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The Kinoform: A New Wavefront Reconstruction Device

Abstract: A new, computer-generated, optical element called a kinoform is described. This device operates only on the phase of an incident wave and forms a single image by wavefront reconstruction without the unwanted diffraction orders characteristic of holograms. The kinoform is also more efficient in the use of spatial frequency potential and reconstruction energy and can be synthesized in considerably less computer time than the digital hologram.

Introduction

The kinoform is a new, computer-generated, wavefront reconstruction device which, like the hologram, provides the display[1-3] of a three-dimensional image. In contrast, however, the illuminated kinoform yields a single diffraction order and, ideally, all the incident light is used to reconstruct this one image. Similarly, all the spatial frequency content or bandwidth of the device is available for the single image. Computationally, kinoform construction is faster than hologram construction because reference beam and image separation calculations are unnecessary.

A kinoform operates only on the phase of an incident wave, being based on the assumption that only the phase information in a scattered wavefront is required for the construction of an image of the scattering object. The amplitude of the wavefront in the kinoform plane is assumed constant. The kinoform may therefore be thought of as a complex lens which transforms the known wavefront incident on it into the wavefront needed to form the desired image. Although it was first conceived as an optical focusing element, the kinoform can be used as a focusing element for any physical waveform, e.g., ultrasound or microwaves.

Figure 1 illustrates the reconstruction from a kinoform of a three-dimensional object. The subject was a set of geometrical figures which when viewed on-axis form a triangle, but which are actually set in three different spatial planes. The triangle view is seen in Fig. 1a and the separation and parallax are shown in the off-axis views, Figs. 1b and 1c. There is only one image (diffraction order) formed in the reconstruction and this is bright enough to be seen in a normally lighted room. The small bright spot or "central order spike" would be eliminated if perfect phase matching were achieved.

In this paper we first discuss the physical basis for kinoforms, then the computational and graphic synthesis of a kinoform, and finally the inherent properties and advantages of kinoforms as compared with computer generated holograms.

Physical basis of kinoforms

The useful information in a kinoform (or in a hologram) is a "coded" description of the wavefront of light scattered from a particular object of interest. On illumination with a reconstruction light beam the device provides display of a three-dimensional image.

The actual scattered wavefront may be expressed as the product of a real amplitude and a phase factor:

$$W(x, y, z_0) = A(x, y, z_0) \exp[i\phi(x, y, z_0)], \tag{1}$$

where the kinoform (hologram) is represented by an area of the x, y plane located at $z = z_0$. In an amplitude hologram the coding is due to the interference of the wavefront W with a reference beam W_0. The transmittance function

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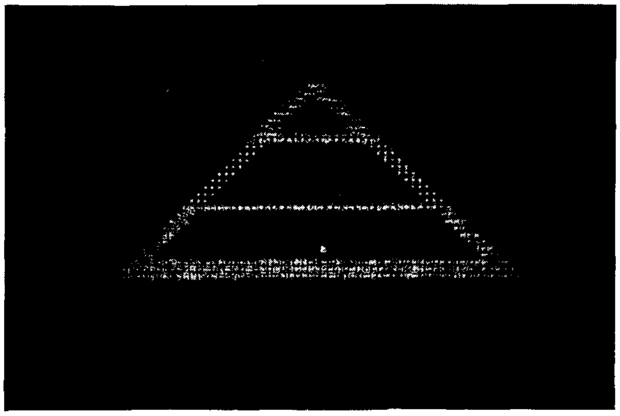
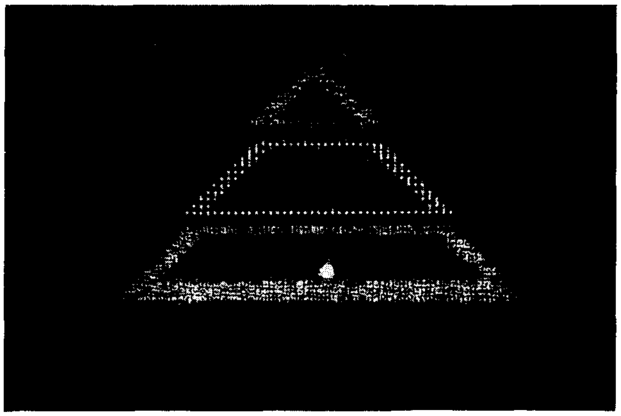


(a)



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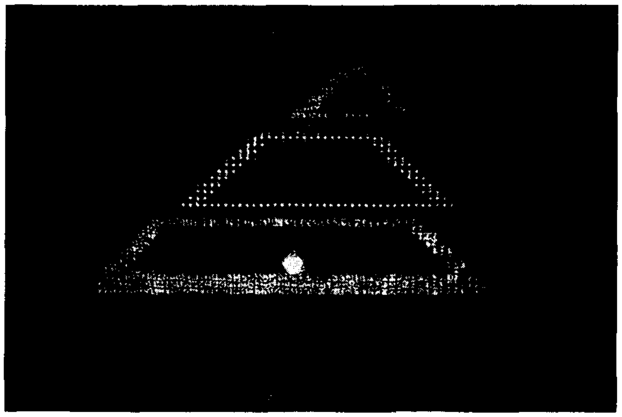


(c)

Figure 1 Photographs of three-dimensional kinoform-generated images; each of the three geometrical figures is located in a different spatial plane: (a) on-axis view; (b), (c) off-axis views showing separation and parallax.

T_a of the hologram is made proportional to a power of the incident light intensity I,

$$T_a \propto I = |W + W_0|^2. \tag{2a}$$

On illumination the hologram reconstructs not only the desired wavefront, W, but also the conjugate wavefront, W^*, and a central beam, $|W|^2 + |W_0|^2$.

Some increase in the intensity of the image can be obtained by bleaching the amplitude hologram to remove the darkened grains in the emulsion ($\rightarrow$ phase hologram), thereby translating the intensity variations into phase variations. The complex transmittance T_p of a phase hologram is exponentially proportional to the recorded light intensity,

$$T_p \propto \exp{(iI)}, \tag{2b}$$

and, although amplitude variations of the original wavefront are not faithfully reproduced, good reconstructions are obtained experimentally. The unwanted diffraction orders are still reproduced, however.

The kinoform was developed using a digital computer and bleaching techniques to provide three-dimensional image display without the superfluous diffraction orders inherent in holograms. It thus provides a more efficient use of the reconstructing light. Unlike the making of holograms, which can be physical recordings of actual wavefront interference patterns or can be photographic reproductions of computer generated patterns, it does not appear possible to create a kinoform using completely optical techniques.†

The kinoform utilizes the functional similarity of the mathematical descriptions of scattered wavefront and bleached hologram transmittance, Eqs. (1) and (2b) respectively. The complex transmittance or, more precisely, the thickness after bleaching, of a kinoform is designed to convert the phase of a known wavefront of unit amplitude (the reconstruction beam) into the phase ϕ of Eq. (1) to produce a virtual image or into $-\phi$ to produce a real image. If the amplitude A in Eq. (1) is constant, the kinoform reproduces exactly the original wavefront and there is only one image or diffraction order into which all the transmitted light is channeled.

A transmission grating with a one-dimensional sawtooth surface structure (Fig. 2) provides a simple example of the phase-shaping technique. The surface relief is expressed as

$$d(x) = d_0 + ax \; (\mathrm{mod}\; d_{max}), \tag{3}$$

where d_{max} is the thickness of the medium that will retard the phase of the incident light by 2π rad. It is easily recognized that, ideally, the emergent plane wave, regarded as the image wave, is (re)constructed by another plane wave incident on the grating at a different angle. The coefficient a is related to the prism angle in the grating and is chosen according to the desired inclinations of the object and

†Although we have made only transmission kinoforms, reflection kinoforms can certainly be constructed. Such reflection kinoforms would have a surface relief somewhat similar to that of the blazed hologram described by Sheridon.[4] In the very simple case of a plane grating the two objects would be identical.

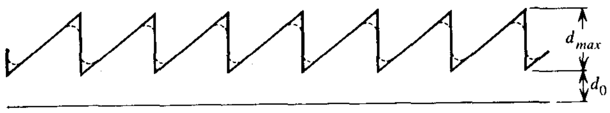


Figure 2 Cross section of a (kinoform) transmission grating showing the surface relief. The solid line represents an ideal grating; the dashed line indicates the effect of using real materials.

reconstruction waves and the index of refraction of the medium. The same phase-shaping result would be produced by a uniformly thick medium with a periodically varying (saw-tooth) index of refraction.

In general the amplitude A at the kinoform plane of the wavefront scattered from the object is not constant. However, with a proper choice of the phase variation at the original object (which is equivalent to illumination through a diffuser), this amplitude is essentially uniform.[5] Since the kinoform is constructed under the assumption $A =$ constant, the quality of the observed image depends on the relative importance of this assumption in the total process of kinoform manufacture and image projection.

Kinoform construction

• *Calculational model*

The problem is to describe mathematically the diffuse illumination of an object and to ensure that information about the object is spread over the whole kinoform plane. The technique used to calculate the wavefront represents the object as a three-dimensional array of point apertures.[6] This model was first introduced for computer-generated holograms because it is economical in the use of computer time and automatically achieves the spatial diffusion of information. Currently we are using an array of up to 99 planar grids, each having 64 × 64 apertures. Each aperture is assigned a transmittance (or reflectance) T,

$$T_{ijk} = t_{ijk} \exp (i\alpha_{ijk}) \tag{4}$$

The values of t_{ijk} represent the transmissivity of the aperture, i.e., zero implies an opaque aperture, unity a completely transparent one, etc.; the values of α_{ijk} model the diffuse scattering from the object, i.e., the phase changes of the wavefront at the object. All the kinoforms discussed in this paper were made using a random number generator to assign values to the α_{ijk}.†

There is preliminary indication from numerical experiments that in some cases a selected non-random phase distribution can yield a scattered wavefront with more nearly constant amplitude in the kinoform plane.

A one-dimensional example will illustrate the actual computations required to prepare a scattered wave pattern. When the extent of the source (here the grid) is small compared with the distance z_0 from the source to the kinoform plane, Kirchhoff diffraction theory[8] provides an expression for the wavefront from a single aperture:

$$W_j(x) \propto T_j \exp \left[\frac{i\pi(x - a_j)^2}{\lambda z_0} \right], \tag{5}$$

where a_j is the coordinate of the jth aperture and λ is the wavelength. [Hereafter Eq. (5) will be regarded as an equality by implicitly redefining W_j; we also define $k \equiv \pi/(\lambda z_0)$.] For an even number m of equally spaced apertures on the (one-dimensional) grid we set $a_j = j\Delta a$; the resultant wavefront is obtained by summation:

$$\begin{aligned} W(x) &= \sum_{j=-\frac{1}{2}m}^{\frac{1}{2}m-1} W_j(x) \\ &= \exp (ikx^2) \sum_{j=-\frac{1}{2}m}^{\frac{1}{2}m-1} T_j \exp [ik(j\Delta a)^2] \\ &\quad \times \exp \left(\frac{-2\pi i x j \Delta a}{\lambda z_0} \right). \end{aligned} \tag{6}$$

To evaluate the wavefront expression (6) at equally spaced points on a grid we let $x \rightarrow x_l = l\Delta x$. We then impose the reciprocal relation

$$\Delta x \Delta a = \lambda z_j / m, \tag{7}$$

where $z_j = z_0$ in the one-dimensional case. This is exactly the condition needed to replace the sum in Eq. (6) by a finite Fourier transform which can be calculated efficiently using the Cooley-Tukey algorithm.[9] If we define

$$U_j \equiv T_j \exp [ik(j\Delta a)^2], \tag{8}$$

then Eq. (6) can be rewritten as

$$W(x_l) = \hat{U}(x_l) \exp (ikx_l^2), \tag{9}$$

where $\hat{U}(x_l)$ is the finite Fourier transform of U_j. (More detail can be found in Ref. 1.)

Equation (7) implies that for a large object the kinoform sample points must be closely spaced. If a large viewing window (i.e., kinoform) is needed there must be many sample points. Since the transform $\hat{U}(x_l)$ is a periodic function of l with period m, the sum involved can be repeated with the coordinate origin translated one period each time until the kinoform is as large and as redundant[1] in information content as desired. The value of $m\Delta x$ should be smaller than the diameter of the lens viewing the kinoform-generated image so that at least one period will be visible at any time. Since $W(x)$ is a continuous function, to sample it correctly Δx must be small enough to satisfy the Nyquist sampling criterion.

†It is interesting to note that in 1880 Rayleigh[7] calculated the distribution of amplitudes that would be observed from a number of harmonic oscillators with the same frequency, but with random phases. The result was a bell-shaped distribution reasonably peaked at the most probable amplitude.

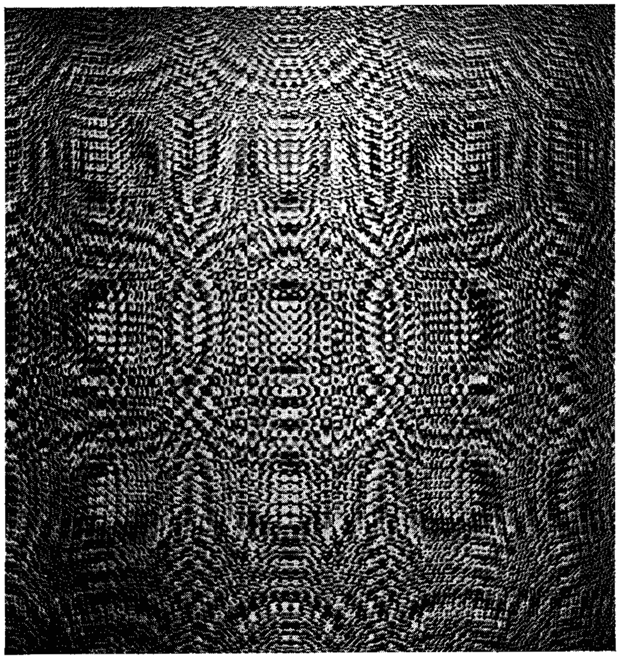


Figure 3 Photograph of the 32-grey-level plot of the computed wavefront phase for the two-dimensional letter B.

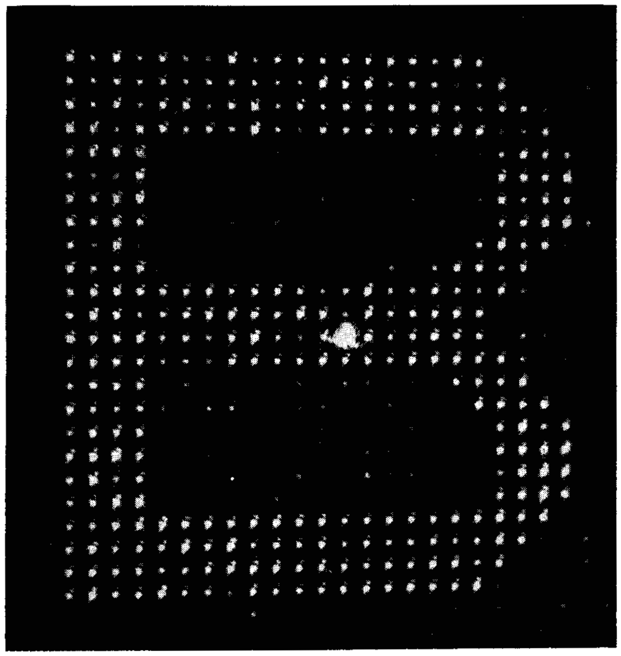


Figure 4 Photograph of the two-dimensional kinoform image of the letter B. Note that there is a small central order spike, but no conjugate image.

The discrete aperture description of the object means that only a relatively small Fourier transform needs to be calculated. In our case we use a basic block of 128×128 points per plane. The grid of 64×64 apertures is embedded in a 128×128 array. This has the effect of interpolation of the $\hat{U}$ array[10]; more points are represented in the wavefront although the image array is not changed. If we were to use a continuous model of the object, in which the object would be divided into 128×128 resolution elements, those elements would have to be mapped into the whole kinoform plane (limited by the plotter to a 12000×8000 array) by the Fourier transformation.

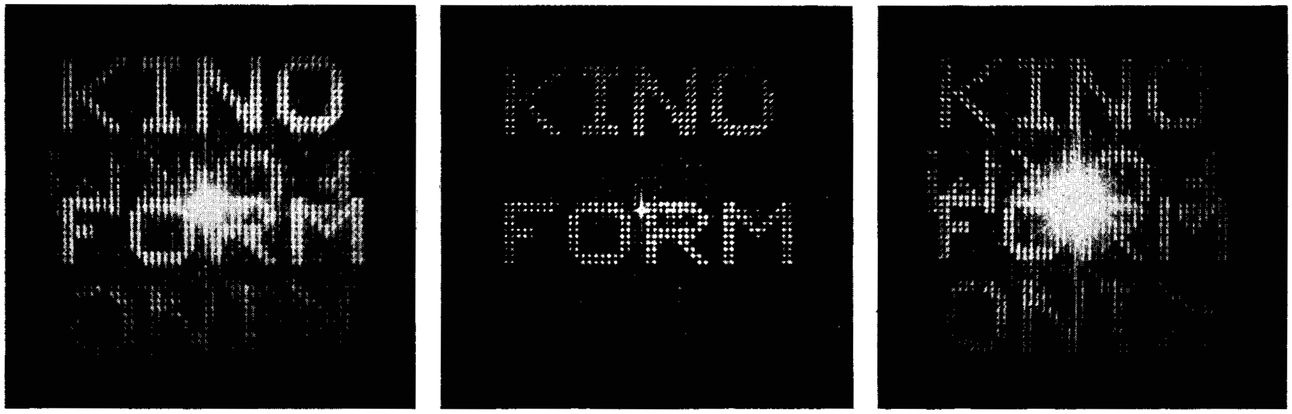
When the object is one- or two-dimensional, once the transform $\hat{U}$ is calculated a further economy is inherent in computer-generated kinoforms as compared with digital holograms. Since the kinoform involves only the phase of the wavefront, the remaining step is a real-number addition of the phase of $\hat{U}(x_l)$ to the quantity kx_l^2 [see Eq. (9)]. If an off-axis kinoform image is required, the phase of an off-axis reference wave can also be added. The complex-number multiplications implicit in Eq. (2a) for holograms are unnecessary. If the object array is three-dimensional, however, the plane-by-plane computer synthesis of the wavefront phase does involve complex multiplication.

• *Graphic synthesis*

The calculated phase of the scattered object wave is reduced modulo 2π and plotted photographically on a GeoSpace Corporation DP 203 (IBM 1780) plotter using a 32-level grey scale. A phase plot for the letter **B** is shown in Fig. 3. On close examination one can see that there are discontinuities in the plot indicating the transition from regions of $\phi = 2\pi$ rad to regions of $\phi = 0$. Such discontinuities are unusual in computer-generated holograms where continuous intensities of diffraction patterns are recorded.

The large-scale plots are photoreduced to sizes appropriate for illumination with visible light. With 12000×8000 resolution elements in a 60-in. $\times$ 40-in. format, the GeoSpace plotter permits the construction of kinoforms 12 cm $\times$ 8 cm with maximum spatial frequencies of 50 line-pairs/mm. Typically such a kinoform is about 2500 times redundant.

Finally the photoreduction is bleached using a tanning bleach[11] to remove the darkened grains in the photographic emulsion. Not only the bleaching, but also the exposure and development of the photoreduction for a kinoform must be performed with much more care than is required for conventional bleached holograms. In particular the bleaching must be carefully controlled so that the surface relief of the end product is such that light incident on a region of phase $\phi = 0$ will be retarded by one wavelength relative to light incident on a region of $\phi = 2\pi$ rad (this step is referred to as "phase matching"). When phase matching is achieved, almost all of the light incident on the kinoform will be present in the projected image. This image is either real or virtual according to whether the photoreduction used is a positive or a negative of the phase plot, respectively.



(a) (b) (c)

Figure 5 Photographs of two-dimensional images produced by kinoforms for which the exposure times of the phase plot photoreductions differed while the bleaching processes were identical: (a) undermodulation (shorter exposure time); (b) nearly correct exposure achieving the best phase matching; (c) overmodulation (longer exposure time).

Figure 4 shows the letter **B** reconstructed by the kinoform evolved from the phase plot, Fig. 3. Note that there is no conjugate image. The small bright spot or central order spike is the result of imperfect phase matching. In general, phase matching can be improved by the use of larger-field reduction lenses and by compensating the nonlinear modulation transfer function of the photographic emulsion.

Discussion

• *Single image*

Two principal factors in the appearance of unwanted images are phase matching and the use of non-ideal materials. When phase matching is not carefully controlled, the kinoform is similar to an in-line hologram in which the real and conjugate images are partially superposed. As phase matching is improved, more light is diffracted into the real image and the conjugate image fades. In the kinoform images already displayed, the only visible remnant effect of imperfect phase matching is the small central order bright spot. In Fig. 5 a series of kinoform reconstructions illustrates the effect of a deliberate phase mismatch.

The effect of using real materials can be seen in an examination of the kinoform grating. A grating made from real materials will exhibit rounding of the cusps of the ideal grating (dashed lines in Fig. 2). If the function d in Eq. (3) describing the surface relief of the ideal grating is decomposed into its (infinite) Fourier series, the use of a real material with a limited frequency response will serve to truncate the series. A numerical evaluation of the kinoform grating in which the spatial frequency cut-off of the material is assumed to be just four times the fundamental grating frequency shows that 96% of the energy diffracted into the 0, ± 1, and ± 2 orders emerges in the -1 order, i.e., in the real image.[12] Kogelnik[13] has calculated that no more than 34% of the light incident on a bleached (phase) hologram can be diffracted into the desired image.†

• *Signal-to-noise ratio*

Experimentally we have observed that the signal-to-noise ratio in images from kinoforms is about that seen in bleached, computer-generated holograms. In both cases some of the noise is introduced by the approximation of assuming that a phase object alone can act on the reconstruction wavefront to form an image.

We have not made a mathematical analysis of the signal-to-noise ratio available from kinoforms; however, numerical experiments indicate that the error introduced by the assumption of constant amplitude is small. For example, the digital kinoform reconstruction of a two-dimensional 64×64 array of open apertures with random phases had a minimum signal-to-noise ratio of 16. In this case the variation of image spot intensity was about $\pm 15\%$ and about 75% of the beam power was used in constructing the image.

It is possible that selected phase distributions other than the random one used in this work will improve the signal-to-noise ratio by improving the constant amplitude approximation. However, experimental procedures relating to plotter alignment, reduction lens quality, bleaching control, etc., are also important in determining the image quality.

†One hundred percent transmission can be achieved through thick dielectric holograms,[13] but there is no obvious way to plot a computer-generated thick hologram.

- *Computational efficiency*

Kinoforms serve for all of the applications of computer-generated holograms, e.g., three-dimensional display, wave conversion, read-only storage, etc. However, kinoforms give a more practical, computationally faster display construction that yields more economical use of the reconstructing energy and that yields only the desired image.

The principal computational advantage of kinoforms as compared with digital holograms is embodied in the fact that all of the spatial frequency content of the device is used in the formation of the real image; none is required for the separation of the real and conjugate images. There is then at least a factor of four reduction in the computer time needed to calculate the wavefront pattern necessary for equivalent image quality. Correspondingly there is a reduction in plotting time for the kinoform.

A further economy is achieved in that no calculations involving a reference beam are necessary. Finally, in the cases of one- and two-dimensional objects only real-number additions are required, once the basic transform is calculated, to determine the wavefront phase for plotting. The corresponding quantity to be plotted for digital holograms is the wavefront intensity which requires multiplication of complex numbers.

Acknowledgments

We gratefully acknowledge the work of D. Richards who has done most of the programming associated with this project. We also thank T. Gipson for his diligent accomplishment of much of the photographic experimentation.

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Received July 29, 1968; revision December 13, 1968