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Some Design Considerations for a Document Sorting Machine

Abstract: A document reader-sorter is analytically studied to determine the effect of document velocity on the number of documents that can be sorted reliably in a given interval of time, and a formula is derived that relates the effect of various design parameters to the throughput of a document sorting machine. One of these parameters, selector response time (i.e., indexing time), is investigated in detail. Both analytical and graphical design techniques are developed to minimize the response time of the selector. These techniques, which are explained by simple examples, are quite general and can therefore be applied to many other incrementing devices.

Introduction

A sorting machine automatically divides a group of documents into smaller groups according to some predetermined characteristic of the data contained on the documents. A simplified diagram of a document sorter, Fig. 1, shows that documents are extracted from a "hopper," routed past a data reading station, and distributed into the appropriate "pockets." The machine throughput, which is the number of documents sorted during a given time interval, is determined primarily by three parameters: document speed, document size, and selector response time, where the selector is a mechanical device whose position can be altered to direct a document either past or into a sorter pocket; the time required to move the selector from one stable position to another is called its response time. A simple mathematical relationship can be derived to show the effect of these parameters on throughput.

It will be seen that relative changes in document speed and size produce greater changes in throughput than does a relative change in selector response time. However, certain considerations (to be noted below) usually prevent the designer from attempting to optimize sorter throughput by varying the first two parameters. Hence, in this paper, we study in detail two methods for optimizing selector response time.

The selector mechanism considered here is one example of a device having intermittent motion. Other examples are devices used for indexing paper in a printer and for starting and stopping rotation of a tape reel. To cause these devices to start, move to the desired position, and stop in as short a time as possible, it is necessary for the designer to optimize the performance of both the device itself and the motor that drives it.

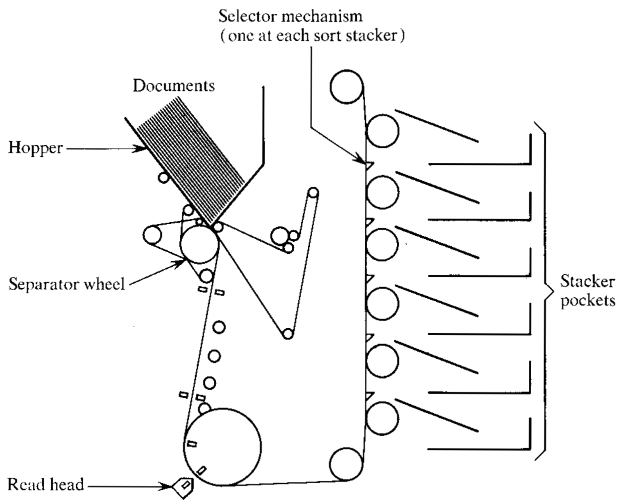


Figure 1 Elements of a document sorting machine.

The analytical design approach presented in this paper can be applied to a variety of indexing devices. However, if the device's shape or material composition is too complex to make an analytical solution practical, a graphical design approach can be used. The mechanism we describe below is an example to which both design techniques have been applied. Both methods are shown to yield similar results and demonstrate that the design of the motor can be performed almost independently from the design of the selector mechanism.

Optimizing machine throughput

A situation analogous in some respects to document flow in a sorting machine is traffic flow through a tunnel. In both cases it is desired to maximize the number of units (documents or cars) traversing the length of the flow path during a given time interval, and it is also desired to assure reliable flow, i.e., to prevent units from disrupting flow by stalling or crashing. (To maintain the traffic analogy, we would have to assume that all cars have the same maximum speed and enter the tunnel at a constant rate.) In the case of tunnel traffic one might intuitively think that throughput will be maximized if everyone drives through the tunnel as fast as his car will go. However, for the cars to get through the tunnel reliably, a safe distance must be kept between them. This distance is proportional to the speed at which the cars are travelling. Thus, the density of cars in a tunnel (which is an inverse function of the distance between cars) is less for a stream of fast-moving cars than for slow-moving cars. Throughput, then, is the speed of the traffic times its density, which in turn is a function of speed; to find the speed that maximizes throughput, it is necessary to solve a nonlinear equation.

Using the insights obtained from the tunnel-traffic analogy, we can derive a relationship to determine the optimum speed of documents in a sorting machine.

As noted in Fig. 1, documents are sorted into various pockets by locating at each pocket a selecting mechanism that diverts each document into one of the two paths. The mechanism must be able to change positions fast enough after feeding one document so that, if need be, it will be able to deflect the very next document to the other path. For analysis it is assumed that the selector moves only in the interdocument gap. The response time required for this selector to change position is governed by both mechanical and electrical constraints. If the document speed is increased beyond the response capability of the selector, then the space between successive documents must be increased to allow sufficient selection time. Thus, for a given selector, the greater the document speed, the greater will be the space required between successive documents.

We have now established that higher document speeds are associated with a lower density of documents, and that document speed can be optimized in a manner analogous to car speed in a tunnel. With this physical model in mind, we derive a mathematical model relating the key design parameters to the throughput. As previously stated, the throughput of a sorter is equal to the number of documents sorted per unit of time, and the document density is equal to the number of documents per unit of length. The inverse of document density is the length of space associated with each document. This length is equal to the document length plus the space between it and the next document, or

$$\frac{1}{D} = L + S \qquad (1)$$

where D is the linear document density (in.$^{-1}$), L is the length of the document (in.), and S is the space between successive documents (in.).

The space between documents is a function of the document speed and the response time of the selecting mechanism; consequently,

$$S = vt_s, \qquad (2)$$

where v is the card velocity (in./s) and t_s is the response time of the selector or the time required for the selector to index (s).

The relation of throughput N to document speed and density is simply

$$N = vD. \qquad (3)$$

Combining Equations (1), (2) and (3), we have

$$N = v/(L + S) = v/(L + vt_s)$$
$$= [(L/v) + t_s]^{-1}. \qquad (4)$$

From Eq. (4), we can see that for a given document length L and a given selector response time t_s, the throughput N increases as the document velocity v increases. However, N increases asymptotically to a value equal to the inverse of t_s, or

$$N = \frac{1}{t_s}\bigg|_{v\to\infty} = N_{\max}.$$

This relationship is shown in Fig. 2, in which the normalized throughput $N/N_{\max}$, where

$$N/N_{\max} = (vt_s/L)\ [1 + (vt_s/L)]^{-1},$$

is plotted as a function of normalized velocity, vt_s/L. As a point of interest, the document space equals the document length when $v = L/t_s$, i.e., when the normalized velocity equals one.

Note from Fig. 2 that throughput is maximized when velocity is maximized. However, the change in throughput for increasing values of velocity is very small once the document velocity exceeds a certain level, and increasing velocity beyond this level would thus be inefficient.

The normalized velocity vt_s/L is approximately equal to two at this level. A typical document length is six inches and a typical selector response time is 15 ms.

Consequently, with these values and a normalized velocity level of two, the document velocity would equal 800 in./s and the throughput would be 2667 doc./min. This speed, however, seems to be much higher than could be obtained at the present state of the art because of document damage and mechanical limitations. Since this "optimum" document velocity cannot be obtained, a document velocity that is as high as possible within design restraints should be used.

At this point it is useful to examine how the other parameters, document length and selector response time, affect throughput. In other words, for a given nominal design, it may be easier to change the value of one parameter than that of another and still obtain a reasonable increase in document throughput. Thus, for a given nominal design (document velocity, nominal document length, and selector response time) we have to decide which of the parameters (v, L, t_s) should be changed to increase throughput most efficiently.

The total differential of N with respect to v, L and t_s, is

$$dN = \frac{\partial N}{\partial v}\, dv + \frac{\partial N}{\partial L}\, dL + \frac{\partial N}{\partial t_s}\, dt_s. \tag{5}$$

Now, if Eqs. (4) and (5), are combined,

$$dN = \left(\frac{L}{v^2}\, dv - \frac{1}{v}\, dL - dt_s\right) \Big/ \left(\frac{L}{v} + t_s\right)^2$$

$$= N^2\left(\frac{L}{v^2}\, dv - \frac{1}{v}\, dL - dt_s\right). \tag{6}$$

Note that the quantity dv/v is the relative change of the document velocity, and similarly, dL/L is the relative change of the document length. Dividing Eq. (6) by N and rearranging terms, we obtain the relative change in throughput as a function of relative changes in velocity, document length, and response time:

$$\frac{dN}{N} = \frac{NL}{v}\left(\frac{dv}{v}\right) - \frac{NL}{v}\left(\frac{dL}{L}\right) - Nt_s\left(\frac{dt_s}{t_s}\right). \tag{7}$$

Throughput is increased most efficiently by obtaining the highest relative increase in throughput for the smallest relative change in any of the controlling parameters. Equation (7) can be used as a guide to efficiently increase the throughput for any sorting machine.

As an example, we select the following nominal values of the controlling parameters:

$v = 240$ in./s.;
$t_s = 0.015$ s.;
$L = 6$ in.;

$$\therefore N = \left(\frac{L}{v} + t_s\right)^{-1} = \frac{1}{0.025 + 0.015} = 25 \text{ doc./s.}$$

For the nominal values chosen, Eq. (7) becomes

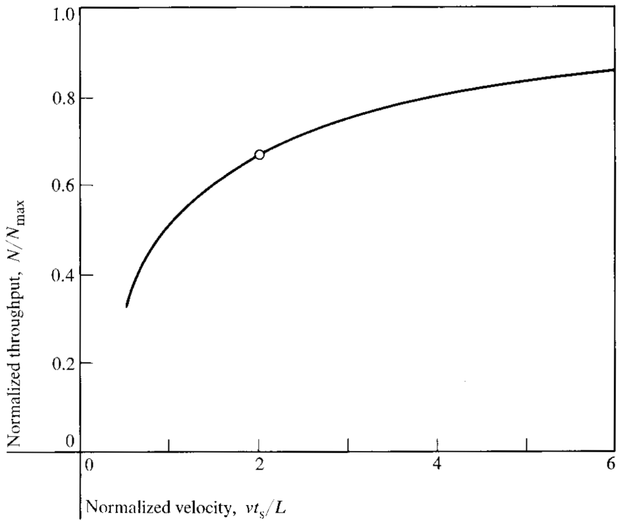


Figure 2 Normalized throughput vs normalized document velocity.

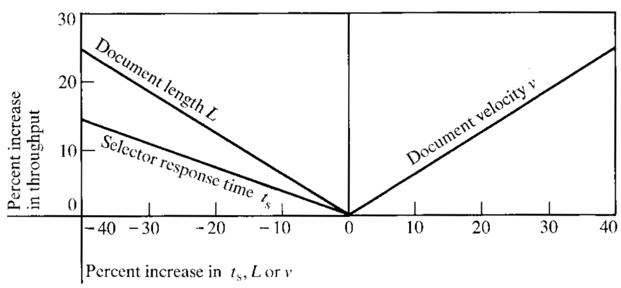


Figure 3 Relative change in machine throughput vs relative change in design parameters (response time, document length, document velocity).

$$\frac{dN}{N} = 0.625\,\frac{dv}{v} - 0.625\,\frac{dL}{L} - 0.375\,\frac{dt_s}{t_s}. \tag{8}$$

Equation (8) shows that a 10 percent increase in velocity ($dv/v = 0.10$) causes a 6.25 percent increase in throughput. Similarly, a 10 percent decrease in card length causes a 6.25 percent increase in throughput. However, a 10 percent decrease in selector response time causes only a 3.75 percent increase in throughput. Thus, for the nominal values chosen, throughput is increased most efficiently by increasing document velocity or decreasing the document length. Because of the nature of the differential relationship of Eqs. (7) and (8), these equations are valid only for relatively small (30 percent or smaller) changes in the parameters from their nominal values. The relationship of Eq. (8) is shown graphically in Fig. 3.

Document length cannot ordinarily be decreased since the designer is rarely encouraged to alter the dimensions

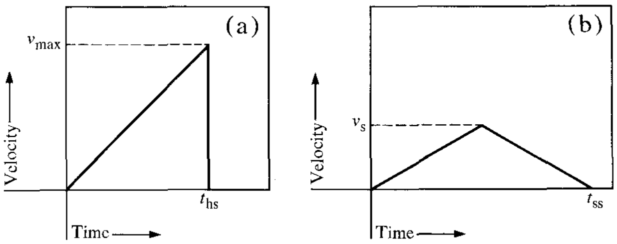


Figure 4 Velocity characteristics for (a) hard-stop and (b) soft-stop indexing.

of the standard rectangular document. However, this does indicate that orienting the document to feed in a direction parallel to its shortest edge might increase throughput. Since Eq. (8) is valid for relatively small changes (30 percent or less), and short edge feeding would dictate a very large change in document length (more than 50 percent), a new set of nominal values should be obtained, which changes the coefficients of Eq. (8).

For the example presented above, document velocity and length have the greatest effect upon throughput. However, the length is not usually subject to control, and increasing the velocity has undesirable side effects on machine life, document energy, etc. We shall therefore concentrate on optimizing the remaining parameter—selector response time.

Optimizing selector response time

• *General theory*

In an indexing machine, a member is incrementally moved through a given displacement by some prime mover such as a motor. In many instances the member must be indexed as rapidly as possible. This is especially important in sorting machines where selector mechanisms must be indexed rapidly to prevent mis-sorting, paper jams, etc. Certain relationships between the driver (prime mover) and the driven member can be obtained so that the response time is minimized. Furthermore, the characteristics of the prime mover that are necessary to minimize response time can also be specified.

Suppose that a given member, initially at rest, is to be indexed from Position A to Position B. To do this as rapidly as possible, we assume that the torque of the prime mover is at its maximum and that the driven member is rigidly coupled to the prime mover. The two possible methods of indexing are the "hard stop" and the "soft stop" techniques [see Figs. 4(a) and 4(b), respectively].

In the case of the hard stop, the prime mover delivers maximum torque for the duration of the indexing operation. When the member has been moved through the proper displacement, the prime mover is turned off and the member is stopped by an idealized shock absorber. The slope of the velocity-time curve is equal to the acceleration of the member. The area under the velocity-time trace is equal to the incremental displacement. For a rotary device, this area is equal to the angular displacement (rotation). During the incrementing period, the following relationship governs the motion:

$$J_T \ddot{\theta} = T_{max}, \tag{9}$$

where J_T is the moment of total inertia of the prime mover and the indexed device (rigidly coupled), $\ddot{\theta}$ the angular acceleration of the device, and T_{max} the maximum torque produced by the prime mover. Thus, the angular acceleration $\ddot{\theta}$ is essentially a constant since the torque produced is assumed always to be the maximum torque of the prime mover.

This fact will be used to obtain the response time of the displaced total inertia in hard-stop indexing. From Fig. 4(a) the angle of increment is as follows:

$$\theta_i = \tfrac{1}{2} v_{max} t_{hs}, \tag{10}$$

where θ_i is the angle of increment and t_{hs} the response time for a hard stop. However,

$$v_{max} = \ddot{\theta} t_{hs}. \tag{11}$$

Therefore, combining Eqs. (9), (10) and (11) we obtain

$$\theta_i = \tfrac{1}{2}(T_{max} t_{hs}^2 / J_T)$$

or

$$t_{hs}^2 = 2 J_T \theta_i / T_{max}. \tag{12}$$

Figure 4(b) describes a soft (or smooth) stop. In this case also, maximum torque is applied by the prime mover. Midway through the increment angle, however, the direction of torque is reversed to decelerate the inertia to a stop. In this case, the increment angle θ_i is equal to

$$\theta_i = \tfrac{1}{4}(T_{max} t_{ss}^2 / J_T)$$

or

$$t_{ss}^2 = 4 J_T \theta_i / T_{max}, \tag{13}$$

where t_{ss} is the response time (incrementing time) for a soft stop.

Thus, for either a hard or soft stop

$$t_s^2 = C J_T \theta_i / T_{max}, \tag{14}$$

where $C = 2$ for a hard stop and 4 for a soft stop. But

$$J_T = J_m + J_s \tag{15}$$

where J_m is the moment of inertia of the prime mover and J_s that of the indexed device.

Combining Eqs. (14) and (15) and dividing numerator and denominator by J_m, we obtain

$$t_s^2 = C[(J_s/J_m) + 1]\theta_i/(T_{max}/J_m), \tag{16}$$

and for a motor,

$$T_{max} = k_T i_{max} \tag{17}$$

where k_T is the motor constant and i_{max} is the maximum current to the coil of motor (determined from electrical and thermal considerations).

Thus, combining Eqs. (16) and (17), we get

$$t_s^2 = C[(J_s/J_m) + 1]\theta_i/(k_T/J_m)i_{max}. \tag{18}$$

From Eq. (18) one obvious conclusion can be made regarding the response time of the device: The torque-to-inertia ratio (T_{max}/J_m) for the prime mover should be as high as possible. (The torque constant k_T, and the source of energy or excitation i_{max} should be as high as possible.) It also seems that the moment of inertia of the incremented device should be as small as possible, but this supposition needs further investigation. If a point on the indexing device must be moved to a certain new location, then the angle of increment is to some extent a function of the geometry of the device. For example, suppose a port must be alternately opened and closed, as depicted in Fig. 5. The Figure shows that the increment angle through which the device must travel to open the port is approximately equal to the width w of the port opening divided by the distance l of the port from the center of rotation of the device, i.e.,

$$\theta_i \approx w/l. \tag{19}$$

Thus, the greater the length l the smaller the increment angle θ_i. However, as this length is increased the moment of inertia of the device also increases. This presents a conflict for minimizing the response time.

The moment of inertia can be minimized by using low density materials and by making the cross-sectional area (area perpendicular to l) as small as possible. If l is reduced, then the device inertia is reduced; however, this does not necessarily reduce the response time t_s (because a larger rotation angle θ_i is required). Thus, when the increment angle is to some degree a function of the geometry of the device, a detailed investigation is needed to determine the optimum length, and hence the optimum moment of inertia, of the device.

The shape of the device is obviously quite important in determining optimum size. For a particular indexing operation, the indexed device could be a disk, a rectangular plate, or any of several other geometric forms. The optimum length will be different for each. Often the device will be a composite of a number of shapes and materials, as in the case of the selector mechanism. Furthermore, the inertia of the device might not easily be

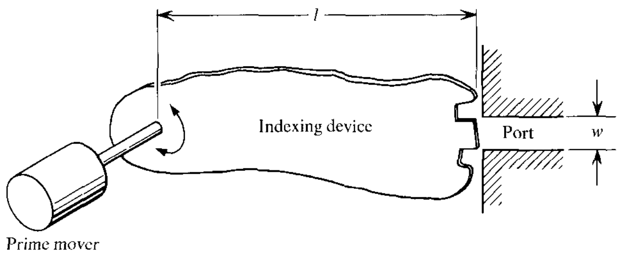
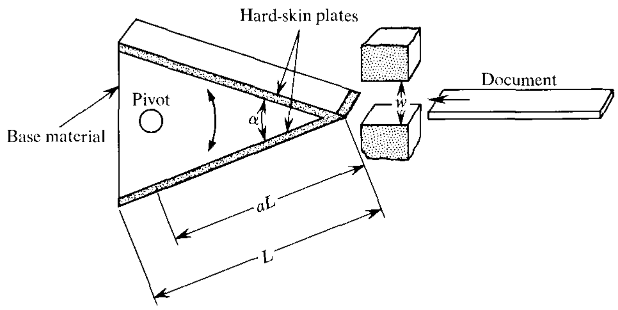


Figure 5 Generalization of mechanism for opening and closing a port.

Figure 6 Schematic of selector mechanism used for the example.



described in a useful mathematical form. For this latter case, a graphical technique has been developed and will be described later.

• *Optimizing a selector mechanism*

Analytical Solution

A selector mechanism, as previously noted, is used to direct a moving document to one of two paths. We have established that the response time of this mechanism should be as short as possible for high machine throughput. The physical shape of the selector mechanism in our application, shown in Fig. 6, was designed to be rigid so that no oscillations would be induced during indexing.

To obtain the optimum response time we first obtain a mathematical relationship for the response time t_s. Equation (18) is still valid; thus

$$t_s^2 = C[(J_s/J_m) + 1] \times [w/aL \cos(\alpha/2)]\,[(k_T/J_m)i_{max}]^{-1}, \tag{20}$$

where $aL \cos(\alpha/2)$ is equivalent to l, the distance from the blade tip to the pivot position, and $a \cos(\alpha/2)$ is thus a fraction of the plate length L. The moment of inertia J_s of the selector (which comprises two hard-skin plates and a base material) about its pivot, is

$$J_s = 2\rho_0 A_0 L^3\left[\frac{1}{3} + a\left(a - \cos\frac{\alpha}{2}\right)\right] + \frac{\rho_I t_I L^3 (L \sin\alpha)}{3}\left[\frac{1}{4} + \frac{1}{2}\cos^2\frac{\alpha}{2} - 2a\cos\frac{\alpha}{2} + \frac{3}{2}a^2\right], \tag{21}$$

where ρ_0 is the mass density of the hard-skin plates, A_0 the cross-sectional area of the plates, L the length of a plate, ρ_I the density of the base material, t_I the width of the selector mechanism, and α the included angle between plates at the apex.

From Eqs. (20) and (21), we can see that the response time t_s can be minimized by minimizing the densities of the plate and base materials, the blade width (i.e. plate width), and the included angle between the plates, while maximizing the torque-to-inertia ratio of the prime mover and the energy input to the prime mover. At this point, however, we cannot clearly see what are the optimum values for the plate length L and the selector pivot location as defined by the quantity a.

Substituting Eq. (21) into Eq. (20), differentiating the resulting relationship for t_s^2 with respect to the plate length L, and setting this result to zero, we obtain

$$0 = \frac{1}{a\cos(\alpha/2)}\left\{\left[\frac{1}{2}\frac{\rho_I}{\rho_0}\frac{L\sin\alpha}{H}\left(\frac{1}{4} - \frac{1}{2}\cos^2\frac{\alpha}{2} - 2a\cos\frac{\alpha}{2} + \frac{3}{2}a^2\right) + \frac{2}{3} + 2a\left(a - \cos\frac{\alpha}{2}\right)\right]\frac{2\rho_0 A_0 L}{J_m} - \frac{1}{L^2}\right\} = \frac{1}{aL^2}\left(\frac{3J_I + 2J_0}{J_m} - 1\right), \tag{22}$$

for relatively small values of α, where J_0 is the moment of inertia of the hard-skin plates, J_I the moment of inertia of the base material about the pivot, and H the plate thickness. Also,

$$J_s = J_I + J_0.$$

Therefore, Eq. (22) becomes

$$0 = \frac{1}{aL^2}\left[\frac{2J_s}{J_m}\left(1 + \frac{J_I}{2J_s}\right) - 1\right] = \frac{1}{aL^2}\left[\frac{J_s}{J_m}\left(2 + \frac{r}{r+1}\right) - 1\right], \tag{23}$$

where $r = J_I/J_0$. From Equation (23),

$$\frac{J_s}{J_m}\left(2 + \frac{r}{r+1}\right) = 1. \tag{24}$$

Equation (24) describes the constraint on the selector necessary to minimize response time. Thus, when the moment of inertia of the base material is small compared to that of the plates, the optimum selector-to-prime-mover inertia ratio is 0.5. When the base material moment of inertia J_I is large compared to that of the plates, J_0, the optimum inertia ratio J_s/J_m is 1/3.

A plot of the optimum selector-to-prime-mover inertia ratio, plotted as a function of the plate-to-base inertia ratio, is depicted in Fig. 7. This curve is independent of the pivot position denoted by the value of a. Thus, if the design is constrained so that the selector can be pivoted only at a particular location, the inertia of the selector about this pivot position should be determined from Eq. (24).

However, if this constraint does not exist, the optimum pivot location for the optimum inertia ratio should be obtained. Combining Eq. (20) and (21), differentiating t_s^2 with respect to the quantity a and setting this result to zero yields

$$0 = \frac{1}{L\cos(\alpha/2)}\left(\left\{\frac{1}{6}\frac{\rho_I}{\rho_0}\frac{L\sin\alpha}{H} \times \left[\left(\frac{1}{4} + \frac{1}{2}\cos^2\frac{\alpha}{2}\right)\left(-\frac{1}{a^2}\right) + \frac{3}{2}\right] - \frac{1}{3a^2} + 1\right\}\frac{2\rho_0 A_0 L^3}{J_m} - \frac{1}{a^2}\right). \tag{25}$$

If Eqs. (25) and (21) are combined, the following is approximately true for values of $\alpha \leq 30°$:

$$\left\{\frac{1}{6}\frac{\rho_I}{\rho_0}\frac{L}{H}\sin\alpha\left(\frac{3}{2}a^2 - \frac{3}{4} + a^2 - \frac{1}{3}\right)\frac{J_s}{J_m}\right\} \times \left\{\left[\frac{1}{3} + a(a-1)\right](r+1)\right\}^{-1} \approx 1. \tag{26}$$

Now we combine Eqs. (24) and (26) and get

$$\frac{3r[(a^2/2) - 1/4]}{[3/4 - 2a + (3/2)\,a^2]} + \frac{a^2 - 1/3}{[1/3 + a(a-1)]} = 2 + 3r. \tag{27}$$

When $a = 1$, Eq. (27) is exactly satisfied for all values of the base-material-to-plate inertia ratio r. Thus the selector should always be pivoted about its extreme end. The length L of the selector should then be adjusted so that Eq. (24) is satisfied. This same conclusion could have been reached by substituting Eq. (24) into Eq. (20), which gives

$$t_s^2 = C\left[\frac{1}{2 + (r/r+1)} + 1\right]\left[\frac{w}{aL\cos(\alpha/2)}\right]\left[\frac{k_T}{J_m}i_{max}\right]^{-1}.$$

Thus, the largest possible value of a, $a = 1$, would minimize the response time.

When the selector design is constrained so that Eq. (24) cannot be satisfied, a new optimum pivot location must be determined from Eq. (26). In Fig. 8, the optimum pivot position is plotted as a function of the selec-

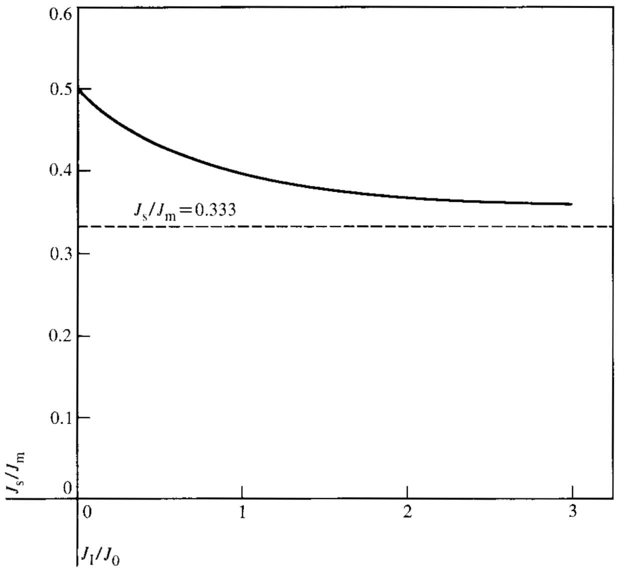


Figure 7 Selector-to-driver inertia ratio vs plate-to-base-material inertia ratio.

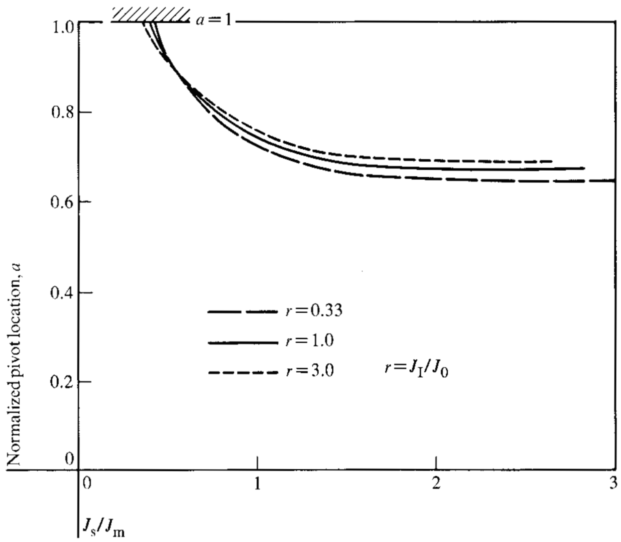


Figure 8 Optimum pivot location for various inertia ratios.

tor-to-motor inertia ratio for various base-material-to-plate inertia ratios. The optimum pivot position under the given conditions seems to be relatively independent of the base-to-plate inertia ratio for the range of values plotted. For the range of values shown in Fig. 8, the value of a corresponding to optimum pivot position varies from about 0.67 to 1.

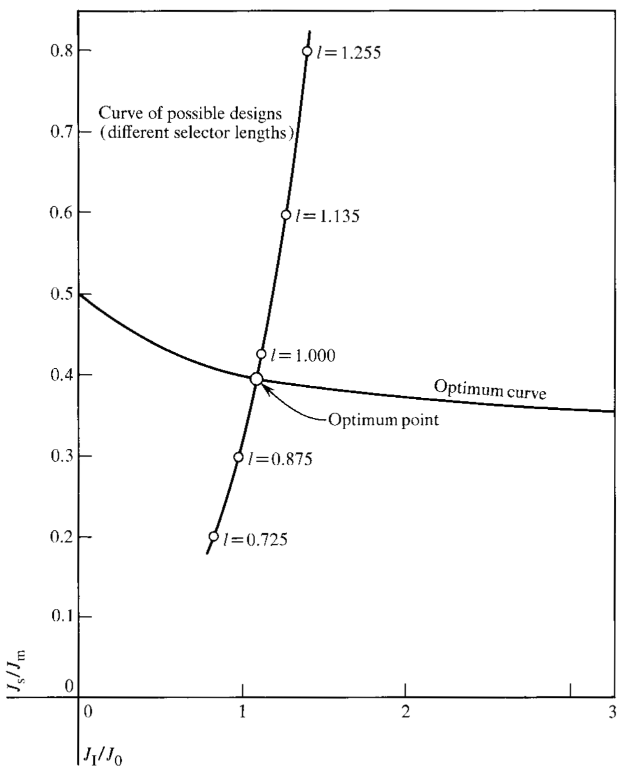
The analytical design procedure is completed by using Fig. 7. A family of curves can be drawn for different combinations of material densities, selector lengths, cross-sectional areas for fixed pivot locations, etc. The intersection of a given design curve with the optimum curve represents an optimum condition for the parameters chosen. In Fig. 9, the curve for one possible design is superimposed on the optimum curve. Each point on the design curve corresponds to a different selector plate length L with all other parameters held constant. The intersection of the two curves determines the optimum selector plate length. Similar curves could have been drawn for various lengths with each point along the curve corresponding to a different material density.

Graphical solution

The graphical solution for optimizing a selector mechanism follows a different course than the analytical solution, but the problem is defined in essentially the same manner. A rotary input device is rigidly coupled to an incrementing device that produces a given output. The output is a displacement at the selector tip that is related to the angular drive input by the constant l.

In our previous examples, optimization has required the differentiation of J_s with respect to l or some other

Figure 9 Illustration of design selection of optimum length for a selector pivoted at its base ($a = 1$).



representative variable. In some design situations this may be impractical or extremely difficult. An example of this is a drive mechanism employing a gear train, where l is changed in steps and the resulting inertia of the system depends upon the designer's choice of gear pairs,

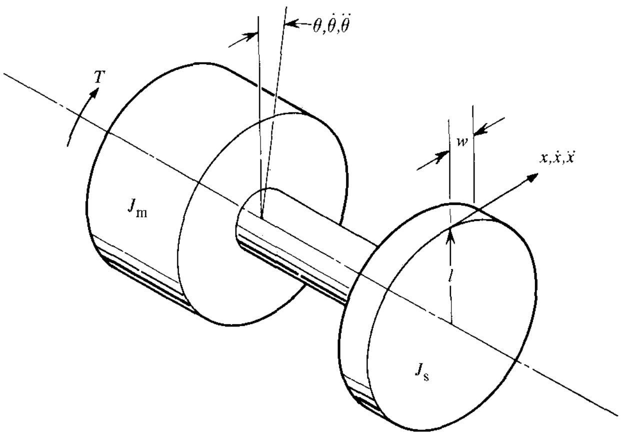


Figure 10 Indexing system. (J_m = driver inertia; J_s = driven equivalent inertia at driver shaft; T = driving torque, assumed to be constant at T_{max}; θ = input drive angle; w = specified incremental displacement; x = output displacement.)

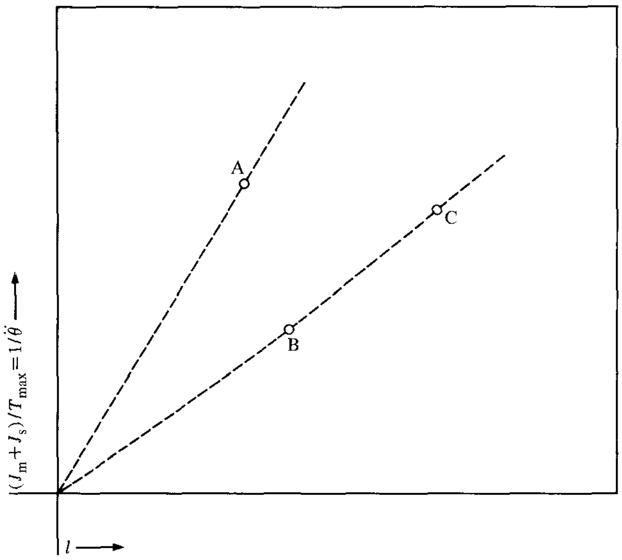


Figure 11 Coordinates for graphical analysis.

mounting hardware, etc. In such a case, the allowable values of J_s and l can most easily be listed in tabular form.

In the case of the selector, the output is defined as a tip displacement w. The time required to move the tip through this distance w must be minimized. We previously defined a relationship between the input angle θ and w (Eq. 19) as $l\theta \approx w$, where l is the distance from the center of rotation to the selector tip. This displacement of the tip is represented by the variable x:

$$x = l\theta;$$
$$\dot{x} = l\dot{\theta};$$
$$\ddot{x} = l\ddot{\theta}.$$

We assume that the device can be represented by the idealized form shown in Fig. 10.

Referring to the hard and soft stop techniques previously described, we find that:

$$t_s^2 = \frac{Cw}{\ddot{x}} = \frac{Cw}{l\ddot{\theta}},$$

where C is the drive constant (2 and 4 for hard and soft stops, respectively), $\ddot{x}$ is the output acceleration and $\ddot{\theta}$ the input acceleration.

For a particular design, C and w are fixed. To minimize t_s, $1/\ddot{x}$ must be minimized (or the output acceleration maximized) and

$$\frac{1}{\ddot{x}} = \frac{1}{l\ddot{\theta}} = \frac{J_m + J_s}{lT_{max}}.$$

Taking the derivative of the above with respect to l, equating it to zero, and reducing, we obtain the criterion for optimum l (corresponding to minimum response time):

$$\frac{1}{l}\left[\frac{J_m + J_s}{T_{max}}\right] = \frac{d}{dl}\left[\frac{J_m + J_s}{T_{max}}\right] \quad \text{or} \quad \frac{1}{l}\left[\frac{1}{\ddot{\theta}}\right] = \frac{d}{dl}\left[\frac{1}{\ddot{\theta}}\right].$$

We can show that this represents a minimum point when

$$\frac{d^2}{dl^2}\left[\frac{J_m + J_s}{T_{max}}\right] > 0.$$

Design limitations sometimes preclude the use of this particular point. As implied previously, $(J_m + J_s)/T_{max}$ (or $1/\ddot{\theta}$) cannot be easily differentiated but can be obtained in tabular form. Consider coordinates representing $(J_m + J_s)/T_{max}$ vs l, as shown in Fig. 11. The slope of a line from the origin to a point in the plane is $1/l\ddot{\theta}$ or $1/\ddot{x}$. It follows then that point B, falling on a line of lesser slope than point A, represents a shorter incrementing time than that represented by point A. Also, point C represents the same incrementing time as point B.

If $(J_m + J_s)/T_{max}$ for a system is plotted from design data on these coordinates, a tangent line from the origin can locate the optimum incrementing configuration (see Fig. 12).

At E:

$$\frac{1}{l}\left(\frac{J_m + J_s}{T_{max}}\right) = \frac{d}{dl}\left(\frac{J_m + J_s}{T_{max}}\right).$$

In the case for which T_{max} is not a function of l (as in the selector) a plot of $J_m + J_s$ vs l is sufficient (see Fig. 13).

Figure 13 differs from Fig. 12 only by a scale factor and produces the same optimum l. Notice in Fig. 13 that the moment of drive unit inertia J_m may be easily separated from the moment of driven inertia J_s. A change in J_m will, in effect, move the origin and shift the optimum point. This feature can be useful in cases for which the

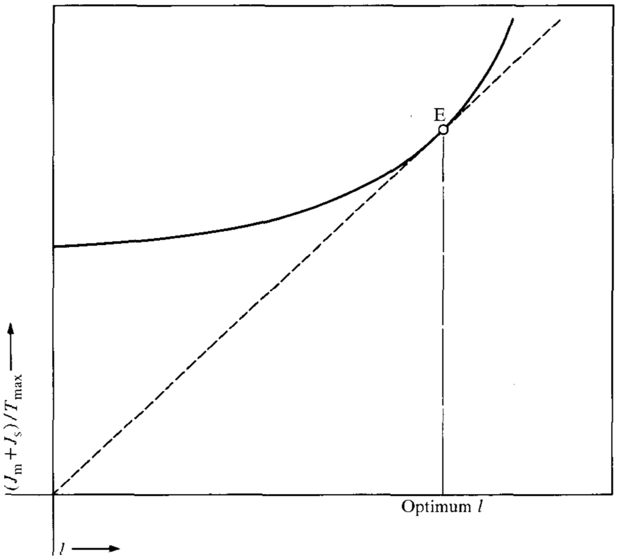


Figure 12 System response plot for graphical selection of optimum l.

designer may be required to design a selector before the drive unit has been chosen. His choice of a selector must minimize the exposure to redesign that might occur when the drive unit is finally assigned.

As an example, Fig. 14 shows a smooth curve of J_s vs l, plotted from computed data points. The points O_A and O_B were chosen to represent the probable extremes of selector motor inertia, and tangents from these two points to the curve will locate the range of optimum radii (l_A to l_B) that can be expected. Now, an early choice of radius l_C can be evaluated to see how far, proportionally, the response time corresponding to a given motor would be from its optimum. For example:

Slope of $\overline{O_BB} = 2.8162$, and
slope of $\overline{O_BC} = 2.8875$.

For motor B, $\dfrac{\text{actual } t_s}{\text{optimum } t_s} = \sqrt{\dfrac{2.8875}{2.8162}} = 1.013.$

Similarly for A, $\dfrac{\text{actual } t_s}{\text{optimum } t_s} = 1.012.$

Therefore, the designer knows that his early choice of l_C will allow him to choose any motor inertia between O_A and O_B and still be within 1.3 percent of optimum select time for the particular motor chosen.

With l_C and J_s fixed, the choice of motor is now simplified to picking the one that produces the greatest value for

$$\frac{Tl_C}{J_m+J_s} = \ddot{x}.$$

A problem similar to the one just described is found in the design of a motor-driven capstan in a magnetic tape unit. Here a point on the tape must assume a given velocity within a minimum displacement to minimize the inter-record gap size. Once again $\ddot{x}$ is to be maximized. The same approach as that used for the selector example can be used to make a good early choice of capstan diameter. This will allow the motor designers and tape transport designers to work independently with a high probability that the final unit will function almost optimally.

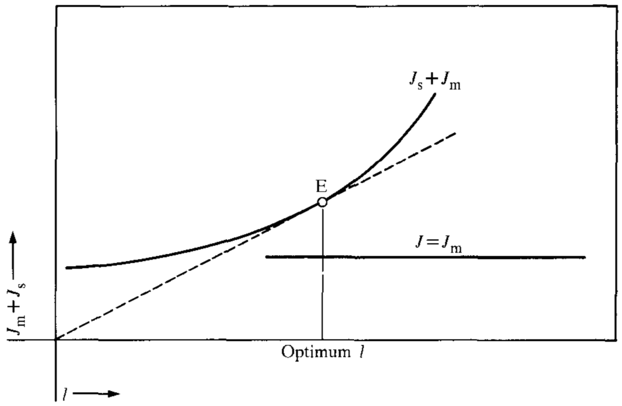


Figure 13 System response plot for T_{max} independent of l.

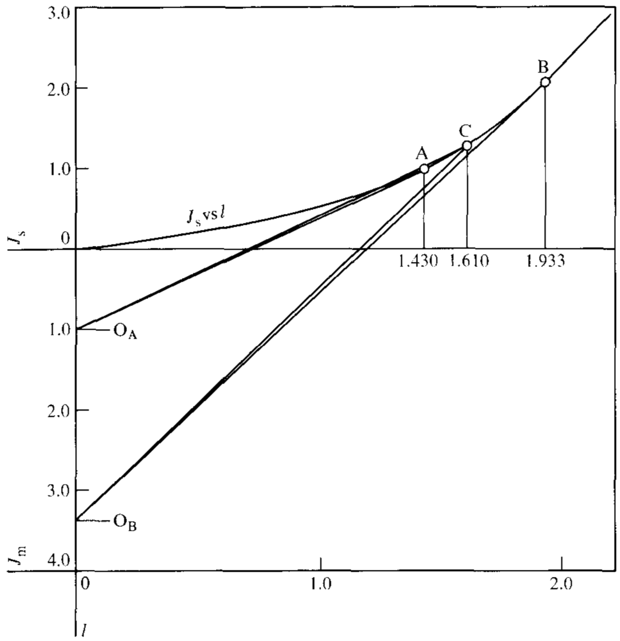


Figure 14 Example of graphical selection of l by estimating the range of J_m.

Conclusions

We have seen that the three parameters controlling sorting machine throughput are document velocity, document length and selector response time. Increasing docuument velocity effectively increases throughput in a typical case. However, this velocity is limited by considerations of machine life, noise levels and document kinetic energy. Document length reduction, though effective in increasing throughput, is generally not possible when document size and reading methods are standardized. This leaves the selector response or indexing time as the principal controlling parameter once the velocity is at its allowable maximum.

A minimum response time can be obtained by proper selection of the prime mover. The variation of the selector inertia as a function of its length, and the dependence of the increment angle upon this length, make an optimization possible in most cases. An analytical and a graphical method have been developed, with which the optimum load-to-prime-mover inertia ratio and the optimum driven inertia pivot location and length can be determined.

These methods also permit the choices of driver and load to be relatively independent of each other in the design cycle, and provide a means for evaluating the consequences of not achieving an absolutely optimum system. Both procedures assume that the driving torque is constant (although it can be either single- or bi-directional), and both can be applied to many similar indexing or incremental-displacement devices where that assumption is valid.

Received July 16, 1971

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