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Analysis of a Nondegenerate Two-Photon Giant-Pulse Laser

Introduction

In a recent article[1] to which we shall refer as [*S&B*], a two-photon laser was proposed and analyzed. This device consists of a cavity resonant at frequency ν_A and containing ions of type B with an inverted population N_B/V between levels separated by an energy difference $h\nu_B$ such that $\nu_B = 2\nu_A$; it is necessary that the system not lase at ν_B, which criterion can be satisfied by low reflectivity of the cavity at frequency ν_B, by strong parasitic absorption in the laser material near frequency ν_B, or preferably by a choice of ion such that the transition ν_B is highly forbidden to a single-quantum process. The authors of [S&B] thus show that a certain priming density of photons of frequency ν_A will provoke the simultaneous emission from the inverted population N_B of pairs of photons ν_A at a rate exceeding the cavity loss, the process diverging until the population inversion is eliminated.

It is the purpose of this communication to show by a very similar analysis that the same system of ions N_B in a cavity resonant at two frequencies ν_A and ν_C, such that $\nu_A + \nu_C = \nu_B$, may be primed at ν_A with a number of photons small compared with N_B and will yield two giant pulses simultaneously at frequencies ν_A and ν_C. We consider the energy level diagram of Fig. 1. The relaxation of the requirement of [S&B] that $\nu_B = 2\nu_A$ leads to the following: (1) it allows the use of metastable levels ν_B such that $\nu_B \gg \nu_A$ and thus makes available high-intensity laser output in a new short-wavelength range;[2] (2) it allows the production of new laser lines in addition to the amplification of known ones; (3) it eases substantially the problem of designing a system to exhibit the unique fast rise-time characteristics of the multiple-photon laser, which are discussed later in this communication; and (4) it allows the ready production of difference frequencies from the interaction, in a suitable nonlinear medium, of the automatically simultaneous giant pulses.

Equations for the photon population

To the accuracy required for our purposes now, the analysis of [S&B, Eqs. (1) to (13)] makes plausible the following rate equations:

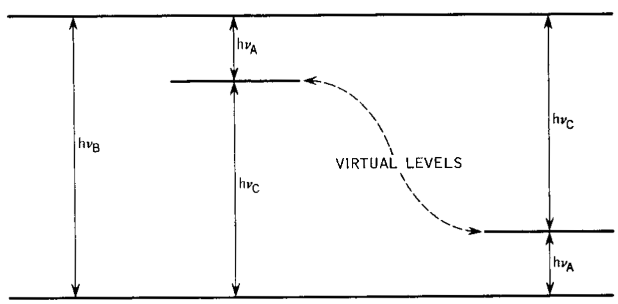
$$\frac{dS_C}{dt} = B_1 S_C S_A N_B - \frac{S_C}{\tau} \qquad (1)$$

$$\frac{dS_A}{dt} = B_1 S_C S_A N_B - \frac{S_A}{\tau} \qquad (2)$$

$$\frac{dN_B}{dt} = -B_1 S_C S_A N_B \qquad (3)$$

in which S_A and S_C are the cavity populations of photons of frequency ν_A and ν_C respectively. The cavity decay-time τ is assumed common for the two sets of photons, and the two-photon coupling constant B_1 is given in [S&B,

Figure 1 **Energy level scheme for the two-photon laser.** *The inverted population N_B is prevented from lasing by means of low cavity Q or by choice of a very long spontaneous lifetime. The cavity has a high Q at both ν_C and ν_A, and the laser will be "primed" with an initial population $S_A(0)$ photons at frequency ν_A. (For simplicity we consider only a single mode at ν_C and ν_A).*



Eq. (16)]. The behavior of the multi-photon laser is simply treated in three time regimes: I, exponential growth of the minority photon population; II, giant pulse, during which S_A and S_C grow together and N_B approaches 0; and III, decay. We shall first discuss Regimes II and III, and then shall treat the priming requirements of Regime I.

Regime II: Development of the giant pulse

In this regime $S_A \approx S_C \equiv S$, and the terms in $1/\tau$ are negligible. Thus Eqs. (1) and (2) become

$$\frac{dS}{dt} \approx B_1 S^2 N_B \approx -\frac{dN_B}{dt} \tag{4}$$

which indicates a maximum logarithmic growth rate

$$\left.\frac{1}{S}\frac{dS}{dt}\right|_{\max} \approx B_1 S N_B(0) \approx B_1 N_B(0)^2,$$

which may far exceed the logarithmic growth rate of a conventional giant pulse laser. Eq. (4) would yield the solution

$$(1/S_0) - (1/S) = B_1 N_B t,$$

showing that the giant pulse total growth time is on the order of $1/[S_0 B_1 N_B(0)]$. Equations (3) and (4) are readily solved together in the form

$$d\sigma/dT = \sigma^2(1 - \sigma) \tag{5}$$

[in which $\sigma \equiv S/N_B(0)$ and $T \equiv B_1 N_B{}^2(0)t$, and in which we have noted that $N_B(t) \approx N_B(0) - S(t)$ according to Eq. (4)] with the indefinite integral

$$-\frac{1}{\sigma} - \ln\left(\frac{1}{\sigma} - 1\right) = T \tag{6}$$

giving rise to the plot of Fig. 2.

From the parameters given in [S&B], $N_B = 2 \times 10^{18}$ and $B_1 = 3.6 \times 10^{-25}$ sec^{-1}, we find the time unit $[B_1 N_B{}^2(0)]^{-1}$ to be 0.7×10^{-12} sec, and from Fig. 2 we see that

$$\left.\left(\frac{1}{S}\right)\frac{dS}{dt}\right|_{\max} \approx \frac{B_1 N_B^2(0)}{4} \approx 3 \times 10^{11}\ \text{sec}^{-1}.$$

Such enormous rates of change of population justify the neglect of the $1/\tau$ terms in this growth regime. Incidentally, they also cast some doubt on the quantitative validity of this model of a homogeneous cavity population in the presence of a growth rate corresponding to ~1 mm travel of light!

Regime III: Decay

As has been shown, the growth of the photon population and the deexcitation of all of the ion inverted population occurs in a time much less than the cavity decay-time τ. Thus Regime III is simply an exponential decay, $S_A \approx S_B \approx N_B\, e^{-t/\tau}$.

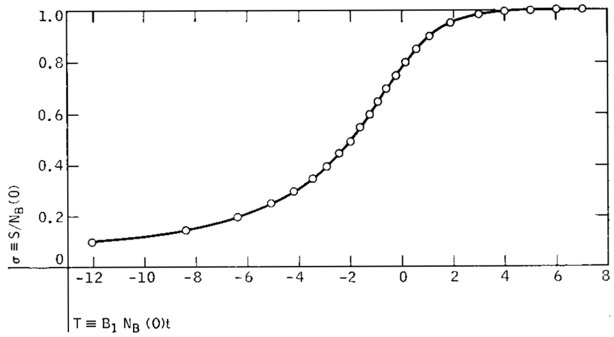


Figure 2 **Detail of Regime II: The fast-rise portion of the giant pulse.** *The number of photons of either frequency ν_A or ν_C as a function of time in the development of the giant pulse.*

Regime I: Priming conditions

• *Method I*

The priming criteria require some special discussion. We assume that the laser is primed with a substantial population $S_A(0)$ of photons ν_A. The condition for growth of the ν_C population, according to Eq. (1), is

$$B_1 S_A(0) N_B > \frac{1}{\tau} \tag{7}$$

or

$$S_A > S_0 \equiv \frac{1}{B_1 N_B \tau}, \tag{8}$$

which is identical with [S&B, Eq. (17)] except for a trivial factor of 2. Thus if Eq. (7) is well satisfied, S_C will grow exponentially with a time constant $1/B_1 S_A N_B(0)$ until S_C is no longer small compared to S_A. More precisely, one scheme for priming is to fill the cavity to a level $S_A(0)$ satisfying Eq. (8), and to allow the ν_A population to decay freely while the ν_C population grows. The condition that $S_C \approx S_A$ before S_A decays below the critical level, Eq. (8), is thus readily seen to be

$$\frac{S_A(0)}{S_0} \geq \ln\left(\frac{S_0}{1}\right) \tag{9}$$

(considering the initial "spontaneous" emission from the $[N_B + S_A(0)]$ system into the ν_C mode as being induced by the zero-point energy of the vacuum).[3] Thus Method I requires an initial priming photon density about 30 times as great as is necessary for the degenerate two-photon laser of [S&B].

• *Method II*

An alternative to Method I is to supply priming photons

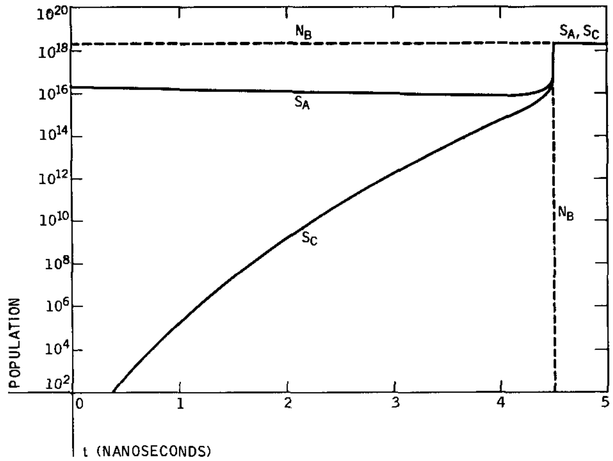


Figure 3 **Course of events in the nondegenerate two-photon laser.** *This hypothetical system of $N_B(0) = 2 \times 10^{18}$ inverted B ions was primed with a total of $\sim 10^{16}$ A photons, a priming energy of 10^{-8} joule.*

ν_A over a period of several cavity decay-times. We can thus calculate the total number of photons ν_A and the corresponding supply time required for reaching Regime II. The result is that the number is a minimum for instantaneous supply as in Method I and is

$$S_{\min} \approx S_0 \ln S_0, \tag{10}$$

but that the total number of priming photons required does not increase by much so long as one pumps well over the threshold, Eq. (8). Thus, $2S_0 \ln S_0$ expresses the number of ν_A photons required if one maintains a photon level $2S_0$ in the cavity for a time $\tau \ln S_0 \approx 10^{-7}$ sec, using the parameters of [S&B]. Normal laser spikes exceed 10^{-7} seconds in duration, so that the nondegenerate two-photon laser can be primed by the same photon source that would be adequate for the degenerate case.

Figure 3 shows the course of the various populations as a function of time, using Method I for priming, whereas Figure 2 shows the steep region of the pulse on a time scale expanded $\sim 10^4$ times.

Discussion

We have analyzed briefly the predicted performance of a nondegenerate two-photon laser. The triggering requirements would be eased by a higher cavity Q for the priming photons ν_A. They seem stiff but not impossible.

The very high logarithmic-growth-rate of the two-quantum laser deserves some comment. Normal Q-switched lasers are limited in growth rate by the condition that the cavity in the low-Q status be stable against the exponential growth of population in the resonant modes. Thus, if the cavity time constant is switchable between τ/q and τ, the above condition requires the build-up time in the absence of loss to be longer than τ/q. For pink ruby, the Q-switched rise time has been shown[4] to be about 2×10^{-9} sec, about three orders of magnitude larger than the rise time calculated above for the two-photon laser. It remains for experiment to demonstrate the magnitude of improvement actually attainable by the two-photon technique.

Acknowledgments

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References and footnotes

1. P. P. Sorokin and N. Braslau, *IBM Journal* **8**, 177 (1964).
2. The priming requirement increases by less than a factor of two for $\nu_C/3\nu_A < 3$, so that the approximate analysis will be made here for $\nu_C \approx \nu_A$.
3. Strictly speaking, unless $S_C \gg 1$, Eq. (1) should be written $dS_C/dt = B_1 S_A N_B(S_C + 1) - S_C/\tau$, i.e. $dS_C/dt = B_1 S_C S_A N_B - S_C/\tau + B_1 S_A N_B$. The last term represents spontaneous emission and is included in the above analysis by starting with $S_C = 1$. Equation (9) is obtained as follows: In the priming phase of Method I we have

$$S_A(t) = S_A(0) e^{-t/\tau} \tag{9a}$$

and

$$\frac{dS_C}{dt} = \frac{S_A(t)}{S_0} \cdot \frac{S_C}{\tau} - \frac{S_C}{\tau} = \frac{S_C}{\tau}\left[\frac{S_A(0)}{S_0} e^{-t/\tau} - 1\right], \tag{9b}$$

which integrates directly to

$$\ln\left[\frac{S_C(t)}{S_C(0)}\right] = \frac{S_A(0)}{S_0}(1 - e^{-t/\tau}) - \frac{t}{\tau}, \tag{9c}$$

which for $S_C(0) = 1$, and $S_A(0)/S_0 \gg t/\tau \gg 1$, gives Eq. (9).
4. L. M. Frantz, *Appl. Optics* **3**, 417 (1964).

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