephone line which has $R=10$ ohms/mile, $L=0.0037$ henry/mile, $C=0.0083 \mathrm{x}$ $10^{-6}$ farad $/ \mathrm{mile}$, and $G=0.4 \times 10^{-6} \mathrm{mho} / \mathrm{mile}$. At a frequency of 1, 000 cps , we have from Eq. (4):

$$
\begin{aligned}
& \quad Z_{o}=\sqrt{\frac{R+j \omega L}{G+j \omega C}}=\sqrt{\frac{10+j 23.2}{(0.4+j 52.1) \times 10^{-6}}} \\
& =\sqrt{\frac{25.3 \angle 66.8^{\circ}}{52.1 \times 10^{-6} \angle 89.6^{\circ}}}=\sqrt{48.5 \times 10^{4} \angle-22.8^{\circ}} \\
& =697 \angle-11.4^{\circ}=683-\mathrm{j} 138 \mathrm{ohms}
\end{aligned}
$$

From Eq. (6) the propagation constant at this frequency is

$$
\begin{aligned}
\gamma & =\sqrt{(R+j \omega L)(G+j \omega C)}=\sqrt{\left(25.3 / 66.8^{\circ}\right)\left(52.1 \times 10^{-6} / 89.6^{\circ}\right)} \\
& =\sqrt{13.2 \times 10^{-4} \angle 156.4^{\circ}}= \\
& =0.0363 \angle 78.2^{\circ}=(0.0074+j 0.0356) \mathrm{p} \quad \text { mile }
\end{aligned}
$$

Therefore, the attenuation constant is

$$
\alpha=0.0074 \text { nepers } / \mathrm{mile}
$$

and the phase constant is

$$
\beta=0.0356 \mathrm{rad} / \mathrm{mile}
$$

It should be noted ${ }^{4}$ that while $\beta$ represents an angle of phase shift, it should not be confused with any phase angle which may exist between the current and voltage at a particular point on a line. In practice it is usually easier to evaluate $\alpha$ and $\beta$ by making use of equations (6) and (7). They also may be expr sed directly in terms of the primary constants $-R, L, G$ and $C$, as follows:

$$
\begin{align*}
& \alpha=\sqrt{\frac{1}{2} \sqrt{\left(R^{2}+\omega^{2} L^{2}\right)\left(G^{2}+\omega^{2} C^{2}\right)}+\frac{1}{2}\left(G R-\omega^{2} L C\right)} \\
& \beta=\sqrt{\frac{1}{2} \sqrt{\left(R^{2}+\omega^{2} L^{2}\right)\left(G^{2}+\omega^{2} C^{2}\right)}-\frac{1}{2}\left(G R-\omega^{2} L C\right)} \tag{9}
\end{align*}
$$

In the foregoing discussion ${ }^{4}$ we considered for the most part the mathematical significance of $\alpha$ ain . Let us now analyze the physical circuit to determine what ually happens as th. urrent is sent from point to point. In o: ce to simplify the ena ysis, we shall start with an actual cyci of emf impressed on tice er ling and of multisection netwos: nd consider separalsly the effecti of inductance and capacitance on the propagation.


FIG. 64. VOLTAGES AND CURRENT IN AN INDUCTIVE CIRCUIT

From our previous study, we know that inductance acts to cause the current I to lag behind the impressed voltage $E$, so that in a circuit made up of series connected resistance $R$ and inductance L we would expect a lagging currer". Figure 64 shows the time relationship between voltages and current in such a circuit, where $E$ is the curve of impressed voltage, and I the current curve. This current sets up a back or induced emf $E_{l}$, which is the sum of the IR drop across the resistance and the IX drop across the inductance L. It combines with the original impressed voltage E to give the resultant voltage $E_{R}$ across the load. The curve $E_{R}$ is obtained by adding $E$ and $E_{1}$ and it will be observed that the resulting curve $E_{R}$ lags behind E , the original impressed voltage.

A circuit containing ${ }^{4}$ resistance $R$ and capacitance $C$, on the other hand, produces a leading current I as shown in Figure 65, and this current produces an IR drop which is trying to oppose the flow of the current I. Now if we combine the IR drop and the voltage $E$, we obtain the resultant voltage $E_{R}$, which exists across both the capacitor $C$ and the load. This voltage $E_{R}$ likewise lags behind the original voltage $E$.


FIg. 65. VOLTAGES AND CURRENT IN A CAPACITIVE CIRCUIT

In both cases we have obtained a resultant voltage $\mathrm{E}_{\mathrm{R}}$ which lags behind the impressed voltage $E$. Bridged capacitance $C$ assists series inductance $L$ in the phase retarding effect in a circuit containing $L$ instead of $R$ in Fig. 65. Due to the presence of reactance, herefore, the voltage $E_{R}$ has been "delayed in time" behind $E$, so tait the maximum of voltage $E_{R}$ reaches any given point along the network later than it would if the reactances were not present.

Each section of the network representing a transmission line absorbs energy and thereby reduces the voltage which can act on the next section due to resistance $R$ and leakage G. Further, the voltage avalable at the next section $\mathrm{E}_{\mathrm{R}}$ lags behind the voltage E impressed on the section, so that as we move away from the generator, the acting voltages $E_{R^{\prime}}$ are lagging farther and farther behind the generator voltage E. Here we have a connecting link between time and geographical distance traveled along the line.

To bring this out clearly, ${ }^{4}$ let us assume that we take our T sections of such a length that, for a frequen of 1000 cycles, the tim lag between voltages $E$ and $\mathrm{E}_{\mathrm{R}}$ can be re ted by 30 degree. per section on the time-voltage diagram. in we simulate by suth section fourteen and three-quarters m1 s of 104 open wire circuit, will obtain such a relationship. In order to make the story complete, we will also assume the reduction in voltage magnitude due to resistance and leakage loss to be such as to give a
ratio of 0.895 between the end and beginning of each section. If we as sume the original voltage $E_{o}$ to be 10 volts, the voltage at the end of the first section, $E_{1}$, will be 8.95 volts, lagging $30^{\circ}$ behind $E_{o}$. $\mathrm{E}_{2}$, at the end of the second section, will be $0.895 \times 8.95$ or 8.01 volts, lagging $30^{\circ}$ behind $E_{1}$ or 600 behind $E_{0}$. If we represent the voltages $\mathrm{E}_{\mathrm{n}}$ at various points by vectors, we will obtain a system of vectors as shown in Figure 66 (B), where the multisection network is shown as Figure 66 (A) and the voltage acting at each junction is directly below.


FIGURE 66. TRANSMITTED CURRENTS AND VOLTAGES AT JUNCTIONS OF A MULTISECTION UNIFORM NETWORK

Since the ratio of voltage to current is constant, ${ }^{4}$ at each frequency, and equal to $\mathrm{Z}_{0}$ it follows that the chart representing currents $I_{n}$ will have the same form, with each vector proportional to $\mathrm{E}_{\mathrm{n}}$ and shifted by an angle $\Theta$ from the corresponding voltage vector $E_{n}$, where $\Theta$ is the angle of the characteristic impedance $Z_{o}$. Thus we may treat a similar figure such as 66 (C) as a "distance-current diagram" where the vectors, $I_{0}, I_{1}, I_{2}$, etc., show the magnitude and relative phase of the currents at the network junctions. If now
we referen the current $I_{n}$ vectors to a common reference point $G$, we will obtain a broken curve such as that of Figure 67 (A), which shows graphically how the currents at various points are related. In this Figure the vector $I_{0}$, shown as $G-0$, is the current entering the first section and $I_{1}=G-1$, the current leavirg that section. Then the vector l-0 must be the current $I_{1}^{\%}$ that asses through the shunt $Z_{2}$ in the first section, because the sum o the current $I_{1}$ through the shunt and the current $I_{1}$ going ahead gives G-0 or $I_{O}$ as the resultaint of the vector diagram. This is perhaps more clearly illustrated by Figure 67(B). For the same reason 2-1 will be the current passing through the second shunt, etc.

We may, therefore, conceive of the total entering, current $I_{0}$ as the resultant of a number $n$ of component currents $I_{n}$ which flow from the generator through the various shunt paths $Z_{2}^{n}$ and back to the generator, each component of a different magnitude and


CIGIRE 67. POLAR DIAGRAM OF THE VECTORS $I_{n}$ AND I' ${ }_{n}^{\prime}$ ' FIGURE 66. (C)
phase. The effect of these components can observed, since at certain junctions the line current is flowing in opposite direction to taken by the entering current $I_{0}$; at othe ints there is a $90^{\circ}$ ase difference between the two; and a\% other points there St ase difference. In other words, urrent vector $I_{n}$ may be consile ed as moving about center $G$, rotating clockwise through 300 for every section traversed and diminishing in value about $10 \%$ in each section.

Figures 66 and 67 show 4 the effective values of the current at certain points along the line and their phase positions with respect to vector $I_{0}$ assumed to be in the initial horizontal position. These diagrams are independent of time, i.e., they are applicable at any and all times. If on the other hand, we select a given instant of time and plot the instantaneous values of the current at some points along the line, we obtain the curve shown in Figure 68 which shows clearly how the current reverses in direction as it passes through the various sections. It also shows how the current decreases in value or is attenuated as it passes along the line. It will be noted that at the selected instant $(t=0)$ shown here the input current is at a maximum value $I_{0}$ flowing in a positive direction; at a point 3 , three sections along the line, zero current is flowing; a.t a point 6 , six sections along the line, maximum current is flowing in a direction opposite to that of the current at the input end; at nine sections there is again zero current while at 12 sections a maximum positive current is flowing.

If it is kept in mind that since all current (or voltage) vectors rotate with the same angular velocity counterclockwise it is possible to draw a series of plots of the instantaneous values of currents at subsequent instants. Figure 69 shows the plot of


FIG. 68. INSTANTANEOUS CURRENT ALONG UNIFORM TRANSMISSION LINE
current $I_{n}$ one twelfth of a cycle later than Figure 68. If more such plots were drawn at several instants, each of which would be one twelfth of a cycle later than the preceding one, it would become
obvious that the wave shown in Figure 68 is moving to the right; this indicates a current or voltage wave traveling down the line starting from the generator.


TRANSMISSION LINE ONE TWELFTH OF A CYCLE LATER THAN IN FIG. 68.

The distance between adjacent points 4 where maximum positive (or negative) current is flowing is known as the wavelength and is represented by the symbol $\lambda$. The time required to set up one wavelength along the line is equal to the time required for the impressed voltage to complete one cycle. The velocity of propagation $V$ of the energy along the line is therefore equal to the wavelength divided by the time required to establish the wave, which is the reciprocal of the applied frequency; thus since $T=\frac{l}{f}$.

$$
V=\frac{\lambda}{1 / \mathrm{f}}=\mathrm{f} \lambda
$$

This velocity lin telephone circuits may range in value from as little as 10,000 miles per second to a maximum approaching the speed of light in a vacuum, some 186,000 miles per second.

The value of the wavelength $\lambda$ is readirnetermined from the alue of $B$. which, as we have seen, depen a primary constants of the line and the frequency. $\beta$ gives the phase shift in adians or delay per unit length of line. The total phase shift for one waverngth is obviously $360^{\circ}$ and since $360^{\circ}$ equals 2 radians we may write the equation.

$$
\lambda=\frac{2 \pi}{\beta}
$$

The equation for velocity may then be rewritten

$$
\mathrm{V}=\frac{2 \pi \mathrm{f}}{\beta}=\frac{\omega}{\beta}
$$

Constants $\alpha, \beta$ and V have been determined for the common types of transmission lines at typical operating frequencies. A few typical values are shown in Table 2.

TABLE 2

## Attenuation and Velocity of Typical Facilities

|  | Attenuation $\alpha$ <br> db per mile | Velocity V <br> miles per second |
| :---: | :---: | :---: |
| 104 mil Copper Open Wire | 0.067 |  |
| 1000 cycles |  | 176,000 |
| 140 kc | 0.308 | 183,000 |
| 19 gauge Toll Cable |  |  |
| 1000 cycles | 1.06 | 47,200 |
| 60 kc | 3.78 | 124,000 |
| 150 kc | 6.02 | 129,000 |
| 19 gauge H88 loaded Cable |  |  |
| 1000 cycles | 0.36 | 14,300 |

You will note in Table 2 that attenuation and velocity are different for each type of facility. This is reasonable when we consider that the propagation constant is determined by the $R, L, C$, and $G$ of a line. Attenuation and velocity also vary with frequency, since the effect on current flow offered by $L$ and $C$ depends on the frequency of the signal. In general the higher the frequency, the greater are both the attenuation $\alpha$. and the greater the velocity V .

The variation of attenuation and velocity with frequency results in two types of distortion:
a. Frequency distortion - Signals of different frequencies suffer different amounts of attenuation $\alpha$ in traversing the line. Hence, they will be received at different relative volume levels.

> h. $\frac{\text { Delay (or phase) distortion - Having different }}{\text { velocities } V \text {, signals of different frequencies which }}$ are transmitted simultaneously will not be received at the same time at the receiving end of the line.

## 4. 5 FACTORS AFFECTING ATTENUATION

Although the attenuation constants $\dot{\alpha}$ expressed in db per mile (as shown in Table 2) are not impressive numerically, they can add up to tremendous losses in transmission over practical distances. Remember that each section of line (say one mile) reduces the signal transmitted by an equal proportion - not an equal amount. One of the great challenges in the development of telephony has been getting a usable amount of signal to the far end of a transmission line. The strong arm approach is to put a large enough signal into the sending end of the line, but very often this is not a realistic solution.

## Example

Consider a 19 gauge cable with a loss of about l db per mile. The frequency of signal is 1 kc . Thirty miles would have a loss of 30 db . Expressed as a power ratio this would be $1 / 1000$. To receive one microwatt, we would have to send 1000 microwatis or 1 milliwatt. This is not unreasonable. Now suppose that the line is a hundred miles long. The attenuation is 100 db , and we must send ten billion times the power we wish to receive. Ten billion microwatts is 10,000 watts. That's enough power to run an electric stove, which is usually hooked up with \#6 or \#8 wire. Obviously, this is not the way to run a telephone system.

The obvious error in this example is that we have chosen the wrong facility for our 100 mile line. Table 2 shows that the attenuation of 104 copper open wire is 0.067 db per mile. Had we used this type of construction, the line would have a loss of 6.7 db . This would be a power ratio of $1 / 4.7$, or we would have had to send only 4.7 microwatts to receive one microwatt. Electrically, the 104 mil open wire and 10 gauge toll cable pair in our example diffe only in the values of their primary constants. These values are compared in Table 3.

TABLE 3
Line constants of 19 ga. Toll Cable and 104 mil Copper Open Wire at 1000 Cycles

Ratio
Units $19 \mathrm{ga} . \quad 104 \mathrm{cu} . \quad$ Cable/ $\frac{\text { perloop mile }}{\text { (2 way) }} \frac{\text { Toll Cable }}{}$ Open Wire Open Wire

Attenuation Constant $\dot{\alpha}$
db
1.06
0.067
15. 8

Resistance R
ohms
84.0
10.0
8.4

Inductance L Capacitance C
millihenrys

1. 13
2. 66
0.3

Leakage Resistance $\frac{1}{\mathrm{G}}$ megohms
0.062
0.0085
7.3

When current passes through a resistance, a portion of the electrical energy is converted into heat. This heat is dissipated, and the energy is lost to the electrical circuit. In the table, the higher resistance 19 ga. cable has a greater attenuation than the lower resistance 104 mil open wire. The relation between line inductance $L$ and attenuation constant $\alpha$ is a little tricky. It is part of the story of loading, and is a subject by itself. It will be discussed later.

Line capacitance C provides a shunt path between the conductors for alternating currents. All currents taking this path return to the sending end of the line without being passed on to the next section. Therefore, capacitance contributes to attenuation. In Table 3, we note that the $0.062 \mathrm{mfd} / \mathrm{mi}$ cable has more attenuation than the $0.0085 \mathrm{mfd} / \mathrm{mi}$ open wire. Line capacitance is primarily responsible for attenuation increasing with frequency. Leakage also constitutes a shunt on the line, but the leakage comparison in Table 3 is a little deceptive. Under normal conditions the present techniques hold leakage resistance to such high values that it has little effect on line loss. However, the reduction of leakage resistance can materially increase attenuation. "Wet cable" and "trouble" are synonymous. Rain increases the attenuation of open wire, but not to the same degree that water affects this loss in cable.

To summarize, if a line is to have low attenuation, it must have:
a. Low series resistance $R$
b. Low capacitance $C$, and
c. High shunt leakage resistance, equal $\frac{1}{G}$

## 4. 6 WAVF PROPAGATION

Having examined some mathematical and quantitative aspects of the long transmission line, it may be proper to consider some of the qualitative effects that we might expect to occur. Figure 70 shows the sine waves of voltage and c rrent in-phase or out-of-phase that exist on an infinite line being $f d$ from a sine-wave generator. The vol tage maxima and current maxima appear at the same points on this line but, because of attenuation, the curves keep diminishing in amplitude down the line. Figure 70 could only be obtained if the voltage and current could be "frozen" at a particular instant in time. An instant later all waves would have moved to the right slightly. Waves exist because it takes certain amount of time for electrons to transfer emergy along a wire by means of their motion.

Electrical waves normally travel at approximately the speed of light which is 186,300 miles per second equal to $300,000,000$ meters per second. Thus, if a source were producing a voltage at $30,000,000$ cycles per second and this voltage were applied to a transmission line, a positive voltage or current peak would travel down the line only ten meters (a little less than 33 feet) before another positive peak would be applied to the end of the line by the transmitter. The distance which the waves move during one ycle of the alternating voltage is known as the wave length. At 3,000 megacycles per second the distance is about four inches; at 186, 000 cycles almost one mile, and at 1,000 cycles 186 miles. At commercial power frequencies of 60 cycles, 3,100 miles would be traversed during one cycle. A comparison of these waves is shown in Figure 71. At a frequency of 1,000 cycles we would find that the current in a non-loaded 19-gage cable circuit is flowing in opposite directions (see Figure 70) at points about 88 miles apart instead of 93 miles due to delay by $L$ and $C$, and that an appreciable time is required for the transmission of energy over that distance along the circuit.

In considering the transfer of electric energy ${ }^{4}$ along circuits at the lower frequencies, we ordinarily think of a continuous flow of current through the conductors under the influence of an applied electromotive force; and, furthermore, in short transmission lines, we ordinarily think of this energy transfer as being practically instantaneous. In the study of long transmission tines and higher frequencies these concepts tend to become inad. .te, if not inco. ect. It is easier now to think of a wave of energy traveling ling the line in the space between and surro nding the conductors, and set ir "ripples" of electron motion the wires as it gocu along. This traveling wave of energy is usually thought of as an electromagnetic field which, in the present case, is associated with




CURRENT WAVE IN WIRES ALONG LINE

FIGURE 70. INSTANTANEOUS CURRENT AND VOLTAGE ON AN INFINITE LINE AT A PARTICULAR INSTANT


$$
\begin{aligned}
& \lambda=\frac{300,000,000}{F}=\text { METERS } \\
& \lambda=\frac{186,000}{F}=\text { MILES }
\end{aligned}
$$



FIGURE 7I. RELATIVE LENGTH OF WAVES ON A LINE
or bounded by the wire conductors but which can also exist in free space where no conductors are present. Under this concept the electromagnetic field is considered as being made up of (1) the electric field whose lines of force radiate at right angles from the charges on the surfaces of the conductors and (?) the magnetic field whose lines of force encircle the conducto s and are always at right angles to the force lines of the electric field as shown in Figure 72. The total energy of the wave is al vays equally divided between the electric and magnetic fields and passes back and forth between them at a rate determined by the frequency of the applied emf. The entire electromagnetic wave travels along the lines at the speed of light if not retarded by inductance or capacitance effects in the conductors.

## 4. 7 REFLECTIONS ON TRANSMISSION LINES

If a transmission line were actually of infinite length and without loss, 4 an electromagnetic wave of energy would obviously travel along the line forever; and this would be true even though the sending-end source of energy was disconnected some time after the wave was started. Practically, of course, we are concerned with delivering power to receiving devices over lines of finite length. It becomes necessary, therefore, to consider what will happen when a traveling wave reaches the end of a uniform line.

For purposes of such analysis, it is convenient to assume a line of such low resistance as to be practically without loss, a situation which is actually approximated in a relatively short line at very high frequencies. Let us investigate first the situation where such a lossless line is open at the far end. In that case, there can be no current flow at the open end and the magnetic field created by the current must disappear there. But the energy which the magnetic field contained must be conserved and will be transferred to the electric field. This energy, added to the energy already contained in the electric field approaching the end of the linc, doubles the total electric field at that point. In other words, the voltage at the open end of the lossless line becomes twice as great as the voltage applied at the sending end. This will be explained in detail, further.

We may then consider that the increased voltage at the open end starts a wave of energy returning alons the line towards the sending end. Thus the traveling electromagietic wave reaching this total discontinuity (open circuit) in the line is reflected in bomewhat the same way a light wave is teflected from a mirror. If the ritial or incident wave is a sine wave the reflected wave will also be a sine wave. Back over the line, the reflected wave will add to the incident wave at all points and the sum must also be a


FIG. 72. CONFIGURATION OF THE ELECTRIC AND MAGNETIC FIELDS OF AN ELECTROMAGNETIC FIELD

sine wave. This is illustrated in Figure 73 where an incident voltage wave is shown reflected from the open end of the line at its positive crest.
3. " Fsumed here that the initial incident voltage wave is reflected withut hange in phase in point $B$, which can be explained in the following manner. Electric lines of force are assumed to extend from one wire of the line to the other. The two wires are at different potential and, since there is no conductir 9 path at the receiving end (open circuit), no interchange of charge: from wire to wire can occur. Thus, there is no change in phase of eflected voltage at an end. Because of that the reflected wa is essentially a contimuat for incident voltage wave so fax as wave action concerned. Thus, if we would draw the continuation of the incident wave to the right beyond point $B$ and then would fold it back at the ordinate of point $B$ we would obtain the dashed reflected wave of Figure 73. This wave is in phase with the incident wave and is of almost the same dimensions. Therefore the resultant wave will be of almost double voltage of the incident wave. This is the case when reflection occurs at the crest of the incident wave.

Figure 74 shows conditions of reflection in the same line which occurs at the zero amplitude of the incident voltage wave at point $B$. If the same reasoning is applied as for conditions shown in Figure 73 it will be seen that the amplitudes of the incident and $r$ eflected waves are of opposite sign and almost equal along the line.

 woltages along the line at thay instant will " $2 \in T$

The two examples shown indicate that the reflected wave then
adds 4 in phase or out-of-phase in the open line at two extreme instants when rotating vector is either at $0^{\circ}$ or $90^{\circ}$ respectively. If all positions of the rotating vector are considered we will obtain the resultant wave known as a standing wave because it does not travel longitudinally along the line. In other words, if an RMS voltmeter is used to make measurements along the line it would read zero at the nodal points and maximum voltages of twice the effective values of the applied voltage at the points where the positive and negative voltage crests are shown. Instantaneously at these points the voltage would be varying from maximum positive value through zero to maximum negative value in accordance with the frequency of the applied voltage. This is illustrated by Figure 75 .

A reflection from an open-end line that results in the development of a standing voltage wave of approximately double magnitude will occur only when the distance from the sending end to the receiving open end is equal to an odd number of quarter wavelengths. The two illustrations given as an example show a line one and a quarter wavelengths long. It may also be noted that if, as we have assumed (Fig. 73), the line is lossless the impedance at the sending end " 0 " in this case is theoretically zero because resultant voltage at this end 0 is always zero. Energy is nevertheless surging back and forth in the line. The line may accordingly be thought of as behaving like a series resonant circuit. For this reason, lines having discontinuities which cause reflections and consequent standing waves are known as resonant lines. A uniform line of


FIGURE 75.
infinte se, one terminated in its characteristic impedance, on the other hand, is a non-resonant line. Of course the complete noloss condition we have assumed for discussion purposes never actually exists, but at very high frequencies the inductive and capacitive reactances may be so large compare with the resistance as to cause the actual situation to al roach closely the no-loss condition.

A Milar analysis 4 of a line consisting of an odd nes. be: of quarter wavelengths and shorted at its distant end will show that the standing voltage wave will have its maximum value equal to the maximum value of the applied voltage at the sending end and will have a nodal point or zero voltage at the shorted end. The sending end impedance of such a line approaches infinity. Lossless lines whose length is an even multiple of quarter wavelengths, on the contrary, have extremely high impedance at the sending end when open at the distant end, and impedance near zero wher shorted at the distant end.

Analogical reasoning applies to the behavior of current in a line consisting of an odd number of quarter wavelengths. If there is a short at the receiving end $B$ of the line, there is no voltage developed there and there is no change in value and in phase between the initial and reflected currents. Thus, wave relatio tips such as shown in Figure 74 apply at $90^{\circ}$ vector angle. The resultant current wave coincides with the abscissa axis at that instant. If or the other hand, the receiving end at $B$ is open, no current can filow there and therefore the initial and reflected current waves must be $180^{\circ}$ out of phase. Similar reasoning applies here as for voltages.

The, extreme situations that we have discussed above ${ }^{4}$ would not occur ir: transmission line designed to carry energy from one point to another. We have considered only lines that were open or shacit. and only at points that were some integral multiple of a quarter- avelength at some specific frequency. Short resonant lines of these types have numerous useful applications in very igh frequency work, but they are not useful as transmission lines in th ordinary sense. Nevertheless, the phenomena of wave reflection must be taken into consideration in any stansmission line where the impedances are not matched at all junction -nints. Whewe there is any such impedance mismatch on any type of. mission line, and re edless of the frequencies being transmit. . we will be some wef ctions. In general, the impedance isworatity will not only $\therefore \quad A^{-}$maximum power transfer at thent of mismatch be Wi- sult in actual loss of energy. his is due to the fact that any actual transmission line munt inde some resistance which will cause $I^{2} R$ losses due to the reflected currents in addition to
the $I^{2} R$ losses of the useful current transmitted to the load at the receiving end of the line. The extent of a reflection loss, of course, depends on the extent of the mismatch.

Summarizing the preceeding discussion, it may be stated that a line which has standing waves present is said to be a "resonant line" (at the particular frequency involved). A line which does not have standing waves is defined as a "non-resonant line." Such a line is either an infinitely long line or a line terminated in its characteristic impedance $Z_{0}$. Since there are no reflections all of the energy coming down the line is absorbed by the load. Nonresonant lines are typical of the telephone transmission lines.

Standing waves on resonant and mismatched transmission lines are usually expressed in terms of the ratio of the maximum $A_{\max }$ voltage to the minimum voltage $A_{\min }$ (Figure 76) on the line which is obtained by measuring the outgoing voltage $\mathrm{V}_{\mathrm{S}}$ and the reflected voltage $\mathrm{V}_{\mathrm{R}}$ and then dividing the sum of the two voltages by their difference as shown in the formula below. This ratio is known as the voltage standing wave ratio or, more commonly, VSWR and equates to 1.0 when the reflected voltage is zero, and infinity when the reflection is 100 per cent.

$$
\operatorname{VSWR}=\frac{\mathrm{V}_{\mathrm{S}}+\mathrm{V}_{\mathrm{R}}}{\mathrm{~V}_{\mathrm{S}}-\mathrm{V}_{\mathrm{R}}}=\frac{\mathrm{A}_{\max }}{\mathrm{A}_{\min }}
$$

since $V_{S}+V_{R}=A_{\text {max }}$ and $V_{S}-V_{R}=A_{\text {min }}$.


FIGURE 76.STANDING WAVE MEASUREMENT

- e SWR may be used to determine the percentage of reflection …

$$
\% \text { REFL }=\frac{(\mathrm{SWR}-1)^{2}}{(\mathrm{SWR}+1)^{2}} \times 100
$$

## $\because 8$ LOSSES DUE TO REFLECTION

It icilows from the previous section that, unless the impedance $Z_{S}$ of the sending circuit is exactly equal to the impedance of the connecting line $Z_{1}$, and the impedance of the receiving circuit: $\mathrm{Z}_{\mathrm{R}}$ is exactly equal to the impedance $\mathrm{Z}_{2}$ of the other end of the connecting line (see Figure 77), there will be reflections and other losses in addition to the usual attenuation loss. Figure 78 represents a simple transmission system in which a sending circuit having an impedance $Z_{o a}$ is connected at points $X$ and $Y$ to a receiving circuit $\mathrm{Z}_{\mathrm{ob}}$. In the sending circuit the voltage to current ratio between points $A$ and $B$ is

$$
\frac{\mathrm{E}_{\mathrm{a}}}{\mathrm{I}_{\mathrm{a}}}=\mathrm{z}_{\mathrm{oa}}
$$



FIGURE 77.

In the receiving circuit the ratio between points C and D is


FIGURE 78.

Before the signal generated in the first section can enter the second section it must adjust itself to a new voltage to current ratio at X - Y. During this adjustment a portion of the signal is reflected. This reflection may be compared to other phenomena where there is a discontinuity in the transmission media such as an "echo." In Figure 78, if $P$ is the power of the signal arriving at the junction $X$ and $Y$, then part of the signal $P_{t}$ is transmitted through and the remainder $P_{r}$ is reflected back toward the source. The reflection loss is the ratio (in db) between the power $P_{t}$ transmitted in a mismatched impedance circuit and the power $P$ that would be transmitted if the impedances match. The reflection loss in db is equal to

$$
\text { Reflection Loss }=10 \log _{10} \frac{P_{t}}{P}=10 \log \frac{P_{t}(\text { Mismatched })}{P \text { (Matched) }}
$$

The preceeding formula may, in turn, be expressed in terms of impedances by writing current equations for Figure 78 and then taking the ratio of these currents to result in:

$$
\text { Reflection Loss }=20 \log \frac{2 \sqrt{Z_{a} \times Z_{b}}}{\mathrm{Z}_{\mathrm{a}}+\mathrm{Z}_{\mathrm{b}}}
$$

If $\mathrm{Z}_{\mathrm{a}}=\mathrm{Z}_{\mathrm{b}}$, reflection loss $=20 \log 1=0$

A- ther consideration of real practical importance in the telephone work is the return loss which is the ratio in db of the power $P$ that would be transmitted if the impedances were matched to the power $P_{r}$ reflected in a mismatched impedance circuit.

$$
\begin{aligned}
\text { Return Loss } & =10 \log \frac{\mathrm{P}}{\mathrm{P}_{\mathrm{r}}}=10 \log \frac{\text { (Matched) }}{\mathrm{P}_{\mathrm{r}} \text { (Refler }} \frac{(\text { in Mismatched Cond. })}{\text { ( }} \\
& =20 \log \frac{\mathrm{Z}_{\mathrm{a}}+\mathrm{Z}_{\mathrm{b}}}{\mathrm{Z}_{\mathrm{a}}-\mathrm{Z}_{\mathrm{b}}}
\end{aligned}
$$

## Example

Calculate return loss when $\mathrm{Z}_{\mathrm{a}}=600$ ohms and $\mathrm{Z}_{\mathrm{b}}=400$ ohms.

Retưrn Loss $=20 \log \frac{600+400}{600-400}=20 \log \frac{1000}{200}=20 \log 5=14 \mathrm{db}$

Note that if the junction is matched ( $Z_{a}=Z_{b}$ ), the retrin loss becomes infinite. Thus, the better the match, the highe the return loss. If there is more than one discontinuity in a transmission system, a portion of the signal reflected at one junction will be re-reflected at the other junctions and give rise to an oscillatory condition called "singing." The impedance irreguliarity can in general be eliminated by an impedance matching device at the junction. The se may consist of inequality ratio transformers or pads.

Within certain limits, impedances may be matched by the insertion of a simple $T$ pad made up entirely of inductors or capacitors. Such T pad introduces relatively little attenuation loss but its effectiveness as an impedance matching device is limited to the frequency range for which its reactance values were determined. Transformers are, therefore, generally used for impedance matchin purposes in the voice frequency range.

## References for Chapter 4

3. From John D. Ryder, NETWORKS, LINES i " FIELDS, 2nd
ition. 1955, by permission of Prentice Ha.n, wac., Eaglewood Cliffs, N.J.
4. (c) Fraterican Telephone and Telegraph Company, 1961.

## CHAPTER 5

## TRANSMISSION IMPROVEMENT

### 5.1 INTRODUCTION

Some of the previous chapters, notably the ones on networks and transformer circuit, have already covered apparatus and techniques employed to improve transmission. In this chapter we shall discuss some additional devices and arrangements used to improve transmission.

### 5.2 THE DISTORTIONLESS LINE

From the equations for attenuation and velocity ${ }^{4}$ developed in the preceding chapter it will be seen that in the general case both quantities are functions of frequency. For telephone transmission over long lines, variation of these quantities with frequency is obviously undesirable because it results in distortion of the trans. mitted signal. Thus frequencies at the upper end of the voice range might suffer more attenuation than frequencies at the lower end of the range; also they might differ from the lower frequencies in the time taken in reaching the receiving point. The seriousness of such distortions in practice would depend of course on the extent of the attenuation and phase shift variations with frequency.

If a line is to have ${ }^{3}$ neither uency nor delay distortion, then $\alpha$ and the velocity $V$ of propagation cannot be functions of fre. quency $\omega$. In view of the fact that

$$
\mathrm{V}=\frac{\omega}{\beta}
$$

then $\beta$ must be a direct function of frequency.

$$
\begin{equation*}
\beta=\sqrt{\frac{1}{2} \sqrt{\left(\mathrm{R}^{2}+\omega^{2} \mathrm{~L}^{2}\right)\left(\mathrm{G}^{2}+\omega^{2} \mathrm{C}^{2}\right)-\frac{1}{2}\left(\mathrm{GR}-\omega^{2} \mathrm{LC}\right)}} \tag{9}
\end{equation*}
$$

shows that if the term under the second radical be reduced to
equal

$$
\left(\mathrm{GR}+\omega^{2} \mathrm{LC}\right)^{2}
$$

then the required condition on $\beta$ is obtained. Expanding the terms under the internal radical and forcing the equality gives

$$
\begin{equation*}
\mathrm{R}^{2} \mathrm{G}^{2}+\omega^{4} \mathrm{~L}^{2} \mathrm{C}^{2}+\omega^{2} \mathrm{~L}^{2} \mathrm{G}^{2}+\omega^{2} \mathrm{C}^{2} \mathrm{R}^{2}=\left(\mathrm{RG}+\omega^{2} \mathrm{LC}\right)^{2} \tag{qa}
\end{equation*}
$$

This reduces to

$$
\begin{array}{r}
\omega^{2} L^{2} G^{2}-2 \omega^{2} \mathrm{LCRG}+\omega^{2} \mathrm{C}^{2} \mathrm{R}^{2}=0 \\
\omega^{2}(\mathrm{LG}-\mathrm{CR})^{2}=0
\end{array}
$$

Therefore the condition that will make $\boldsymbol{\beta}$ a direct function of fre-
quency is

$$
\mathrm{LG}=\mathrm{CR}
$$

A hypothetical line might be built to fulfill this condition. The line would then have a value of $\beta$ obtained by use of

$$
\beta=\omega \sqrt{\mathrm{LC}}
$$

The velocity of propagation is then

$$
\mathrm{V}=\frac{1}{\sqrt{\mathrm{LC}}}
$$

which is the same for all frequencies, thus eliminating delay distortion

Equation for $\alpha$

$$
\begin{equation*}
\alpha=\sqrt{\frac{1}{2} \sqrt{\left(\mathrm{R}^{2}+\omega^{2} \mathrm{~L}^{2}\right)\left(\mathrm{G}^{2}+\omega^{2} \mathrm{C}^{2}\right)+\frac{1}{2}\left(\mathrm{GR}-\omega^{2} \mathrm{LC}\right)}} \tag{8}
\end{equation*}
$$

may be made independent of frequency if the term under the internal radical is forced to reduce again to

$$
\left(\mathrm{RG}+\omega^{2} \mathrm{LC}\right)^{2}
$$

Identical analysis shows that the condition of Eq. (10), $L G=C R$, will again produce the desired result, so that it is possible to make $\alpha$ and the velocity $V$ independent of frequency simultaneously. Applying the condition of Eq. (10) to the expression for $\alpha$ gives

$$
\alpha=\sqrt{\mathrm{RG}}
$$

which is independent of frequency, thus eliminating frequency distortion on the line.

The characteristic impedance of any line at any frequency is given by equation (4) in Chapter 4.

$$
Z_{0}=\sqrt{Z_{1} Z_{2}}=\sqrt{\frac{R+j \omega L}{G+j \omega C}}
$$

At radio frequencies $\omega L$ and become so much greater than $R$ and $G$ that the latter may be nefected, and accordingly

$$
Z_{o}=\sqrt{\frac{R+j \omega L}{G+j \omega C}}=\sqrt{\frac{L}{C}}
$$

where $L$ is the series inductance of the line in henrys, $C$ is the shunt capacitance; $Z_{o}$ will be the characteristic impedance in ohms and will be a pure resistance. The values of $L$ and $C$ may be for any (but the same) length of line.

The developments of the preceding formulae were made by Oliver Heaviside in England about 1890; he showed theoretically that, if the linear line constants (or parameters) were so related that LG $=C R$, transmission conditions would be improved. Thus, if $L G=C R$, the wave velocity, the attenuation, and the characteristic impedance are independent of frequency as was mentioned before. If complex voltage waves, such as of speech, are impressed on this type of circuit, the current entering the line (determined by $\mathrm{Z}_{\mathrm{O}}$ ) will be independent of the frequency, the rate at which the various components travel along the line will be independent of frequency, and
the attenuation will be independent of frequency. Thus a complex wave will be transmitted without a change in wave form (without distortion), and therefore a transmission circuit in which $L G=C R$ is known as a distortionless line.

Other characteristics of the distortionless line are that the attenuation $\alpha$ becomes the least possible value, and the characteristic impedance $Z_{o}$ becomes a pure resistance and is increased to a high value.

Unfortunately, a hypotetical transmission line having such optimum characteristics is not attainable in practice with distributed parameters, but the preceding analysis points the way to the solution. To achieve the condition

$$
\begin{equation*}
L G=C R \text { or } \frac{L}{C}=\frac{R}{G} \tag{10}
\end{equation*}
$$

requires a careful evaluation.
The value of shunting conductance $G$ is very small in normal transmission line, otherwise losses would be large; G cannot be artifically changed because it would increase attenuation. The value of capacitance C likewise cannot be changed because the spacing between wires depends on the type of insulation. To attain the optimum condition shown by the equation (10) it would be necessary either to increase the value of inductance $L$ or to decrease the value of resistance of wires R. The latter method is not practical. To reduce $R$ means to raise the diameter and cost of the conductors above economic limits, so that the hypothetical results cannot be achieved. However, it is possible to increase the value of inductance $L$ to a limited degree and thus approach the ideal condition. This practice is known as loading.

### 5.3 LOADING

Heaviside suggested that the inductance $L$ be increased. Attempts to follow his suggestions did not prove successful at first.

Pupin successfully solved the problem of what is called inductive loading. He suggested adding series inductance, in the form of carefully constructed coils, at regular and comparatively short intervals.

At about this same time Campbell, working independently, also developed a theory of inductive loading. Priority was adjudged Pupin, and his patent rights were acquired by the Bell System, whose engineers are largely responsible for the application of loading to modern communication.

Loading was fret appied to telephone open-wire lines in about 1900. By the use of loading, talking distances were approximately doubled. By 1925, most loading in the United States had been removed from open-wire lines. This was because of the development of the vacuum-tube amplifier. Loading is expensive, and on open-wire lines is particularly susceptible to impairment and damage by heavy transient currents, such as are induced by lightning sometimes. At the present time, only cables are loaded in the United States.

In practice, loading coils having an impedance of $\frac{1}{2} Z_{L}$ are installed in series with each cable conductor at spacings $S$ miles apart as indicated in Fig. 78.


FIG. 78 METHOD OF INSERTING LOADING COILS IN A CABLE PAIR. THE COILS AT EACH LOADING PCIT ARE INDUCTIVELY COUPLED

## Loading Advantages

Adding inductance $L$ to the line results ${ }^{4}$ in an increased resistance $R$ which partially offsets the beneficial effect. Material net reduction in the value of $\alpha$ (attenuation) is obtained by proper loading, that is increase of $L$ as can be seen from the formula.

$$
\alpha=\frac{\mathrm{R}}{2} \sqrt{\frac{\mathrm{C}}{\mathrm{~L}}}
$$

Practical loading also decreases phase distortion $\beta$ and increases the value of the characteristic impedance $Z_{0}$. Iacreasing the characteristic impedance gives the effect of transmission at a higher voltage and lower current. Thus inductive interference is reduced.

## Loading Disadvantages

Despite the advantages of loading described above, there are important practical limitations to the usefulness of loading. One disadvantage is its effect in decreasing the velocity of propagation.

$$
\mathrm{V}=\frac{1}{\sqrt{\mathrm{LC}}}
$$

When relatively large amounts of inductance are added, the time delay of propagation over very long circuits may be great enough to introduce disturbing effects. Much more important, however, is the fact that practical loading imposes a sharp limitation on the total range of frequencies that can be transmitted. This would not be true if it were feasible to add the required loading inductance on a continuous uniformly distributed basis by wrapping each conductor with a spiral of magnetic material. This is done on long submarine telegraph cables where only a single conductor is involved but is obviously impractical to apply to the large number of conductors of a telephone cable. "Lumped" loading is applied as mentioned before by inserting inductance coils at regularly spaced intervals along the lines. This effectively breaks the loaded circuit up into network sections, the major electrical constants of each of which are the series lumped inductance and the shunt capacitance.

A network of this type has the characteristics of a 'low-pass filter" which means that it tends to block frequencies above some critical value. This filtering action is due to resonance effects. The critical frequency where the attenuation of the loaded circuit begins to increase rapidly is known as the cutoff frequency and is determined from the following equation:

$$
f_{c}=\frac{1}{\pi} \sqrt{L C}
$$

where $L$ is the inductance of the line between coils in henries and $C$ is the capacitance of the line between coils in farads.

In the design of loading systems the value of this cutoff frequency may be varied considerably by varying the spacing of the loading points (and thus C) and the amount of inductance $L$ added. It is not feasible in practice, however, to design a loading system in which the cutoff frequency is much higher than about 30,000 cycles. Loading, therefore, cannot be applied to circuits on which broadband carrier systems are to be superimposed. Its present application in long distance telephone practice is limited to toll cable circuits on which carrier systems are not superimposed, and to the relatively short entrance cables connecting to open-wire facilities.

## Loading Symbols

Loading symbols ${ }^{4}$ have been coded to indicate many things in terms of transmission. The first letter of the code indicates the spacing between coils (in feet). The number following the letter gives the inductance in millihenrys. The next letter $\mathrm{S}, \mathrm{P}$ or N indicates whether the circuit is side, phantom or non-phantom. (See Table 5 - 1). The code is further extended so that phantom group loading may be indicated by a letter followed by two numbers, and the letters $S$, $P$ or $N$ may be omitted. Thus, $\mathrm{B}-88-50$ represents a phantom group in which the phantom circuit is loaded with 50 millihenry coils and the side circuits with 88 millihenry coils, both spaced at 3000 feet intervals. If the spacing between phantom and side circuit differs, two letters are used as the first symbol of the code. For example, BH-15-15 denotes a phantom group where both coils are 15 millihenrys, but the side circuit coils are spaced at 3000 feet intervals while the phantom coils are at 6000 foot intervals. The following table Fig. 79 shows the load coil spacing code.

| Code | Spacing in |
| :---: | :---: |
|  |  |
| A | 700 |
| B | 3000 |
| C | 929 |
| D | 4500 |
| F | 5575 |
| FS | 2787 |
| H | 640 |
| X | 6000 |
| Y | 680 |
|  | 2130 |

*Spiral four cable.
FIG. 79 LOADING COIL SPACING CODE

### 5.4 ATTENUATION EQUALIZERS

Another aspect ${ }^{4}$ of the frequency-attenuation characteristic which affects the quality of voice transmission is the unequal attenuation of the currents of different frequencies. For example, the attenuation of a non-loaded open-wire circuit is greater for the higher frequencies than for the lower, and this difference in attenuation is directly proportional to the length of line. When long circuits are used, it is frequently necessary to make use of attenuation equalizers to correct this distortion. These equalizers are usually associated with the repeaters which must be included to assure a satisfactory volume of sound at the receiving end.

Table 5-1
Characteristics ${ }^{4}$ of Standard Types of Paper Cable Telephone Circuits at 1000 Cycles Per Second.

| TYEOF gIRCUIT | WIRE GAGE A. W.G. | TYPEOF LOADING | SPACING OF LOAD COILS MILES | LOAD COIL CONSTANTS PER LOAD SECTION |  | VELOCITY <br> MILES PER <br> SECOND <br> v | CUT-OFF FREQUENCY $F_{c}$ <br> (APPROX.) | TRANSMISSION EQUIVALENT db PER MILE (CALCULATED) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | R OHMS | $L$ <br> HENRYS |  |  |  |
| SIDE | 19 | ARLS. | - | - | - | 46330 | - | 1.08 |
| - | 19 | H-31-S | 1. 135 | 2.7 | . 031 | 23331 | 6700 | . 56 |
| * | 19 | H.44-S | 1.135 | 4.1 | . 043 | 20022 | 5705 | . 49 |
| " | 19 | H-88-S | 1. 135 | 7.3 | . 088 | 14319 | 3997 | . 36 |
| * | 19 | H.172.S | 1.135 | 13.0 | . 170 | 20326 | 2878 | . 28 |
| ' | 19 | H-174-S | 1.135 | 16.1 | . 171 | 10288 | 2870 | . 29 |
| ' | 19 | B-88-5 | 0.568 | 7.3 | . 088 | 10157 | 5655 | . 28 |
| " | 16 | N.L.S. | - | - | - | 64506 | - | . 73 |
| " | 16 | H.31.S | 1.135 | 2.7 | . 031 | 23818 | 6700 | . 29 |
| ${ }^{\prime}$ | 16 | H.44-S | 1.135 | 4.1 | . 043 | 20048 | 5705 | . 26 |
| " | 16 | H-88-S | 1.135 | 7.3 | . 088 | 14365 | 3997 | .19 |
| " | 16 | H.172.S | 1.135 | 13.0 | . 170 | 10331 | 2878 | . 16 |
| " | 16 | H.174.S | 1.135 | 16.1 | . 171 | 10297 | 2870 | . 17 |
| " | 16 | B.88.S | 0.568 | 7.3 | . 088 | 10165 | 5655 | . 16 |
| PHANTOM | 19 | N.L.P. | - | - | - | 51525 | - | . 96 |
| ' | 19 | H.18-P | 1.135 | 1.4 | . 018 | 23781 | 6959 | . 46 |
| " | 19 | H-25-P | 1.135 | 2.1 | . 025 | 20621 | 5916 | . 40 |
| ' ${ }^{\text {d }}$ | 19 | H.50.P | 1.135 | 3.7 | . 050 | 14861 | 4193 | . 30 |
| ، | 19 | H.63.P | 1.135 | 6.1 | . 063 | 13334 | 3738 | . 29 |
| ${ }^{\prime}$ | 19 | H.106.P | 1.135 | 8.2 | . 107 | 10252 | 2871 | . 23 |
| " | 19 | B. 50.P | 0.568 | 3.7 | .050 | 10590 | 5936 | 24 |
| " | 16 | N.L.P | - | - | - | 70604 | - | . 65 |
| $\cdots$ | 16 | H-18-P | 1.135 | 1.4 | . 018 | 24129 | 6959 | . 24 |
| " | 16 | H-25-P | 1.135 | 2.1 | . 025 | 2 | 5916 | . 21 |
| $0 *$ | \% | H. 50.P | 1.135 | 3.7 | . 050 | 18996 | 4193 | .16 |
| " | 16 | H.63-P | 1.135 | 6.1 | . 063 | 13354 | 3738 | . 16 |
| " | 16 | H-106.P | 1.135 | 8.2 | . 107 | 10257 | 2871 | . 14 |
| " | 16 | B. $50-\mathrm{P}$ | 0.568 | 3.7 | . 050 | 10597 | 5936 | . 14 |
| PHYSICAL | 19 | B-22 | 0.568 | 1.28 | . 022 | 19790 | 11276 | .47 |
| " ${ }^{\text {d }}$ | 16 | B-22 | 0.568 | 1.25 | . 022 | 20010 | 11276 | . 24 |

Attenuation equalizers are networks consisting of inductors, capacitors and resistors of such value that when connected to the circuit (line) their attenuation frequency characteristics are complementary to the line characteristics that produce the distortion. In brief, the total loss of the Iine plus that produced by the equalizer will be substantially the same for all frequencies in the voice band as shown in Figure 80.


FIG. 80. LINE AND EQUALIZER CHARACTERISTICS

One of the simplest types ${ }^{4}$ of equalizers shown in Figure 81 is bridged directly across the line to be corrected.

At higher frequencies we come closer to the resonant frequency of the LC circuit, the bridging impedance of the equalizer increases and therefore causes little loss due to mismatch with the line. At frequencies considerably lower than the resonance the parallel LC circuit behaves like an inductance and thus the LC circuit offers to the line a lower bridging impedance and the loss due to mismatch is greater. The changes of the bridging impedance, particularly at the low frequencies provide a substantial loss. This introduces an impedance irregularity into the circuit of a sizeable value. The use of bridged equalizers of this type therefore has definite limitations in practice.

In long circuits equipped with repeaters, the equalizing effect is obtained by inserting the equalizing networks at the mid-point of the input transformer primary winding. This arrangement changes the overall gain-frequency characteristic of the repeater to match closely the loss-frequency characteristic of the line. For the frem quencies where the line loss is high, the repeater gain is high and vice versa.


FIG. 8I. SIMPLE BRIDGED EQUALIZER
Both of the above methods ${ }^{4}$ of equalization give satisfactory results where the amount of attenuation distortion to be corrected is relatively small. To use either of these methods to correct a large attenuation distortion, might result in an impedance irregularity of such a magnitude as to more than offset the benefits obtained by equalizing. To equalize for these relatively large amounts of attenuation distortion, a somewhat more complex equalizing network, in the form of a bridged T-structure, may be used. This equalize is designed to have a constant impedance over the entire frequ ncy band transmitted.

As its name implies, the bridged $T$-equalizer is built in the general form of a T-network, but it has an additional impedance path bridged across its series elements. This latter path controls the loss of the equalizer. The elements of the bridged T-equalizer are connected in a Wheatstone bridge arrangement, and the principle of its operation may be best grasped by first referring to the ordinary Wheatstone bridge circuit illustrated in Figure 82.


FIG. 82 WHEATSTONE BPIDGE CIRCUIT

Here a generator $E$ is connected ${ }^{4}$ to the two opposite points 1 and 4 of the bridge through the impedance $R$. And a detector $G$ is connected across the other two points 3 and 5 through an impedance R. The bridge is balanced and therefore no current flows through the impedance $R$ in series with $G$ when the following proportion holds true for the Wheatstone bridge:

$$
\mathrm{R}^{2}=\mathrm{Z}_{11} \mathrm{Z}_{21}
$$



CONSTANT RESISTANCE " $T$ " NETWORK WHEN $Z_{11} Z_{21}=R^{2}$

FIG. 83. RRIDGED T-EQUALIZER

Now let us rearrange this same bridge circuit in the form of a $T$-network where the series elemer $\Delta R$ and $R$ are bridged by the impedance $Z_{11}$, as illustrated in Figure 83. The $T$-network proper is formed by two horizontal resistors $R$ and $Z_{21}$ with $Z_{11}$ as the bridging impedance while two vertical $R$ resistors become the input and output impedances, respectively. It is obvious that this $T$-network is balanced when $R^{2}=Z_{11} Z_{21}$. When the bridge is balanced no current flows in the right horizontal resistance $R$ (Fig. 83), and for purposes of analysis we may therefore simplify the network by removing this $R$ resistance giving us the network ${ }^{4}$ of Figure 84. Looking from the generator across terminals 1-2 of this circuit, we now see two parallel paths which present an input impedance $Z_{\text {in }}$ of

$$
\begin{aligned}
Z_{i n} & =\frac{\left(R+Z_{11}\right)\left(R+Z_{21}\right)}{R+Z_{11}+R+Z_{21}} \\
& =\frac{R^{2}+R Z_{11}+R Z_{21}+Z_{11} Z_{21}}{2 R+Z_{11}+Z_{21}}
\end{aligned}
$$

or substituting $R^{2}$ for $Z_{11} Z_{21}$,

$$
Z_{i n}=\frac{R\left(2 R+Z_{11}+Z_{21}\right)}{2 R+Z_{11}+Z_{21}}=R
$$

In other words, when the bridge is balanced, that is when $Z_{11} Z_{21}=R^{2}$, the input impedance of the equalizing network is a pure resistance $R$. Moreover, since the $T$-network is symmetrical, the same reasoning can be applied at the output terminals $3-4$, and the impedance will also be found to be a pure resistance, $R$, for the balanced condition.


FIG. 84

As in the case ${ }^{4}$ of any other circuit, the loss produce by this network may be determined by the ratio of the current $I_{b}$, received in the output impedance $R$ before the network (shown in dashed lines) is inserted, to the current $I_{a}$ received after the network is inserted. Thus, the current, $I_{b}$, in the output before the network is inserted will be

$$
I_{b}=\frac{E}{R+R} \text { or } \frac{E}{2 R}
$$

After inserting the network, the output current of the generator will remain the same $I_{b}$ because the impedance of the network as seen at terminals $1-2$ is still $R$. As the input current $I_{b}$ divides into the two parallel paths (Figure 84), the branch current flowing in the output (terminals 3-4), $\mathrm{I}_{\mathrm{a}}$, is

$$
I_{a}=I_{b} \times \frac{R+Z_{21}}{R+Z_{c 1}+R+Z_{11}}
$$

> Then from the las: equation

$$
\frac{I_{a}}{I_{b}}=\frac{R+Z_{21}}{2 R+Z_{11}+Z_{21}}
$$

or using the reciprocal of this formula

$$
\frac{\mathrm{I}_{\mathrm{b}}}{\mathrm{I}_{\mathrm{a}}}=\frac{2 \mathrm{R}+\mathrm{Z}_{11}+\mathrm{Z}_{21}}{\mathrm{R}+\mathrm{Z}_{21}}
$$

Since we are considering the balanced condition where $Z_{11} Z_{21}=R^{2}$, then $Z_{21}=$ $R^{2} / Z_{11}$. Substituting this in the last equation for $I_{b} / I_{a}$, we get the loss

$$
\begin{aligned}
\frac{I_{b}}{I_{a}} & =\frac{R^{2}+2 R Z_{11}+Z_{11}^{2}}{R^{2}+R Z_{11}}= \\
& =\frac{R+Z_{11}}{R}=1+\frac{Z_{11}}{R}
\end{aligned}
$$

This shows ${ }^{4}$ that, as long as the balanced condition is maintained, the loss of the network is determined by $\mathrm{Z}_{11}$. This is also apparent from an inspection of Figure 84 because this impedance is in series with the receiving circuit, and any value of loss may be secured without affecting the input or output impedances, providing the balanced condition is maintained.

To summarize, the bridged T -equalizer has a constant impedance, as seen from both terminals, equal to $R$ when $Z_{21}$ is the
inverse of $Z_{11}\left(Z_{21} Z_{11}=R^{2}\right)$, and its overall loss-frequency characteristic is determined by the $b$ dged series impeaance network, $\mathrm{Z}_{11}$.

Both $\mathrm{Z}_{11}$ and $\mathrm{Z}_{21}$ represent generalized impedances which may be resistances, capacitances, inductances, or any combination of them. The one and only requirement is that established by the balanced condition $\left(Z_{11} Z_{21}=R^{2}\right)$, which means there must always be
an inverse relationship between $Z_{11}$ and $Z_{21}$. If $Z_{11}$ is a pure inductive reactance represented by $j \omega L$, then $Z_{21}$ must be $R^{2} / j \omega L$ or $-j R^{2} / \omega L$, which represents a capacitive reactance, $-j / \omega C$, where $C=L / R 2$. On the other hand, if $\mathrm{Z}_{1}$ is a capacitance, $\mathrm{Z}_{21}$ must be an inductance, which is the reverse of the above case. If $\mathrm{Z}_{11}$ is a resistance, then $\mathrm{Z}_{21}$ will also be a resistance. When $\mathrm{Z}_{11}$ is a network, $\mathrm{Z}_{21}$ is a network with the same number of elements but each element is the inverse of the corresponding element of $\mathrm{Z}_{11}$ as illustrated by the following table:

## When $\mathrm{Z}_{11}$ is:

Inductive reactance. Capacitive reactance. Resistance.
Series inductance. Series capacitance. Parallel resonance. Seies resonance.
$\mathrm{Z}_{21}$ becomes:
Capacitive reactance. Inductive reactance. Resistance Parallel c , sacitance. Parallel is uctance. Series re nance. Parallel resonance.


SERIES NETWORK IMPEDANCE, $Z_{11}$


FIG. 85 INVERSE SERIES AND SHUNT NETWORKS

This inverse relationship ${ }^{4}$ is further illustrated in Figure 85 where the series network, $\mathrm{Z}_{11}$, and its inverse shunt network, $\mathrm{Z}_{21}$, are shown at left and right, respectively. Here the advantages of using the two-digit subscript for $Z$ become more evident. The first digit of the subscript indicates whether the element belongs to the series $L$ or shunt impedance 2 , while the same second digit designates the corresponding inverse elements of the two networks. Therefore, in Figure $85 \mathrm{C}_{21}$ is the inverse of $\mathrm{L}_{11} ; \mathrm{C}_{22}$ is the inverse of $L_{12} ; L_{21}$ is the inverse of $C_{11}$; and $R_{21}$ is the inverse of $\mathrm{R}_{11}$.

In designing a bridged $T$-equalizer for a specific use, the attenuation-frequency characteristic of the $\mathrm{Z}_{11}$ network must be complementary to the attenuation -frequency characteristic of the circuit to be corrected. This is true because, as we have seen, the loss-frequency characteristic of the bridged $T$-equalizer is controlled by the series impedance networi. Z...

## 5. ${ }^{\text {OICE }}$ FREQUENCY REPEATERS

Thdern telephone practice ${ }^{4}$ requi. . types oi ancilifers to meet its various needs. An important application is the relatively simple voice-frequency amplifier commonly
known as the telephone repeater. This device is often inserted at certain intervals in long voice-frequency telephone circuits to offset line attenuation.

A voice frequency repeater circuit is an arrangement of electron tube or semiconductor amplifiers and associated apparatus capable of receiving a voice frequency current and retransmitting it, without appreciable distortion, at a greater magnitude. Voice frequency repeaters are used mostly in local plant and on shorter toll lines to extend the range of transmission, to provide transmission more economically and to improve the quality of transmission.

Repeaters, according to use, can be divided into three types: (a) through line repeater; (b) switched-in repeaters; (c) cord-circuit repeaters. Through line repeaters are permanently associated with a particular toll line; switchedoin repeaters are automatically associated with a particular toll line as a result of relay action; cordcircuit repeaters, before they became obsolete, could be placed in any connection for which they are specified by the toll board operator, by means of switchboard cords.

In general, four-wire telephone repeaters use two one-way amplifiers to provide transmission gain and are equipped with regulating devices for adjusting gain to meet operating requirements. Hybrid coils are used for adapting one-way amplifiers to two-way transmission in the 24 type repeaters. A balancing network is employed to approximate closely the impedances of each line of the circuit and its transmitted associated frequency band, thereby maintaining the degree of balance requi: sor the proper functioning of the hybrid coil. That is, energy (at voice frequencies) from the output of one amplifier must be prevented from reaching the input of the other, since this would impair the quality of the transmission, or even cause "singing." Filters are used to filter out unwanted frequencies. Other miscellaneous apparatus and circuit features are used to adapt repeater circuit to standard operating practices and become more or less a part of the repeater.

The function of such apparatus associated with repeaters is to: match impedances; connect 2 -wire to 4 -wire lines; equalize the unequal attenuation at different frequencies; adjust for variations in attenuation due to varying conditions; by-pass low-frequency or d-c signaling; and devise phantom channels separately. The selection of this associated apparatus is sometimes determined more by the line to which it is assigned than by the amplifier with which it is used. For example, a repeater is designed to have a nominal impedance (e.g. 600 ohms), so the repeat coils have different ratios because of different line impedances on both sides. Also, low-frequency ringing does not pass through the amplifier at all, so the means of deriving the signaling circuits are controlled by the line not by the amplifier.

T maximum overall gain of repeaters is determined by the amplifier uscu, ut adjustments, for example, by means of the slidewire potentiometer and resistance pads, can bring this overall gain to any lower value. Usable gain is, however, determined by circuit and cross-talk considerations. It is well o remember, however, that one-half of the energy is lost eac time it must pass through each hybrid coil circuit. This means th the actual power gain of each amplifying element must be at leas ó db greater than the overall ower gain required. This is compensated in the calibration of the repeater potentiometers.

## 5. 51 V -TYPE REPEATERS

This is a two- or four-wire repeater with 2 pentode amplifiers (in the older repeaters) arranged for negative feedback with a maximum available gain of about 35 db . The feedback causes a reduction of gain by 10 db from 45 db . All hybrid coils, equalizers, filters and regulating networks that are associated with the line are provided separately and are not part of the repeater in the older V -type repeaters. It is possible to transfer the repeater from one line to another without equipment rearrangement. The $V_{1}$ repeater is replaced by the $V_{3}$. The $V_{3}$ repeater uses miniature pentode tubes and has plug-in amplifiers. The $\mathrm{V}_{4}$ repeater is a development which utilizes transistorized amplifiers. All components are comt ied in one plug-in shelf. It is applied similarly to the $\mathrm{V}_{3}$ repeater.

## 5. 52 E-TYPE REPEATERS

All ${ }^{4}$ of the repeaters (except E-type) are "double-track" devices, employing separate amplifiers for each direction of transmission. E-repeater of radically different design is widely used to improve transmission in relatively short circuits such as the interlocal trunks in toll-connecting trunks and special service lines of exchange plant. It is known as a negative impedance repeater or converter and in the Bell System is coded as an "E-type telephone repeater". A general schematic of the mostrecent design ${ }^{4}$ of this type of repeater connection (E6) is given in Figure 86. Here it may be noted that, instead of being inserted in the line as in the case of other types of repeaters, the gain unit is so connected as not to break the continuity of the line for the transmission of d-c supervisory signals or other low-frequency signaling currents. This is effected by coupling the series converter ths ough a transformer and by the inclusion of a blocking capacitor $C$ in $0 \%$ the bridged leads to $r$ shunt converter. The gain unit consists of the converters and cain adjusting networks $Z$. The impeda se of the latter equals $Z$.


## FIG. 86 LINE CONNECTION OF EG REPEATER

The line building out (LBO) networks (in the E6 repeater) are included for impedance matching purposes. They are made up of resistors, inductors and capacitors which can be adjusted to fit the electrical characteristics of the particular line with which they are associated so as to present toward the gain unit the impedance of a 900 ohm resistor in series with a 2.14 microfarad capacitor at all frequencies in the voice band.

The converters are dynamic devices which have the ability to convert positive impedances to negative impedances. A series converter is a push-pull, class A amplifier with feedback. Its operation is such that it amplifies signal current "i" introduced to it from the line E via the transformer and returns the amplified current "I" in phase with the injected signal current "i". If the signal voltage in line $E$ is designated by $e$, then this line's impedance $Z_{E}$ is

$$
\mathrm{Z}_{\mathrm{E}}=\frac{\mathrm{e}}{\mathrm{i}}
$$

The signal current amplified in the converter was desig-
nated by I. Now the impedance will be

$$
\mathrm{Z}=\frac{\mathrm{e}}{\mathrm{i}}
$$

Since e is assumed constant and $\mathrm{I}>\mathrm{i}$, therefore $\mathrm{Z}<\mathrm{Z}_{\mathrm{E}}$.
Thus the converter reduced $Z_{E}$ to $Z$, and it may be said that the converter added a negative impedance. This is equivalent to stating that current "i" was amplified to the value of "I'".


FIG. 87. SERIES NEGATIVE MMPEDANCE CONVERTER

Amplifiers ${ }^{4}$ meeting special requirements specified in the above discussion are readily designed. A push-pull arrangement of two vacuum tube triodes in a grounded-grid connection provides a good approximation. A pair of junction transistors in a groundedbase connection meets the requirements even more closely. Figure 87 shows the essential elements of the series converter of the E6 repeater. The feedback connection here is through the relatively large capacitances $C_{1}$ and $C_{2}$. The amount of negative impedance effectively inserted into the transmission line is directly controlled by the value of the impedance $Z$ of the gain adjusting network Z .

The amount of net gain that can be realized with the series converter alone is limited in practice by the fact that it tends to introduce a substantial impedance irregularity in the line at the point where it is connected. The severity of this irregularity can be minimized if $Z$ is made up of a rather complex arrangement of resistors, inductors and capacitors which can be connected in various combinations to best adjust to the impedance characteristics of a particular line.

Better impedance matching and consequently higher gains, can be secured by including in the repeater a second converter which is bridged across the center taps of the lin transformer, as indicated in Figure 86. This "shunt converter" e? गys somewhat dit rent circuitry than that shown in Figure wi perates on the me principle to introduce a negative adm: ance which, together witi. egative impedance of the series cunverter, presents net impedance o 900 ohms plus 2 Mf toward the line in both directions. With this arrangement where LBO networks are used, the adjustabie $Z$ networks in both converters can be made up entirely of resistors
which can be easily adjusted ${ }^{4}$ in steps of $\frac{1}{2} \mathrm{db}(0.1 \mathrm{db}$, if desired) to provide an overall repeater gain of any desired value up to a maximum of about 12 db . Because of these advantages, later vintages of E-type repeaters include both series and shunt converters and no provision is made for using either converter by itself.

### 5.53 APPLICATION OF REPEATERS

In practice, after the location of the individual repeater stations has been selected, the amount of gain of each repeater must be determined. If the level of message current is too low, two effects may occur. First, small noise currents that are induced from external sources will cause excessive interference when they are amplified along with the message currents. Second, if this "low energy" pair is adjacent to a "high energy" pair of another circuit, a possible crosstalk condition exists (to be discussed later in this chapter) and the high energy circuit will transfer a quantity of energy. However, while strong message current seems desirable, the gain required of a repeater to attain this current may bring with it considerable distortion in the amplifier or crosstalk in adjacent circuits.

The ordinary limits ${ }^{4}$ of 22 repeaters (the latter were not described in this text since they are no longer manufactured) are as follows. If the volume of transmission at the switchboard end at the sending terminal of a circuit is defined as zero transmission level, a 22 repeater may ordinarily be operated to deliver a volume of transmission not exceeding the zero 1 nerel by more than 6 db at the output with a maximum net gain of nc more than 18 db . The output of V -type repeaters may be as high as 10 db above the zero level and total gain may be as high as 25 db . Repeaters using 4 -wire cable circuits may be operated at higher gains but crosstalk considerations prevent use of maximum possible gain. The permissible limits are 25 db below zero level (input) to 10 db above zero level (output) on an overall gain of 35 db . Under this condition the energy delivered is nearly 3160 times as great as the energy received. This high energy ratio is the reason for the crosstalk limitation.

In laying out circuits ${ }^{4}$ containing a number of repeaters in tandem an energy level diagram is prepared which shows the losses in each section, the gain in each repeater, and the level of the voice energy at each point as compared with input energy (zero level). Figure 88 is an illustration showing a diagram for a typical 2 -wire cable circuit. The figures at the end of the ordinates represent energy levels in db above and below zero level. The gains of repeaters are represented by vertical lines and attenuation losses in lines by lines sloping downward in the direction of transmission. The effect


FIG. 88. ENERGY LEVEL DIAGRAM FOR 2-WIRE CIRCUIT
of the 22 -type repeater is shown by a single vertical line to indicate the net gain including hybrid coils. In the case of $V$ type repeaters, the hybrid coil losses are shown separately from the amplifier ga. :.

### 5.6 CROSSTALK

### 5.61 INDUCED EFFECTS IN TELEPHONE CUITS

One of the factors ${ }^{4}$ upon which the telligibility of a telephone con ation depends is the absence ons......ssive noise and c: stalk. If each tulephone circuit was completely isolated from all other tel.phone circuits and from other sources of electrical interference,
including earth currents and atmospheric charges, we would not expect any potentials to exist in the telephone circuit other than those deliberately introduced for the purpose of the desired signal transmission. However, this is in fact a purely hypothetical situation as, in practice, nearly every long telephone circuit is in close proximity to other telephone circuits, and sometimes to other electric circuits such as power lines. It is necessary, therefore, that telephone circuits not only be efficient in transmitting electric energy without distortion and without too great a loss, but also that they ${ }^{4}$ be protected against induced voltages caused by adjacent telephone or other electric circuits.

As a matter of fact, any two long paralleling telephone circuits that are not "balanced" against each other by means of transpositions, or otherwise, will exhibit crosstalk to such a degree as to seriously interfere with their practical use. Furthermore, because crosstalk is largely an inductive effect due to magnetic and electric induction, its magnitude tends to increase with (l) the length of the paralleling circuits, (2) the strength (energy level) of the transmitted currents, and (3) the frequency of the transmitted currents. It follows that the use of telephone repeaters is likely to increase the crosstalk possibilities because the se devices permit longer circuits and at the same time increase the level of the energy at certain points along the line. Crosstalk possibilities are also increased by the use of carrier systems because of the higher frequencies employed.

Of course, crosstalk can be caused by the direct leakage of current from a disturbing to a disturfed circuit. With properly maintained lines, however, insulatio is usually sufficiently good to make this a negligible factor. The crosstalk coupling which presents the real problem in practice is due to the electric and magnetic fields set up by the voltages and currents in the disturbing circuit. The effects of these two fields are not entirely alike, although their results are generally similar as far as crosstalk is concerned.

### 5.62 CAUSES OF CROSSTALK

The effect of the magnetic field ${ }^{4}$ of one circuit on a second paralleling circuit is called magnetic induction. Similarly, the effect on the second circuit of the electric field of the first circuit is called electric or electrostatic induction. How magnetic induction causes crosstalk can be seen by referring to Figure 89. This shows the four wires of two telephone circuits running parallel to each other in the usual flat configuration employed on standard open-wire lines. If we assume that an alternating voltage is applied to the left end of the disturbing circuit, A, which is made up of wires 1 and 2 , the magnetic field existing about a short section $S$ of this circuit may be represented by lines of magnetic induction as shown.

At a particular instant in the alternating cycle, the current in wires 1 and 2 may be represented by the equal and opposite vectors designated $I_{a}$. As $I_{a}$ increases or decreases in value, the


FIG. 89.
associated lines of magnetic induction will cut ${ }^{4}$ wires 3 and 4 of the paralleling telephone circuit, B. But with the relative spacing of the wires shown in the diagram, more lines will cut wire 3 than cut wire 4. Accordingly, the voltage induced by the magnetic field in wire 3 will be somewhat greater than that induced in wire 4 . The voltages induced in both wires are in the same direction at any given instant, so that they tend to make longitudinal currents circulate in the circuit $B$ in opposite directions. If they were equal, therefore, their net effect would be zero. But in so far as the induced voltage $e_{3}$ exceeds the induced voltage $e_{4}$, there is an unbalance voltage $e_{3}-e_{4}$, tending to make a current circulate in circuit $B$. If circuit $B$ is terminated at both ends in its characteristic impedance, $Z_{0}$, the current resulting from this unbalance voltage induced in a short section of the B circuit may be written as

$$
\mathrm{i}=\frac{\mathrm{e}_{3}-\mathrm{e}_{4}}{2 \mathrm{Z}_{0}}
$$

The summation of the circulating currents caused by the voltages induced in each small section of the line $B$ may result in an appreciable cusent through both terminals of the circuit, which is due entirely to magnetic induction.

It should be noted that although the energy in the disturbing circuit was considered in the above as being transmitted from the left end to the right end of the circuit, the crosstalk current in the disturbed circuit appears at both ends of the circuit B. The crosstalk appearing at the left end of the disturbed circuit is known as near-end crosstalk and that appearing at the right end is known as far-end crosstalk.

Now turning our attention ${ }^{4}$ to electric induction, Figure 89 may also be used to show the equipotential lines of the electric field established about circuit $A$ under the same conditions as in the previous example. This electric field will set up potentials on the surfaces of wires 3 and 4 and, with the configuration and spacing of the wires shown in the drawing, these potentials will not be equal. The resultant difference in potential between wires 3 and 4 will tend to cause crosstalk currents to flow to both ends of circuit B.

The crosstalk effect of electric induction may also be analyzed by consideration of the capacitive relationships between the wires of the disturbing and disturbed circuits. Thus, referring to Figure 90, we know that in any unit length of the two circuits there is


FIG. 90.
a definite capacitance between wire 1 and wire 2 and between wire 3 and wire 4. Moreover, if the wires are equally spaced as shown, the separation a between wires $2-3$ is the same a as that between wires 1-2 or 3-4, and there is therefore the same capacitance between wires $2-3$ as between the wires ${ }^{4}$ of either pair. This is represented in the Figure by the small capacitor designated C. Similarly, the capacitances between wire 1 and wire 3 and between wire 2 and wire 4 are designated by capacitors, $C^{\prime}$, less in value than $C$ because the separation between these wires is greater.

There rew ins also the still smaller capacitance between wire 1 and wire 4 , which is indicated by $C^{\prime \prime}$. Now if we assume an alternating current flowing in circuit A, there will be a difference of potential between wires 1 and 2 , which will tend to cause small currents to flow through the several capacitances to the wirs 3 of circuit B. The net effect can best be analyzed by redrawing the diagram of Figure 90 in the form of a Wheatstone bridge network as $s$ Jwn in Figure 91. A study of the capacitance values of the arms of $t$ is bridge shows that the impedances of the arms are not such as to give a balancer ron.dition


WIRE 2

FIG. 91.
and, consequently, current flows through the 2 impedances $Z_{0}$. In other words, a current is set up in circuit $B$ which will manifest itself as crosstalk at both ends of the circuit.


FIG. 92.

As shown ${ }^{4}$ in Figure 92, the crosstalk due to electric induction may be thought of as being caused by a small generator $G_{e}$, connected across the disturbed pair $B$, while the crosstalk due to magnetic induction may be thought of as being caused by a generator $G_{m}$ connected in series with the disturbed pair. Under these conditions, it will be noted that the currents established by the two generators flow in the same direction in the left portion of the line, but in opposite directions in the right portion. In other words, the crosstalk effects of magnetic and electric induction are additive in the case of near-end crosstalk, but opoosed to each other in the case of far-end crosstalk.

### 5.63 PRINCIPLES OF CROSSTALK REDUCTION

There are several possible ways ${ }^{4}$ to eliminate, or at least substantially reduce, crosstalk induction. In an effort to keep the crosstalk in long toll circuits to a reasonable level, the effects of the basic design of long-line circuits had to be taken into account. These design features ${ }^{4}$ will, in general, apply to both open wire and cable alike. One very important feature is the effect of the location of repeaters on crosstalk. Repeaters should be placed (i. e spaced) on a line so that adjacent lines do not have such differences in energy level that crosstalk occurs. Another element in circuit design in most of the longer voice-frequency cable circuits and in all carrier circuits, is that the effect of near-end crosstalk is minimized by the use of separate paths for transmission in the two directions. In cable circuits, the pairs carrying the transmission in two directions are separated as mutas possible by placing them in different layers in the cable; or, in the case of "K" carrier circuits in different cables. A similar separation is obtained in open-wire carrier circuits by using different bands of frequencies for transmission in each direction.

In the case of open-wire lines, crosstalk reduction depends upon three main factors, namely: wire configuration on the poles, resistance balance and transpositions.

It is usually assumed that the two wires of each pair are electrically identical which is ordinarily the case in practice. However, there is always a possibility in open-wire lines of the series resistance, or the insulation resistance, of one wire in a pair being slightly different from that of its mate due to imperfect joints defective insulation etc. Any such resistance unbalance would cause induced voltage in one wire to be different from one in the other. As a result of the difference of the voltages a crosstalk current would flow through the terminal. It is very important, accordingly, that the two wires of every talking pair be so constructed as to always have identical electrical characteristics.

Resistenc unbalance ${ }^{4}$ is particularly important when two pairs are used to create a phantom circuit. Resistance balance is primarily a question of adequate maintenance and ordinarily does not present too much difficulty.

The use of high-frequency carrier syste 1 , with their much greater crosstalk possibilities, has led to the d felopment of new configurations of open-wire line in which the sr cing of the individual conductors in a pair is closer together and the pairs are snaced further apart.

There are two standard ways of placing transpositions along a pole line. They are most commonly known as the "point type" and the "drop-bracket," or "J-bracket," transpositions. These types of transpositions are shown in Figure 93 and 94 respectively.


FIG. 93. POINT-TYPE TRANSPOSITION


PIG.94. DROP-BRACKET TRANSPOSITION

The point type ${ }^{4}$ is widely used on lines carrying carrier systems because it does not change the configuration of the wires i. the adjacent spans, as does the drop bracket type. Where very high frequencies are used, as in the Type-J carrier system, this becomes extremely important. In fact, the sensitiv... of these carxier systems to crosstalk is so great that every possible eft has to be made to avc deven slight deviations in the amount of jaf he wires in the pans between poles.

The principle of transposition involved here can be understood by referring to Figure 95, which shows the same four wires as were indicated in Figure 89. In this case, however, the relative pin positions of wires 1 and 2 are interchanged by means of a transposition in the middle of the short section, $S$. Under these conditions, it will be evident that when equal and opposite currents are flowing in the two wires of circuit $A$, the voltages induced in wires 3 and 4


FIG. 95.
will be in opposite directions on the two sides of the point where circuit A is transposed. Thus, as indicated in the Figure, while e ${ }_{3}$ is larger than $e_{4}$ and $e^{\prime} 3$ is larger than $e^{\prime} 4, e_{3}$ is exactly equal and opposite to $e^{\prime} 3$ and $e_{4}$ is exactly equal and opposite to $e^{\prime} 4$. Therefore, no net voltage is induced in either wir 3 or wire 4 and, consequently, no crosstalk from circuit A into circuit B.


FIG. 96.

The same net effect would be obtained by inserting the transposition in the disturbed circuit B and leaving the wires of the disturbing circuit A rurning straight through, as shown in Figure 96. In this case, the voltage induced in the wire nearer wire number 2 is broken into two equal parts represented by the vectors $e_{3}$ and $e^{\prime} 3$. Similarly, the voltage induced in the wire farth : away from wire number 2 is broken into two parts, $e_{4}$ and $e^{\prime} 4$. Sut with the transposition as shown, voltage $e_{3}$ combines with vo age $e^{\prime} 4$ and voltage $\mathrm{e}_{4}$ combin:s with voltage e'3. The induced or crosstalk current in. the section, therefore, is

$$
i=\frac{\left(e_{3}+e_{4}^{\prime}\right)-\left(e_{4}+e_{3}^{\prime}\right)}{2 Z_{0}}
$$

But with the transposition ${ }^{4}$ in the center of the section as shown, it is obvious that -

$$
e_{3}+e_{4}^{\prime}=e_{4}+e^{\prime} 3
$$

The numerator of the first equation is therefore equal to zero and there is no resultant crosstalk, or $\mathrm{i}=0$.

Either of the transpositions discussed above would be equally effective in reducing crosstalk due to either magnetic or electric induction. It may be noted, however, that a transposition at the same point in both circuits would have no such effect.

For voice-frequency transmission, where the frequencies are relatively low and the wavelengths correspondingly long, it is not difficult to obtain a sufficiently good transposition. Where high-frequency carrier systems are used, on the other hand, the wavelengths are so short as to require closely spaced transpositions. In openwire lines, spacirgs as close as every second pole are used in practice where the wires are carrying frequencies up to 140 kc . The degree of effectiveness of such closely spaced transpositions is of course dependent upon accurate spacing of the poles themselves, and is finally limised in practice by economic rather than theoretical factors.

In the case of cable, several techniques to reduce crosstalk are employed. In addition to the process of manufacture in which the cable conductors are thoroughly transposed by twisting the two wires of each pair together, the two pairs on sanh group of four wirfs are twisted together to form quads whicl spiraled in oppcisite directions around the cable coze. At voice-irequency, gnetic induction (inductive coupling) hetw en circuits in a cable is noif... y small. But the same is not irue of electric induction (capacitive coupiing), which is large. Despite the most careful manufacturing methods, the capacitance unbalances between conductors usually remain great enough to cause objectionable crosstalk
in long circuits. To reduce this crosstalk, additional balancing techniques are used at the time the toll cable is installed. One technique involves splicing the lengths of cable in such a way that no two quads are adjacent to each other for more than a short part of the total length. Within a quad there are two main ways to correct the capacitance unbalance. One is effected by measuring unbalances at several equally spaced splices within a section, and splicing the given unbalance in one section to an equal and opposite unbalance in the adjacent section. In the second method the unbalances are counteracted by connecting small balancing capacitors in the circuits at one or two of the splicing points in each loading section.

The crosstalk coupling between the amplifiers is equal to the db difference between the measured outputs of the disturbing amplifier A and the disturbed amplifier B at given gain settings (usually both amplifiers are set at full gains) when there is no signal input to the disturbed amplifier B except crosstalk from A. In practice the following set up is often used to measure the cross coupling; this set up is shown in the form of a block diagram in Figure 97.

AMPLIFIER A


FIG. 97

B hamplifiers are set to full gain and are left connected exactly as in Pre under normal conditions, except that:

1. Across the input side of $A$ there is connected an audio oscillator set to 1 kc , as weil as an artificial lc $\mathrm{l}_{\mathrm{d}} \mathrm{Z}_{1}$. Both are so connected as to simulate closest possible the $n$ :mal input impedance.
2. Across the output of $A$ there is con ted a high impedance AC VTVM designated $\mathrm{V}_{1}$ with a db scale and a oscillograph. The oscillograph is used to monitor the distortion. There is aiso vonnected an impedance $Z_{2}$ simulating the normal load.
3. Across the output of $B$ there is connected the same type of AC VTVM designated $\mathrm{V}_{2}$, as well as across the input and output the loads $\mathrm{Z}_{3}$ and $\mathrm{Z}_{4}$.

The method of measurement is as follows. There is no input to $B$ and the 1 kc input voltage to $A$ is set to such magnitude as to be close to but below the value at which distortion of output voltage starts. The difference in db between $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ is the crosstalk coupling between amplifiers A and B .

The crosstalk coupling results from two main caused as follows: (1) magnetic coupling between input and output trax aformers of two amplifiers which accounts for practically all crosstal. at low frequencies (up to approximately 300 cycles); and, (2) electzostatic coupling between grid leads which is the main factor at frequencies in the region of 1000 cycles. A shield is provided between adjacent amplifiers in different repeaters to reduce the coupling between grid leads and tubes' plates to avoid excessive crosstalk. The loss in the crosstalk path between adjacent amplifiers in different repeaters should be 70 db or more when measured as described before. The coupling between amplifiers in the same repeater is somewhat higher due to the absence of shields between the grid leads. However, when the repeater is used in accordance with standard arrangements for message service, the crosstalk between amplifiers is applied on the same circuit and results only in negligible effect on internal balance and echo.

## CH. 5 -TRANSMISSION TMPROVEMENT

### 5.7 ECHO



FIG. 98. ECHO PATHS IN A LONG FOUR-WIRE CIRCUIT

Another series ${ }^{4}$ of problems, largely peculiar to the longer cable circuits, arises from the fact that the velocity of propagation over such circuits, as now loaded, is relatively low. The time required for transmission over circuits of this type may thus become quite appreciable. If, when a conversation is being carried on, some portion of the talker's voice is returned toward him from a sufficiently distant point, the effect will be like that of an ordinary echo. This will obviously be rather disconcerting to the speaker if the time factor is great enough so that he hears entire syllables repeated back to him. In any case, such an echo effect tends to degrade the quality of transmission and it must be guarded against in long, lowvelocity circuits.

Figure 98 (A) shows ${ }^{4}$ schematically a long 4 -wire circuit layout. When the person at the east terminal is talking, the voice currents are sent through the $4 \sim$ wire terminating set to both sides
of the cisc :Transmission over the lower (receiving) side from east to west stops at the output side of the terminal repeater, but the voice currents are transmitted over the upper side until the 4 -wire terminating set at the west terminal is reached, where the energy is divided between the balancing network ad the 2 -wire line west connected at that terminal. This transmiss in is indicated by the heavy line in Figure 98 (B) marked "direct tr ismission". With perfect balance between the network and the $2-$ w: e circuit at the west termis no further effects will be produced.

As we have already noted, however, there is always some unbalance at the 4 -wire terminating sets and, consequently, a small current passes into the lower branch of the circuit at the west terminal and is propagated back to the talking station at the east terminal, as is indicated by the line marked "lst echo-talker". This is heard at the east terminal either as side-tone, or as a distinct echo if the time of transmission around the circuit is great enough. Such currents are called unbalance or echo currents affecting the talker.

Due to unbalance at the east terminal, another current, derived from the first echo affecting the talker, is propagated from the east to the west, forming another echo which is heard by the listener. This is indicated by the line marked "lst echo-listener" in Figure 98 (B). Such currents are called unbalance or echo "rrents affecting the listener. The first echo current affecting the listner through the unbalance at the west terminal gives a "2nd echo-talker" current at the east terminal. This action may go on indefinitely.

If the total loss ${ }^{4}$ around the unbalance current path, including the loss through the 4 -wire terminating sets, is greater than the total gain of the repeaters, the successive echoes die out rapidly. With a small margin, there may be several echoes of sufficient magnitude to affect the persons at each end of the circuit and if the loss becomes equal to or less than the gain, the circuit will sing.

The degree of subscriber annoyance due to echo is determined both by the loudness of echo, and by length of delay. On a 4 -wire circuit all echoes, except the "1st Echo-Talker", are normally atten"ated to a point where they do not cause trouble. The loudness of the "lst Echo-Talker" is determined by three factors (see Fig. 99):


FIG. 99. ATtEMUATION IN THE ECHO PATHS

1. The circuit loss from the talker to the listener's end. That is, the loss (ATT. 1) at which the circuit is operating in that direction.
2. The loss across the four-wire terminating set at the listener's end due to the impedance mismatch between the network and the 2 wire terminal facilities. The greater the mismatch, the lower the return loss, and the greater the echo current.
3. The circuit loss (ATT. 2) from the listener's end back to the talker. This is the loss at which the circuit is operating in that direction.

The delay of this "Echo-Talks." is the time it takes the talker's speech to go to the far end of the circuit and back again. Quantitavely, it is twice the circuit length divided by the velocity of propagation.

People will tolerate a much louder echo if the delay is short than they will if it is long. If the delay is long, the echo volume must be much lower (i. e. circuit loss must be greater) if the talker is to be kept happy. There is also a very wide difference between individuals as to the relative loudness and delays which they will tolerate. Table 5-2 gives the average tollerance to talker echo in terms of the minimum rowd trip loss in the echo path required to attenuate the echo just enough for different round trip delays in milliseconds. In other words this table shows the relation between round trip delay and the minimum round trip loss required to attenuate echo enough to provide commercially tolerable conditions in the judgment of an average listener.

Table 5-2
Loss and Delay for Average Tolerance to Echo

Round Trip Delay (Milliseconds)

Minimum Ioss in Echo Path in db 'Satisfactory to Averas Observer)
1.4
11.1
17.7
22.7
27.2
30.9

While the 4 -wire circuit offers ${ }^{4}$ only one round-trip path for echo currents, a 2 -wire circuit involves a large number of such paths. If the sections of line between 2 -wire repeaters were perfectly constructed, if the impedance of the repeaters were such as to form perfect terminations for the sections of line, and if the networks balanced these perfect lines perfectly, transmission could take place from one end of the circuit to the other without setting up any unbalance currents except those reflected from the terminals. Such an ideal circuit would give the same performance as a 4 wire circuit with equal terminal unbalances.

In practice, however, in the 2 -wire circuit there is some unbalance on each side of each repeater so that, as the direct transmission passes through each successive repeater, an unbalance current is set up which travels back toward the talker, giving an echo current for each repeater in the circuit. Each of these first echo currents in turn sets up an echo current traveling toward the listener at every repeater that it encounters, and each of the se echo currents sets up another echo current at each repeater it encounters. This process continues indefinitely but the successive echo currents are attenuated rapidly to inappreciable magnitudes. The action, however, does give rise to a very large number of echo currents. The final effect upon the talker or listener, naturally, depends upon the resultant of all these currents.

These unbalanced currents increase volume as the overall net equivalent attenuation of a circuit is decrea $\}$, due to raising the peater gains. For each circuit, therefcre, che will be a
tain minimum permissible net overall eqi valent attenuation becaus: f the unbalances which are pres it in the circuit. Ar. further incsease in the repeater gains will cause the unbalance currents to become too large to be tolerated. Furthermore, since
each additional repeater in the circuit increases the number of unbalance paths, the total echo effect tends to limit the total number of repeaters that can be operated in tandem in a 2 -wire circuit having a practicable overall net equivalent.

The methods that can be used in general to control echo are:
a. Reduce impedance mismatches (i.e. improve circuit by reducing reflections or by increasing return losses).
b. Control the circuit net loss, ATT. l and ATT. 2, in Figure 99.
c. Choose facilities having a suitable velocity of propagation.

Singing, i.e. sustained oscillations or "repeater howl," is another factor in circuit design. Singing might be thought of as echo which is completely out of control. This can occur at the frequency at which the circuit is resonant. Under such conditions, the circuit losses at the singing frequency are so small that oscillation will continue even after the impulse that started it has ceased to exist. This can happen also when the total round trip loss of the current is smaller than sum of the repeater gains.

In case of 4 -wire circuits ${ }^{4}$ the possible minimum net equivalent attenuation, in so far as echo effects are concerned, is limited only by the extent of the unbalance at the two terminals. If the minimum net equivalent attenuation is still too high, echo suppressors may be inserted in the circuit to break the echo paths and thus permit reducing the overall equivalent to the desired value.

The principle of operation of this so called "full echo suppressor" may be understood by referring to Figure 100. Hybrid coils 1 and 2 are inserted in each side of a 4 -wire circuit (or in each side of a 2 -wire circuit). The bridge points $N, M$ of hybrid coil 1 are connected to amplifier $F$ and the bridge points $H$, $G$ of the coil 2 to amplifier E. The outputs of E and F are connected to a common rectifier R. The rectified d-c output of the latter is fed through the windings of relays $B$ and $A$ to ground, as shown. Relays A and B in this circuit operate differentially.


FIG. 100. SIMPLIFIED SCHEMATIC OF ECHO SUPPRESSOR

When there is no energy, or only low levels of speech or noise in both sides of the 4 -wire circuit, the d-c rectifier output current will be at its normal value which lies between 3.9 mA , the minimum operating current for relay A , and 1.48 mA , the maximum release current of relay B. Thus both resistance-capacitance networks connected to the series windings 5 and 6 of the odd hybrid coil 2 and to windings 7 and 8 of the even hybrid coil 1 are short circuited by the closed contacts of relays A and B. Relatively high speech energy in the odd side of the circuit, when speech is transmitted from $Z_{1}$ to $Z_{2}$ will cause the rectified current to decrease below $1 .{ }^{4 ?} \mathrm{~mA}$. In this case the normally operated : $\quad \mathrm{y}$ B releases, while A i unaffected. Therefore, the ground connection is removed from the even network D. The simplified diagrand of Fig. 101 shows the circuit cemations at the hybrid coil 1. Eerein coils $7,8,9,10,11 \& 12$


FIG. 101.
shown in Figure 100 are all wound on a common core; coils 9 \& 11 are combined in one coil $9^{\prime}$ and coils $10 \& 12$ in $10^{\prime}$ respectively. This is permissible since we have a loop ANM (consisting of G; $Z_{3}$; 11; $\mathrm{Z}_{\mathrm{F}}$ and.9) in which 9 \& 11 are on the same core; thus we can shift the position of 11 and combine $9 \& 11$ into $9^{\prime}$. The same modification is performed on $10 \& 12$.

When a party from the $Z_{1}$ side talks to party on the $Z_{2}$ side over the odd line there will be alst Eciu-Talker which will try to return and pass from $Z_{3}$ to $Z_{4}$ over the even line. This echo energy is represented by generator $G$ in Fig. 101; it will produce current I3 which will develop the voltage $\mathrm{V}_{\mathrm{Z}_{\mathrm{F}}}$ across the impedance $\mathrm{Z}_{\mathrm{F}}$. Due to transformer action between coils $9^{\prime}, 7,8$ and $10^{\prime}$ a voltage $\mathrm{V}_{10}{ }^{\prime}$ across coil $10^{\prime}$ will be developed. Choosing proper coil turn-ratios and winding directions and proper values for $\mathrm{Z}_{\mathrm{F}}$ and $\mathrm{Z}_{\mathrm{N}}$, the voltages $V_{Z_{F}}$ and $V_{10}$ are equal and opposite and the result is zero current $\mathrm{I}_{4}$ in $\mathrm{Z}_{4}$. Thus there will be no transfer of energy between $\mathrm{Z}_{3}$ and $\mathrm{Z}_{4}$ and echo is stoped in the even line. In practice there would be about 40 db attenuation here.

When a party from the $\mathrm{Z}_{1}$, side stops talking to the party on $\mathrm{Z}_{2}$ side there is a return to the original conditions. The d-c from the rectifier becomes now larger than 1.48 mA (but still smaller than 3.9 mA ) and relay B operates grounding the right side of the D network. Now $\mathrm{Z}_{\mathrm{N}}$ becomes zero, current $\mathrm{I}_{4}$ starts flowing and there will be an energy transfer from $\mathrm{Z}_{3}$ to $\mathrm{Z}_{4}$, with about 1.8 db of loss.

Now going back to Fig. 100 when speech is transmitted in the even line from $Z_{3}$ to $Z_{4}$, the rectified current will increase above 3.9 mA and the relay A will operate while $B$ will be unaffected. As a result ground will be disconnected from the right side of odd network C. Transmission of echo coming from Z: will be blocked for exactly the same reasons as explained before.

Experience indicates that generally se sfactory operation will not obtained with more than two echo suppressors operated in tandem, because clipping and lockout result in high percentage in such cases.

Echo suppressors are used where delays of more than 45 milliseconds are encountered. An echo suppressor is a voiceoperated device which, while one subscriber is talking, inserts as much as 40 db loss in the opposite direction of transmission - the path over which the echo would return. Although they effectively suppress echoes, echo suppressors introduce their own impairments by sometimes clipping the beginnings and ends of words. Another more serious problem occurs on multilink connections where 2 or more circuits equipped with echo suppressors must be switched together. It is possible for each subscriber to talk simultaneously and gain "control" over the echo suppressor nearest him. In this case both directions of transmission will have a high loss inserted in them and a condition of "lock-out" is said to exist, in which neither subscriber can be heard by the other. Because of these effects, echo suppressors are used sparringly in the plant. (Under the present toll switching plan, the suppressors are used only between the Regional Centers ( $R C$ ) or on trunks between regions which would have a VINL design of more than 2.5 db . When echo suppressors are used, the VNL is set equal to 0.5 db . The Via Net Loss or VNL will be discussed later in Chapter 6.)

## References for chapter 5

3. From John D. Ryder, NETWORKS, LINES AND FIELDS, 2nd Edition. 1955, by permission of Prentice-Hall, Inc., Englewood Cliffs, N. J.
4. (C) American Telephone and Telegrapl Company, 1961.

CHAPTER 6

## TRANSMISSION DESIGN

## 6. 1 INTRODUCTION

The telephone customer wants to enjoy an easy exchange of information and to recognize a familiar, natural sounding voice. The circuit to satisfy this desire must deliver speech sounds with a satisfactorily high volume and bandwidth and a tolerable, low amount of disturbing distortion, echo, crosstalk and noise. Part of the problem, as a matter of fact, is to determine or arbitrarily set the maximum or minimum values for these factors (i.e. establish "objectives") and then, to adjust or correct each transmission facility until these objectives are reasonably met.

Over the years, a number of transmission "standards" have been adopted. They all established a limiting transmission loss between any two subscribers, and provided for the distribution of this over-all loss among the several types of circuits making up a connection. The only variable under the control of the transmission engineer was loss, and every reduction in loss was purchased with heavy expenditures for copper. Each end (central) office (EO) was assigned a maximum local loop loss which was determined by a "loop and trunk study." These studies were made for each exchange, to determine what divisior of loss between loops and trunks would give the lowest over-all cost. Naturally, this balance was influenced heavily by the trunking plan and the local office's relation to the national switching network. The advent of improved station instruments, e. g. type 500 subsets, and relatively inexpensive repeaters and carriers permitted a new look in transmission design for they gave the transmission engineer a new tool - cheap gain. Thus, volume requirements become relatively easily attained and quality and other factors become the primary criteria in transmission design.

With regard to distortion and noise as contributors to good quality (or lack of it), their control is pretty much a matter of application of the most sophisticated apparatus available and proper maintenance. This, plus the conclusion that the greater the perfection the higher the cost, can be seen from the remarks below about noise and distortion.

Noise, as it was mentioned before, has been defined as unintelligible or unwanted sounds in a transmission system which tend to mask the desired speech transmission and can be subdivided into two general classes-internal and external.

Internal noise is generated within the $t$ nsmission system and consists generally of the following compoi.ents:
a. Thermal noise due to random movement of electrons.
b. Contact noise caused by minute electrical arcs (e. g. switching)
c. Noise from generators used for charging, etc.

External noise is induced into the transmission system from external sources and consists of the following general types:
a. Ambient (or room) acoustic noise at the talker and listener locations.
b. Impulse noise from the operation of dial office or telegraph equipment.
c. Atmospheric noise.
d. Induction from radio systems.
e. Power line induction.

The general approach to noise is to find ways to eliminate it rather than to adjust circuit losses to compensate for it.

Distortion, to some degree, is always present since there are no electrical transmission systems which permit the perfect reproduction of intelligence. As was mentioned before, the three basic types of distortion are:
a. Phase distortion (delay distortion) due to differences in the speed of propagation for various frequencies.
b. ttenuation distortion due to differences in circuit losses for various frequencies. This distortion is sometimes designated as the frequency response.
c. Non-linear distortion due to the presence of nonlinear impedances.

While the human ear is relatively inser. ${ }^{\prime \prime}$ ve to phase distortion; this type of distortion can be contrc.e... : y the use of phaue equalizers if necessary.

Attenuation distortion, which tends to impair the intelligibility of speech transmission, may also be controlled by the use of equalizers. (Other methods used in the carrier systems are frequency inversion and frogging). Non-linear distortion may arise when elements are present in the circuit in which the relation between voltage and current is not substantially linear, such as in overloaded vacuum tubes or transistors, saturated core transformers, and varistors. Prevention of distortion is accomplished, in general, by proper design and maintenance of equipment.

Having discussed noise, distortion and volume in general terms, at least as far as this text is concerned, let us examine some of the other factors affecting transmission quality. We will examine them as they apply to specific segments of the total telephone plant. An example of the large variety of facilities that might be used in handling a telephone call is illustrated in Figure 102.

## 6. 2 SUBSCRIBER LOOP DESIGN

Loop design is very important, since 2 loops are a part of every telephone connection and,in addition,loops account for a high percentage of the plant investment.

A typical pattern of loops is shown in Figures 102 and 103. Before the advent of 500 type telephone set both transmission (voice) and signaling had to be considered in the loop design. The improved performance of the type 500 telephone set permits us to neglect transmission considerations al to determine loop length on the basis of the maximum DC resistance which will enable supervision (ON/OFF hook signals) and pulsing to reach the control office equipment capabilities of central office equipment. Therefore present subscriber loop design is called "resistance design". In the newer crossbar offices the total subscriber loop can be 1500 ohms. Of this 200 ohms is for subsets, and about 100 ohms for temperature effects, loading coils, etc., leaving about 1200 ohms for the actual wire pair.

Very long suburban loops present a special problem. Meeting the office resistance range at the end station may require considerably heavy gauge cable. Here an economic balance must be struck between a cheaper high resistance loop, with less copper and the expense of adding long line equipment such as repeaters.

Although, as said before, the resistance design is the factor which limits gauge and length it is still necessary to find the $d b$ loss (or rating) of the subscriber loop to determine if overall objectives are met.

| E0 | $\begin{aligned} = & \text { END OFFICE } \\ & \text { (CLASS 5) } \end{aligned}$ |
| :---: | :---: |
| TOLL | TOLL CENTER (CLASS 4) |
| PRI | $=$ PRIMARY CENTER (CLASS 3) |
| $s$ | $\begin{aligned} & =\text { SECTIONAL CENTER } \\ & (C L A S S ~ 2) \end{aligned}$ |

TAN = TANDEM OFFICE


FOR CLARITY IN THIS DIAGRAM, NONSTANDARD SYMBOLS ARE USED FOR THE END OFFICES.


FIG. 102 A SMPLIEPET TM


FIG. 103. POSSIBLE LAYOUT OF LOOPS AND TRUNKS IN PART OF AN EXCHANGE AREA.

## 6. 3 EXCHANGE AREA TRUNK DESIGN

These trunks are also called interlocal trunks or interoffice trunks. By exchange area is understood the territory included within the boundaries of an exchange (one city, town or village with environs).
2. due to variety of components which comprise the make-up of this segment the problem is a little broader than in the case of subscriber ${ }^{8}$ s loop. The loss of trunk is usually assumed to be made up of wire losses, originating and terminating office losses, junction losses and other. In practice trunk los 3 can be taken as a complete facility loss, minus the gain (negativ loss) that is inserted if repeaters are used.

The design objective is to make all "trunk losses" as 'cos as practicable and independent of "loop" losses. With this approach and taking costs and new instrumentalities in account, the design objective for the next few years is a 4 to 6 db maximum loss for exchange area trunks and for the time after that it is about zero db . In metropolitan areas, over-all economy can frequently be secured by routing all or a portion of the interlocal calls through one or more intermediate switching points. The design of these tandem and intertandem trunks should be such that the sum of the losses of the 2 or more trunks in any possible connection between local offices (EO) will not exceed the 4 to 6 db objective. A further limitation is also imposed: none of the possible connections between 2 stations in the exchange area should have a transmission contrast over 5 db .

Note: Transmission contrast is the difference in thet loss of the exchange station pairs having maximum net loss and those having minimum loss.

The signalling on virtually all exchange area trunks is still on a DC basis. Trunk range varies between about 600 ohms and 3,000 ohms depending upon the type of Central Office. Knowing the transmission requirement (db) and maximum DC resistance allowable, the problem is to meet these objectives in the most economical manner. Transmission is controlled by the choice of conductor gauge, type of loading, whether or not the trunk is repeatered, and if so, the repeater type and locations. The resistance limitation must be satisfied by conductor gauge selection, or this limitation must be modified by the use of long trunk equipment or signalling repeaters.

## 6. 4 INTERTOLL TRUNK DESIGN

These trunks connect two cities or $\downarrow \times 0$ exchange areas.
The present day design of intertoll maks is intimately reted to the Nationwide Dialing Plan. Und . Che plan, two terminal links c... up to eight intertoll circuits, camected in tandem, a ay be encountered. Further, as a result of alternate routing, different quantities and makeups may be used on successive calls between
two points. Thus, an intertoll trunk may be operated in either of two conditions:
a. the "via condition," when the trunk is an intermediate line in a switched connection and its both ends are extended by other intertoll trunks;
b. the "terminal condition," when the intertoll trunk is terminated in an end (local) office at one or both ends. (It can then be called terminal link or tollconnecting trunk).

The loss of each link which may be used in a connection must be low, in order to provide adequate transmission on all calls and to avoid large differences in transmission on successive calls between the same two places. While the ideal would be to operate all circuits at zero loss, to make the transmission (volume) independent of the number of cirucits used in a connection, the distances in the U.S. and Canada are so great that even the best types of transmission facilities must be operated at some loss to insure suitable transmission characteristics. This minimum loss is that value in db which will provide satisfactory service in the VIA condition with a tolerable level of echo, singing, noise and crosstalk and is referred to as "Via Net Loss".

As previously stated, we can best control noise and crosstalk in the design, layout and maintenance of associated plant-not by adjusting circuit losses. In other wo-*s the preferable approach is to get at the basic cause of noise anc crosstalk and not to cure only the symptoms. Generally speaking, circuit losses that satisfactorily control echo will prevent singing. Therefore, from a practical standpoint, echo is the controlling factor in determining Via Net Loss and in intertoll trunk design.

We have already noted that the amount of echo that a customer will tolerate is governed by two facets: loudness and delay. Echo delay is determined by the propagation velocity and length of the circuit. For a facility of given length, the faster the circuit, the less the delay. Since the bulk of our circuits including intertoll trunks of any appreciable length are on carrier and carrier has speeds that approach that of light, the theoretical maximum, echo loudness and not the delay is the variable which offers the best promise for control. For a given circuit and for a given talker volume, echo loudness is fixed by the losses in the "round trip" echo path. Since circuits are usually operated at the same loss in both directions, we can say that the talker echo loss is twice the oneway loss of the circuit plus the return loss at the listener ${ }^{5}$ s termination, caused by reflections etc.

If this echo loss results in an echo loudness within the average talkeris tolerance limit, we have the design requirements for the loss of an intertoll trunk. Stated algebraically:

Round-trip circuit loss + Terminal (lis ener) return loss = The talker's minimum loss in echo pat? satisfactory to the average observer.
or:
One-way loss = 1/2 (Minimum loss in echo path satisfactory to the average observer - Terminal listener return loss)

Determination of the "minimum loss" (under "via" and "terminal" conditions) is discussed later.

## 6. 41 SWITCHING PADS

An intertoll trunk may be operated as was mentioned before in the VIA or Terminal condition. In the latter the trunk is connected to a local office (EO) at one or both ends and it is then called the toll connecting trunk. When in the VIA condition, a trunk is either switched on a 4 -wire basis or a 2 -wire basis in an office where care has been taken to insure a good impedanc. termination. In either case, the possible echo return path the switching office has been effectively eliminated. This is not the case when a trunk is in the terminal conditions.

Experience has shown that a trunk must have a higher loss in the terminal than in the via condition. For practical operation of the intertoll network, we must be able to use the same trunk for terminal and VIA service. This dual operation is made possible by introducing a loss at the end of an intertoll circuit when it is in the terminal condition (connected to a local office). Until recently, this loss was provided by a pad in the intertoll trunk equipment. The equipment was arranged so that the pad remained in the connection when the intertoll trunk was switched to a toll connecting trunk connected to local office. However, the pad was manually remove? when the intertoll trunk was switched to another intertoll trunk, since it served no useful purpose. In later types of equipment this is done automatically. These pads are known as switching pads or simply "S" pads. The early circuits requia ad pads of 3 or 4 db but improved circuits of today require 2 db pads. ee benefits obtained by $3^{\prime \prime}$ pads are:

1. Prevention of amplifier ovazoading due to exces ive talker volume on short loops.
2. Control of crosstalk, again due to excessive talker volume on short loops.
3. Improvement, that is increase, in return loss by twice the loss of the S pad, for echo currents must pass through the pad twice while speech currents pass through the pad only once. Thus echo is considerably reduced and conditions look more like matched.

## 6. 42 VIA NET LOSS AND VIA NET LOSS FACTORS

We specified before the condition which established the loss in an intertoll trunk, which is known as Via Net Loss. It was defined as: The minimum loss (in db ) which will provide satisfactory service in the VIA condition with a tolerable level of echo, singing, noise and crosstalk.

For practical purposes, a procedure has been developed for determining Via Net Loss (VNL) for each type of facility in the VIA condition as a function of the length of the link. The procedure makes use of a set of factors called Via Net Loss Factors (VNLF). The appropriate VNLF, multiplied by the one way circuit length gives the VNL at which each circuit can be operated regardless of the number of links in the connection. Thus:

$$
\text { (VNLF) } x \text { (CIRCUIT LENGTH) }=\text { VNL }
$$

In Figure 104 via and terminal losses are shown. The total of the Via Net Losses of the links a-r the losses of the two switching pads is then defined as the Termial Net Loss (TNL). Thus:

$$
\mathrm{TNL}=\sum \mathrm{VNL}+4 \mathrm{db}
$$

Like the loss of the pads, each db of VNL is worth 2 db in suppressing echoes, attenuating first the signal on its way to the "mirror", and then attenuating the echo on its return trip.

In chapter 5, table 5-2 was introduced to illustrate, in a general way, the relation between minimum loss in echo path in db (or echo loudness) and round trip delay that would be tolerated by an average telephone user. This same information is plotted as curve X in Fig. 105. However, here the permissible working one way net loss is plotted instead of round-trip losses, versus roundtrip delay in milliseconds.


LEGEND:


FIG. 104. VIA AND TERMINAL NET LOSSES

Now that we have explored the echo problem more fully, we can make a better interpretation of curvo X. For each value of round-trip delay, the loss indicated by the urdinates of curve X is an empirical evaluation of the equation devel before:

$$
\begin{aligned}
\text { Une way loss } \mathrm{db} & =\frac{1}{2}(\text { Minimum Loss in Tcho Path - Terminal } \\
& \text { Return Loss) } \mathrm{db} .
\end{aligned}
$$

In making these empirical evaluations, statistical techniques were used to consider the following factors, assuming that the connection has a single intermediate link:

1. The manner in which "echo tolerance" varies among a large number of talkers.
2. The manner in which a large number of measurements of actual terminal return loss at Toll Center varied from the empirically found average 11 db .
3. The manner in which circuit losses have been found to depart from assigned values.

And finally, the equation was so weighted, that considering all variables, the probability of a talker being slightly disturbed by echo is only one chance in a hundred (1\%).

Now let's see how curve $X$ can be used in determining Via Net Loss Factors.

Suppose we start by considering the via net loss necessary in a single link circuit having zero round trip delay. Since our circuit has one intermediate link, this link will be in the terminal condition, connected at both ends to local office, and thus will have an S pad at each end. As was discussed before, consideration of noise and crosstalk, singing and echo has indicated that in the present state of the telephone art an S pad of 2 db is the best value of switchable loss. So our circuit will have a loss of at least $4 \mathrm{db}-$ the loss of two $2 \mathrm{db} S$ pads at each end of the link. However, our intermediate link must be capable of operating in tandem with other links. Statistical studies indicate that an additional 0.4 db per each link intermediate or terminal must be allowed if links are to be operated in tandem, since there is an increased probability that the over-all loss will deviate from the assigned value. This 0.4 db added to 4 db for two S pads makes the minimum loss 4.4 db for a circuit with a single link and a zero round trip delay. We have established now the minimum practicable loss for the shortest possible circuit. We designate this condition by point A on Fig. 105.

Now let us consider the longest circuit that we may have to operate at Via Net Loss.


RESULTS OF TALKER ECHO , LLERANCE TEST

FIG. 105
6.12

In Chapter 5 Echo Suppressors were discussed. It will be recalled that this is a voice operated device which eliminates talker echo. Experience has shown that satisfactory operation cannot be expected on connections with more than two echo suppressors in tandem, because of clipping and lockout in a high percentage of cases. Under the present view of the toll switching plan, it appears that circuits must be operated up to about 2500 miles one way without echo suppressors if the possible number of echo suppressors is to be limited to two. A 2500 mile carrier circuit will have an estimated round-trip delay of about 45 milliseconds. This is the upper limit for VNL operation that we are seeking. Longer circuits can be operated at a loss independent of echo restrictions by inserting an echo suppressor. By designing all circuits with less than 45 milliseconds delay echo suppressors are not required. Thus the switching plan will permit connection between any two telephones in the United States without having more than two echo suppressors in tandem in case circuits are longer than 2,500 miles. Having determined the range of round-trip delay, 0 to 45 milliseconds, over which circuit loss must be dependent upon echo limitations, we can turn to the development of Via Net Loss Factors.

Referring again to Figure 105, and starting at point A, which is at 4.4 db on the ordinate axis, a straight line $Y$ can be drawn through the point $A$ and another point $B$. At point $B$ the ordinate at 45 millisecond intersects the curve $X$. It will be seen that the slope of line $Y$ is the one-way net loss in db per millisecond round-trip delay ( $0 \cdot 1 \mathrm{db} / \mathrm{ms}$ ). The one-way net loss corresponding to any selected point on iine $Y$ has an ordinate:

$$
Y=4 \cdot 4 \mathrm{db}+0.1 \frac{\mathrm{db}}{\mathrm{~ms}} \times \underset{\text { (milliseconds } \text { delay) } .}{\text { round }-t r i p ~}
$$

With the velocity of propagation $V$ known for the various types of facilities employed, the slope of line $Y$ can be reevaluated in terms of db per mile instead of db per millisecond, a much more workable form. This is done by dividing 2 times the slope of line $Y$ by the velocity of propagation $V$. We use here 2 times the slope instead of just the slope since db's in the slope are for oneway while the delay of echo is for round-trip.

The resultant db per mile figure is called the Via Net Loss Factor (VNLF).

## Example

Intertoll trunks on $K$ carrier have a velocity of propagation V of 105,000 miles per second or 105 miles per millisecond. The circuit is 1000 miles long, one-way. Find Via Net Loss for it. The solution is:

$$
\text { VNLF }=\frac{2 \times 0 \cdot 1 \mathrm{db} / \mathrm{ms}}{105 \mathrm{miles} / \mathrm{ms}}=0.0019 \mathrm{db} / \mathrm{mile}
$$

To determine the lowest loss at which an intertoll trunk can be operated satisfactorily from an echo standpoint, it is only necessary to multiply the VNLF of the facility by its one-way length and add the factor 0.4 db . Table 6-1 gives the Via Net Loss Factors of commonly used long-haul facilities in the telephone plant. As an example, the VNL of this 1000 mile trunk composed of K carrier facilities will be $(0.0019 \times 1000)+0.4=2 \cdot 3 \mathrm{db}$, when just obtained VNLF is used. The Terminal Net Loss of this trunk will be:

$$
\mathrm{TNL}=\mathrm{VNL}+2 \mathrm{~S}=2 \cdot 3+4=6 \cdot 3 \mathrm{db} .
$$

## TABLE 6-1

## VIA NET LOSS FACTORS FOR REPRESENTATIVE TELIEPHONE FACILITIES USED IN THE BELL SYSTEM

## FACILITY

19H 88-50 side
19 H 88-50 phantom
19B 88-50 side
19H $44-25$ phantom
19H 44
$19 \mathrm{H} 44-25$ phantom -
Open wire, voice frequency $\quad 01$
Open wire carrier (all types)
Type K or N carrier - 0015
.

Type L carrier -. . 0015
Carier circuits on radio 0015
H8o n paired exchange type cables, any gauge

04
D88 on
any gauge
. 03

The Via Net Loss Factors for 4 wire trunks are derived from the lowest losses permitted from a talker echo standpoint which, under conditions previously discussed, also meet the current objectives for singing, crosstalk and noise. The factors for 2-wire trunks have been increased over the echo requirements on a judgment basis to allow for the effect on singing as well as on talker echo of additional return loss paths at intermediate repeater points. In 2wire voice frequency facilities it is necessary to support the required repeater gains by adequate impedance balancing to permit working at the losses determined by the Via Net Loss Factors. It should be pointed out that 2 -wire intermedia,te trunks are not equivalent in terms of loss and delay to high velocity 4 -wire trunks even though the 2 -wire link is operating at "Via Net Loss".

## OFFICE BALANCE

The Via Net Loss Factors shown in table 6-1 assume that all interconnection of intertoll trunks at intermediate toll offices will encounter no appreciable echo paths at the switching centers. In 4 -wire switching offices such an echo path is automatically eliminated by retaining 4 -wire operation through the switches. By careful engineering and maintenance, 2 -wire switching systems can be made to give satisfactory transmission performance.

On connections through an intermediate switching point, echo can arise due to unbalance between the 2 -wire office equipment and associated wiring on one hand and the balancing network $A$ on the other hand. See Figure 106. The lat'r may be located either in the terminal repeater (in some repeaters) or in the 4 -wire terminating sets. By using capacitors for balancing office cabling as outlined below, echo can be held to such small values as to cause little or no impairment on the 2 -wire switched connections.


FIG. 106.

In order to interconnect a 2 -wire circuit at random at switching points (see Figure 106), a single type compromise balancing network A must balance any of the 2 -wire circuits in the toll switching office. It follows that the impedance of all the 2 -wire circuits terminati $g$ in the toll switching office must be equal to that of the A network within reasonable limits of precision. A nominal toll -ffice impedance of 600 ohms (looking into 2 -wire circuits) wat selected some time ago. Since the present standard outgoing trunks are usually H-88 loaded cable, crossbar tandem office impedance for toll switching is considered to be 900 ohms. The 4-wire circuit terminal impedances $C$ are designed to match the nominal impedance $B$ of the 2 -wire office. However, since supervisory signals are often transmitted nver the tallking path conductors, capacitors are required in cerk "cations to isolate su-ervisory path of the signalling circuit. ( ) mently the toll sw ching office impedance $B$ is assurved : 600 or 900 ohms Aomonding on the type of office) in serims a 2 mf capacitom, and the . rcuit impedances are so desig. .

$R_{2} C_{1} C_{1}=$ PARTS OF BALANCING NETWORK
$C_{2}=$ "DROP BUILDING-OUT" CAPACITORS IN TRK CKT
$D=$ REPEATER

PIG. 10\% TYPICAL CONNECTION OF INTERTOLL TRUNRS AT A SWITCHING OFFICE

The office cable required (see Figure 107) to extend the circuit terminal to the switching point (switches or switchboard) modifies the input impedance $B$ of the 2 -wire circuit. Also there are different lengths of cable in different 2 -wire cirucits and this difference may be great enough to impair the office return loss. Where the impairment is too great the capacitance of each switching path (only one is shown) is adjusted to a uniform value by means of the capacitors $\mathrm{C}_{2}$ shown in Fig. 107. These are known as "drop building out capacitors". Having restored uniformity to the "terminal impedance", the capacitor $C_{1}$ (known as network building out) is adjusted to a value such that the compromise network will balance the 2 -wire circuit terminal impedance $B$ as modified by the office cable and building ant capacitances $C_{2}$ on the two circuits that are interconnected.
6. 17

The overall effect is satisfactory if the offices are engineered so as to keep the length of cable to moderate amounts. In particular, if long series runs are necessary, the use of larger conductors is advantageous. Except in a very small office, conductors smaller than 22 gauge should not be used; 19 gauge is preferable in the tie cables. The practical design limitation is to restrict total loop resistance (between 2 termine 1 points and not a subscriber's loop) of all wiring and equipment hetween 4 -wire terminating sets to 45 ohms in 600 offices and ; ohms in 900 ohm offices.

With the exception of a few isolated cases the following summarizes the nominal switching impedance currently used in Bell System:

Local offices 900 ohms No. 5 crossbar-local 900 ohms
Manual toll offices 600 ohms
No. 5 crossbar-toll 600 ohms Step-by-Step (SXS) intertoll 600 ohms No. 4 type toll crossbar 600 ohms Crossbar tandem 900 ohms

If the average return loss for all the intertoll trunks between cities or exchange areas is equal 27 db or more, the 2 -wire switching office may be considered to be equivalent to 4 -wire switching. If this degree of balance is not achieved a " $B$ " factor is assigned to the switching office. The "B" factor is the additional loss assigned to each "intertoll trunk" terminating at a 2 -wiwa switching office to provide the same echo performance as with 4wire switching. In other words, there is a transmission penalty whenever switching is done on a 2 -wire basis unless the office caf be balanced properly and maintained in balance. The B factors are as follows:

> | Average |
| :---: |
| Office Balance |

## B factor (db)

| 27 | 0 |
| :--- | ---: |
| 25 | 0.1 |
| 23 | 0.2 |
| 21 | 0.3 |
| 19 | 0.5 |
| 17 | 0.8 |
| 15 | 1.2 |
| 13 | 1.7 |

It is desirable to avoid the need of assigning a $B$ factor since this adds loss in each switched connection and, if occuring at $i$ nimber of offices on a connection, will inc ease the number of calls that would be unsatisfactory due to over-all transmission loss being too high.

## References for Chapter 6

1. From Fletcher's Speech and Hearing in Communication, Copyright, 1953, D. Van Nostrand Co., Inc., Princeton, N.J.
2. From Everitt \& Anner's COMMUNICATION ENGINEERING. 3 ed. Copyright 1956 McGraw-Hill, Inc. Used by permission of McGraw-Hill Book Company.
3. From John D. Ryder, NETWORKS, LINES AND FIELDS, 2nd Edition. 1955, by permission of Prentice-Hall, Inc., Englewood Cliffs, N. J.
4. (c) American Telephone and Telegraph Company, 1961.

[^0]:    -ae to the properties of the AC Wheatsto: Age in balanced .ndition, there will be no flow of $\epsilon$ uer: vewee, the output of and the input to $Z_{4}$, as well as bet se output of $Z_{4}$ nd who to $\mathrm{Z}_{3}$. Disregarding gains $C . \angle_{3}$ and $\mathrm{Z}_{4}$ amplifiea , the power quiput of the complete repecter is approximately only 25 of that which would be obtained if these amplifiers were used in tandem and in one direction.

