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Using a Desk-Top Computer for an On-Line Flood Warning System

Abstract: The paper deals with the development of an adaptive model that is applicable to real-time forecasting of hydrologic processes. The rainfall-runoff process is considered here. In this model the discharge was modeled as autoregressive with past discharges and a moving average representation on the precipitation. The model makes use of the Constrained Linear Systems (CLS) technique to split the precipitation into two rainfall inputs by using a threshold based on an antecedent precipitation index. This technique can be thought of as a piecewise linearization of a nonlinear process. The real-time forecasting model is a time invariant linear state model where the state variables, discharge and rainfall, are estimated by the Kalman filtering algorithm and the unknown model parameters by using the instrumental variables approach. This technique was applied in a case study using data from the Ombrone River Basin, Italy, and was implemented on a small desk-top computer.

Introduction

In most countries of the world, flood warning systems, if they exist at all, are the responsibility of some large central authority and are limited to major streams and population centers. For the myriads of small towns and cities clustered along the banks of minor rivers, flood warning systems are either nonexistent or of such a nonspecific nature as to be of little practical value to the residents. It is the purpose of this paper to show that modern technology has advanced to the stage where small local civil defense or similar authorities could consider installing and maintaining their own "flash flood" warning systems. Further, it is suggested that local control of a flood warning system combined with the education and involvement of the area residents is likely to lend more credence to flood forecasts, and lead to fewer and less serious recriminations following faulty forecasts.

To issue flood warnings, local authorities need a relatively cheap, reliable forecast system, combined with some knowledge of the variance associated with their forecasts. In this paper we develop an algorithm suitable for use on a desk-top computer which, when interfaced with some telemetering equipment connected to upstream sensors, could form the basis for such a local forecast system. The algorithm is applied to a flood that occurred in the Ombrone River Valley catchment of southern Tuscany, Italy.

Ombrone River Valley

The total catchment area for the Ombrone River is about 3500 km^2 (see Fig. 1), most of which has fairly low permeability. From a geologic point of view the basin is very heterogeneous and much faulted although three main geologic provinces are evident. In the north the soils have developed from marine clay and sandstones; further south are limestones, shales and quartzite conglomerates, while the southern portion of the basin is covered with nonmarine alluviums [1]. The region can be classified as moderately hilly, with a maximum altitude of 1734 m and a mean of 346 m above mean sea level. The land use is principally agricultural, with a few forested areas and many small villages and towns.

Grosseto, a town of approximately 70 000 people, is situated at the mouth of the Ombrone River some 250 km southwest of Florence. The Florence flood of November 4, 1966 received world-wide attention, but the flood which inundated Grosseto to a depth of 3 m on the same day received little publicity outside Italy. This paper is concerned with the problem of making flood forecasts for just such small towns situated at the mouths of minor rivers.

Meteorologic and hydrologic data base

When Grosseto was flooded in December, 1964, there were 11 recording raingauges functioning in the catchment area above the Sasso d'Ombrone streamgauging station (see Fig. 1 for their locations). The raingauges were all of international standard with orifices of 1000 cm^2. Precipitation records in two-hour increments were available for these stations and form part of the data base for this study.

The catchment area above the Sasso d'Ombrone streamgauge amounts to 2657 km^2. If one considers that storms may move across the basin at a rate of 50 km/h, and that interspersed amongst the rain clouds are intermittently functioning lenses yielding intense precipitation, one quickly realizes the magnitude of the sampling problem that exists in the rainfall component of any rainfall-runoff model that is to be developed.

In this study the individual bi-hourly rainfall measurements were combined into a single weighted average bi-hourly precipitation measurement by using the Thiessen method [2]. This method attempts to allow for the nonuniform distribution of raingauges within the basin by assigning weights to the measurements proportional to their surrounding sub-areas, on the assumption that the precipitation between stations varies in a linear manner. Other weightings are of course possible, but no investigation of alternative raingauge weighting systems was made in this study. Further, from the results obtained to date it would appear that the benefits that might accrue from using some other arbitrary weighting system are not likely to have an overpowering influence on the accuracy of the flood forecasts.

Quantitative precipitation forecasts for the area above Sasso d'Ombrone are needed if one wishes to make long-term flood forecasts for the town of Grosseto. Synoptic meteorologic forecasts of precipitation are notoriously inaccurate, and in any case are not available for the Ombrone region. Further, scanning radar which would allow for at least an accurate forecast of the cessation of precipitation is not available for this isolated and hilly region. However, during the intense rainstorms that cause floods in this region, the previously mentioned Thiessen weighted precipitation index has been found to be highly positively autocorrelated (in excess of 0.9); this property forms the basis of the precipitation forecasts that were developed for use in this study.

The streamgauge at Sasso d'Ombrone is well situated in a rocky gorge. The stage/discharge relationship, which yields the discharge, given the level measurements, is regarded as reasonably accurate, and in addition there is a float gauge for measuring flood crests in the vicinity of Grosseto. Mathematical models for routing flood waves are highly developed [3] and of sufficient accuracy to make the Sasso d'Ombrone/Grosseto portion of the Grosseto flood warning system a technically trivial problem of model calibration. The travel time for a flood from Sasso d'Ombrone to Grosseto has been estimated at three hours; therefore, for forecasts of range greater than three hours, the center of attention must be the rainfall-runoff modeling for the region upstream from Sasso d'Ombrone.

All rainfall-runoff models that do not have an on-line updating facility produce forecasts that are either too high or too low for long periods of time [4]. Most modelers in

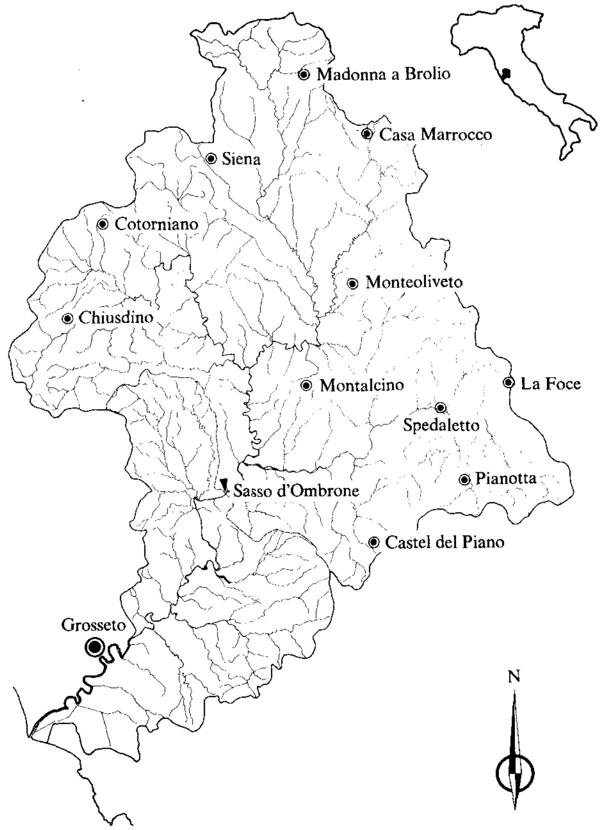

Figure 1 Map of the Ombrone River catchment showing the location of the gauges.

fact modify the rainfall input functions or the evapo-transpiration (E.T.) losses in order to adapt the model to the measured discharges and to thus update the forecasts in real time. In this paper an objective technique for updating the model parameters in real time, and thus the discharge forecasts, is presented which uses the Constrained Linear Systems (CLS) threshold concept [5] in a Kalman filter formulation. The concluding sections of the paper give the mathematical framework for this approach and illustrate its implementation on a small desk-top computer.

CLS approach to rainfall-runoff modeling

Following Dooge [6], a simplified catchment model can be represented by the scheme shown in Fig. 2. Some of the precipitation p infiltrates the ground, depending on the soil moisture content. The remainder, referred to as the excess rainfall, is then responsible for a relatively quick surface water response, while some of the infiltrated water reappears at the outlet of the catchment after a longer period of time. Thus soil moisture, as it is depleted by evapo-transpiration (E.T.) and deep losses, acts as a controller of the saturation threshold S, which determines

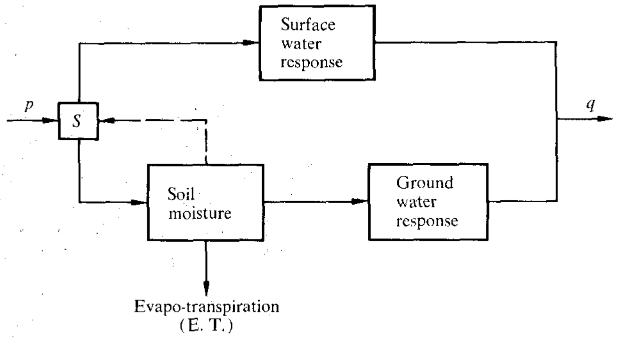

Figure 2 Simplified catchment model; p represents the precipitation collected by the catchment area, and q the measured outfall discharges.

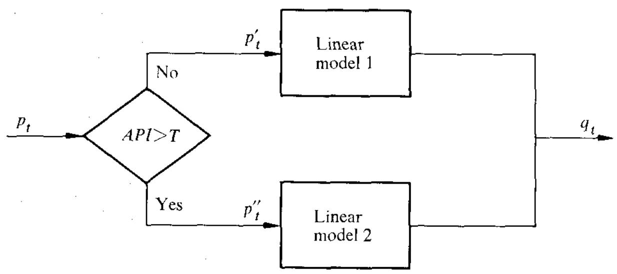

Figure 3 CLS model scheme ($p_t = p'_t + p''_t$, $\forall$ t).

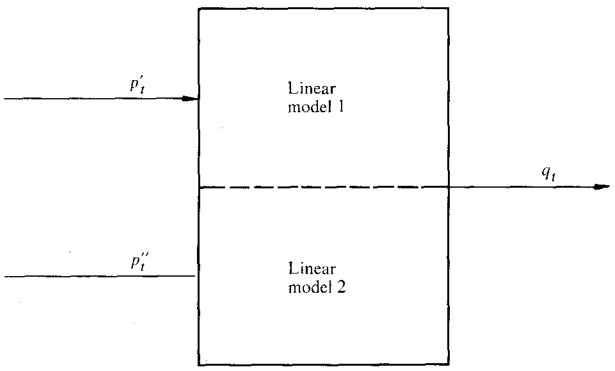

Figure 4 Scheme of the MISO representation of CLS for single threshold T.

how much water percolates and how much flows directly to the drainage network. Though both the surface water effect and the ground water effect can be represented by linear systems, the overall effect is highly nonlinear, since it depends on the relative saturation of the soil during storm events.

Todini and Wallis [5] solved this nonlinear problem by piecewise linearization using the so-called CLS approach. One of the basic ideas of CLS is that the precipitation p_t (t = time) is transformed into runoff q_t by a series of mutually exclusive linear systems depending on the initial condition of the soil.

For the Ombrone model only two linear systems represented the rainfall runoff phenomenon, depending on the value of the antecedent precipitation index (API), which is a function of time:

$$API_t = K \cdot API_{t-1} + p_{t-1},$$

where K is an exponential decay factor that accounts for the depletion of water in the soil due to evapo-transpiration losses and deep percolation.

If the initial condition of the soil is dry, the value of API_t is equal to or smaller than a threshold value T; p_t will then be transformed into q_t by the first linear model. When the soil is wet, API_t is larger than T; therefore, the effect of the precipitation p_t on the discharges q_t is higher. Thus the second linear model, with a larger impulse response, transforms the precipitation p_t into runoff q_t. The logic scheme of CLS is represented in Fig. 3. Though the overall effect is nonlinear, once the threshold parameters have been fixed, the system's behavior can be represented as a MISO (multiple input, single output) model (Fig. 4).

A computer program, CLSB, which allows for the estimation of the unknown parameters of the MISO linear model for each given set of threshold parameters, is available [7] and was used in the identification phase of the Ombrone model.

ARMAX representation of the model

The MISO model already described can be easily reformulated as an ARMAX model of the following general structure [8]:

$$\begin{aligned}(1 + \delta_1 B + \cdots + \delta_r B^r) q_t \\ = (\omega'_0 + \omega'_1 B + \cdots + \omega'_s B^s) p'_{t-b} \\ + (\omega''_0 + \omega''_1 B + \cdots + \omega''_s B^s) p''_{t-b} + w_{1t},\end{aligned} \qquad (1)$$

where B is the backward shift operator, so that $B^r q_t = q_{t-r}$ [9]; r is the order of the autoregressive term, s is the order of the moving average on the exogenous variable, b is the time lag of the input function, and w is the system noise. The δ and ω are parameters to be determined. [The term ARMAX is used to point out that these models, as opposed to the pure ARMA (auto regressive moving average) models, also accept deterministic inputs. Following Young et al. [10], the X may stand for exogenous input variables.]

As stated previously, the lumped rainfall input has been found to be highly autocorrelated; therefore, a model to forecast the rainfall during storm events was set up to allow for a more than one step ahead forecast:

$$(1 + \alpha_1 B + \cdots + \alpha_s B^s) p_t = w_{2t}, \tag{2}$$

where the α are parameters to be determined.

Kalman filter representation

A model to be used for early warning must be very simple, with small CPU requirements, but also efficient with respect to estimating future discharges and providing information on the probability with which some given warning level may be exceeded. All these requirements, together with the possibility of updating in real time the parameters of the model, can be met by using a Kalman filter representation of the above-mentioned ARMAX models.

The Kalman filter is a recursive estimator that performs unbiased and minimum variance estimates of the quantities (state variables) chosen to represent the evolution of a dynamic system. The basic idea is to provide a one step ahead extrapolation of the state variables and of the variance-covariance matrix of the errors of estimate, and to update the forecast, taking into account the information provided by the new noisy measurements. It should be noted that the advantage of the Kalman filter really emerges when real-world data are considered in structural models.

All the models where a physical cause and effect may be hypothesized are in fact structural models rather than regression models [11] with input variables (in our case, e.g., the lumped rainfall input p_t) which are noise corrupted. The regression hypothesis that the input (exogenous) variables be perfectly measured is nullified and therefore the system model should be expressed in terms of the true unknown quantities. A measurement model,

$$q_t^0 = q_t + v_{1t};$$

$$p_t^0 = p_t + v_{2t}, \tag{3}$$

where v_{1t} and v_{2t} represent the measurement errors, must be added to the system model represented by Eqs. (1) and (2). The resulting model expresses a cause and effect relationship between the true unmeasurable quantities, thus allowing a filtering of the measurement noise. It can be rewritten with the Kalman filter notation as

System model $\mathbf{x}_{t+1} = \mathbf{\Phi}_t \mathbf{x}_t + \mathbf{\Gamma}_t \mathbf{w}_t;$

Measurement model $\mathbf{z}_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{v}_t.$ (4)

The state transition matrix $\mathbf{\Phi}_t$, the noise transition matrix $\mathbf{\Gamma}_t$, and the measurement matrix $\mathbf{H}_t$ are amplified below.

Model identification and parameter estimation

The identification of the model structure was performed by using a continuous rainfall-runoff record 160 days long during the fall/winter season of 1964–1965, sampled with two-hour time intervals Δt. Trial and error use of the CLSB program [5, 7, 9] gave a threshold value T of 475.2 million cubic meters that was found to be the most appropriate, together with $K = 0.98$ for the API function.

From the analysis of the cross-correlation functions between q_t^0 and p_t^0 using the standard identification approach discussed in Box and Jenkins [9], the most appropriate values for the parameters were found to be $r = 2$, $s = 2$, and $b = 1$. Therefore,

$$q_t = -\delta_1 q_{t-1} - \delta_2 q_{t-2} + \omega_0' p_{t-1}' + \omega_1' p_{t-2}' + \omega_2' p_{t-3}'$$
$$+ \omega_0'' p_{t-1}'' + \omega_1'' p_{t-2}'' + \omega_2'' p_{t-3}'' + w_{1t};$$

$$p_t = \alpha_1 p_{t-1} - \alpha_2 p_{t-2} - \alpha_3 p_{t-3} + w_{2t}; \tag{5}$$

and

$$q_t^0 = q_t + v_{1t};$$

$$p_t^0 = p_t + v_{2t} \tag{6}$$

are in this case the expanded form of the Kalman filter notation previously given as Eq. (4).

We therefore have

State vector $\mathbf{x}_t = [q_t \; q_{t-1} \; p_t' \; p_{t-1}' \; p_{t-2}' \; p_t'' \; p_{t-1}'' \; p_{t-2}'']^T;$

Measurement vector $\mathbf{z}_t = [q_t^0 \; p_t^0]^T;$

System noise $\mathbf{w}_t = [w_{1t} \; w_{2t}]^T;$

Measurement noise $\mathbf{v}_t = [v_{1t} \; v_{2t}]^T;$

State transition matrix

$$\mathbf{\Phi}_t = \begin{bmatrix} -\delta_1 & -\delta_2 & \omega_0' & \omega_1' & \omega_2' & \omega_0'' & \omega_1'' & \omega_2'' \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha_1 & -\alpha_2 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha_1 & -\alpha_2 & -\alpha_3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix};$$

and

$$\mathbf{\Gamma}_t^T = \mathbf{H}_t = \begin{cases} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \text{for } API_t \leq T; \\[2ex] \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} & \text{for } API_t > T. \end{cases}$$

Further assumptions are

$$\mathbf{w}_t \cong \mathrm{NIP}(\bar{\mathbf{w}}, \mathbf{Q});$$

$$\mathbf{v}_t \cong \mathrm{NIP}(\bar{\mathbf{v}}, \mathbf{R}),$$

where NIP stands for Normal Independent Process, and

$$\mathrm{E}[(\mathbf{w}_t - \bar{\mathbf{w}})(\mathbf{v}_t - \bar{\mathbf{v}})] = 0,$$

where $\mathrm{E}[\cdot]$ denotes the expectation. The linear formulation of the Kalman filter is an unbiased minimum variance estimator if and only if these assumptions are fulfilled.

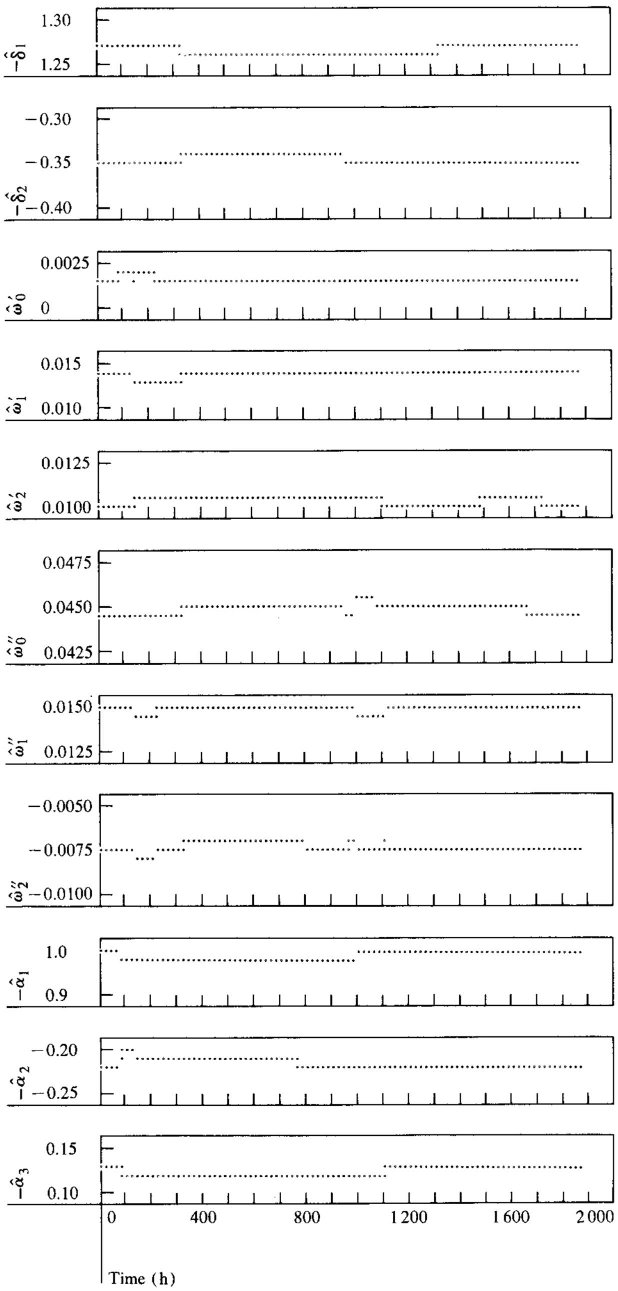

Figure 5 Computer plots of recursive parameter estimates vs increasing length of historical record.

The estimation of the parameters could be made by means of regression, but as previously stated [11–14] these are structural models rather than regression models, since even the independent variables are noise-corrupted, leading to inconsistent estimates of the parameters. Therefore, according to [12], any of three main alternative approaches could be selected:

a. the classical approach, which requires fairly strong assumptions about the probability distributions of the error terms;
b. other approaches based on grouping the observations and making less stringent assumptions about the error terms; or
c. the use of instrumental variables.

The last approach was chosen because the Kalman filter itself can provide at each step in time the instrumental variable (IV) vector that fulfills the requirements of being highly correlated with the unobservable true variables q_t and p_t but totally independent of the measurement errors v_{1t} and v_{2t}. The Kalman filter provides the necessary IV vector that is the state vector estimate to be used for the recursive parameter estimation. The algorithm used is the recursive IV algorithm proposed by P. C. Young [13].

The instrumental variables estimation improves its efficiency according to the quality of the instrumental variable vector provided by the Kalman filter; therefore, the estimation procedure, which is carried out in parallel with the Kalman filter and in which a recalibration of all parameters is performed, is repeated several times on the same data set until further iteration yields negligible changes in the parameter estimates. Note that this calibration of parameters is only performed once with the initial historical data set.

In Fig. 5 the plots of all the parameter estimates vs the number of data used for the estimation are shown, revealing, after a certain number of iterations, that a high degree of stability is obtained. The final values of the parameter estimates were the following:

$$
\begin{array}{lll}
-\delta_1 = 1.267 & -\delta_2 = -0.350 & \\
\omega_0' = 0.002 & \omega_1' = 0.014 & \omega_2' = 0.010 \\
\omega_0'' = 0.045 & \omega_1'' = 0.015 & \omega_2'' = -0.008 \\
-\alpha_1 = 0.999 & -\alpha_2 = -0.221 & -\alpha_3 = 0.126
\end{array}
$$

Use of the model for short-term forecasts

Once the parameters have been estimated by using the historical record, the model is ready to be used for on-line rainfall-runoff prediction. If we denote by

$$
\mathbf{Z}_t = [\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_t] = \begin{bmatrix} q_1 & q_2^0 & \cdots & q_t^0 \\ p_1^0 & p_2^0 & \cdots & p_t^0 \end{bmatrix} \tag{7}
$$

the historical record up to time t, $\hat{q}_{t+\Delta t}$ and $\hat{p}_{t+\Delta t}$ [the predicted values for the discharges and rainfalls at time $(t + \Delta t)$] are only a function of the information content of the past. We can therefore write

$$
\begin{aligned}
\hat{q}_{t+\Delta t} &= f(\mathbf{Z}_t); \\
\hat{p}_{t+\Delta t} &= f(\mathbf{Z}_t),
\end{aligned} \tag{8}
$$

but in general, predictions $1\Delta t$ in advance are of a range too short for effective warning and control purposes, and predictions $2\Delta t$ or greater in advance are often needed. For the Ombrone, $3\Delta t$ advance predictions (Δt = two hours) are needed at the Sasso d'Ombrone gauging station, which provides nine hours' advance early warning for the city of Grosseto, given the three hours for flood wave travel time from Sasso d'Ombrone to Grosseto.

By using the Kalman filter without updating to extrapolate the predictions, it is easy to show that

$$\hat{q}_{t+2\Delta t} = f(\mathbf{Z}_t, \hat{q}_{t+\Delta t}, \hat{p}_{t+\Delta t});$$
$$\hat{p}_{t+2\Delta t} = f(\mathbf{Z}_t, \hat{p}_{t+\Delta t}); \tag{9}$$

and that

$$\hat{q}_{t+3\Delta t} = f(\mathbf{Z}_t, \hat{q}_{t+2\Delta t}, \hat{q}_{t+\Delta t}, \hat{p}_{t+2\Delta t}, \hat{p}_{t+\Delta t});$$
$$\hat{p}_{t+3\Delta}t = f(\mathbf{Z}_t, \hat{p}_{t+2\Delta t}, \hat{p}_{t+\Delta t}). \tag{10}$$

It should be considered that without updating, the variance of the errors of estimate of the state variables $q_{t+i\Delta t}$ and $p_{t+i\Delta t}$ increases with the increasing lag i of prediction due to the system errors, or in other words due to our system simplified model.

Another consideration comes from the physical analysis of the rainfall-runoff process: discharge estimates that are unreliable compared to the previous ones may be obtained if we try to forecast beyond the time to peak of the impulse response. This latter time is a characteristic of the catchment. We therefore must find a tradeoff between our need for forecasting with maximum lag and the physical behavior of the phenomenon. Fortunately, since we may have continuous rainfall-runoff measurements, after $1\Delta t$ we have

$$\mathbf{Z}_{t+\Delta t} = [\mathbf{Z}_t, \mathbf{z}_{t+\Delta t}] = \begin{bmatrix} \mathbf{Z}_t, & \begin{matrix} q^0_{t+\Delta t} \\ p^0_{t+\Delta t} \end{matrix} \end{bmatrix}, \tag{11}$$

and therefore the filter can be recalibrated and the states at time $(t + 2\Delta t)$ can be forecast by using all the information up to time $(t + \Delta t)$. Because the filter is a recursive estimator, only the known quantities at time t and the new measurements at time $(t + \Delta t)$ are needed to perform the recalibration of the model parameters (the unknown coefficients of the state transition matrix $\mathbf{\Phi}_t$) which are then used to perform the forecast at time $(t + 2\Delta t)$:

$$\hat{q}_{t+2\Delta t} = f(\mathbf{Z}_{t+\Delta t});$$
$$\hat{p}_{t+2\Delta t} = f(\mathbf{Z}_{t+\Delta t}). \tag{12}$$

Thus, after updating with the new information content, we are able to move further in time to predict

$$\hat{q}_{t+3\Delta t} = f(\mathbf{Z}_{t+\Delta t}, \hat{q}_{t+2\Delta t}, \hat{p}_{t+2\Delta t});$$
$$\hat{p}_{t+3\Delta t} = f(\mathbf{Z}_{t+\Delta t}, \hat{p}_{t+2\Delta t}); \tag{13}$$

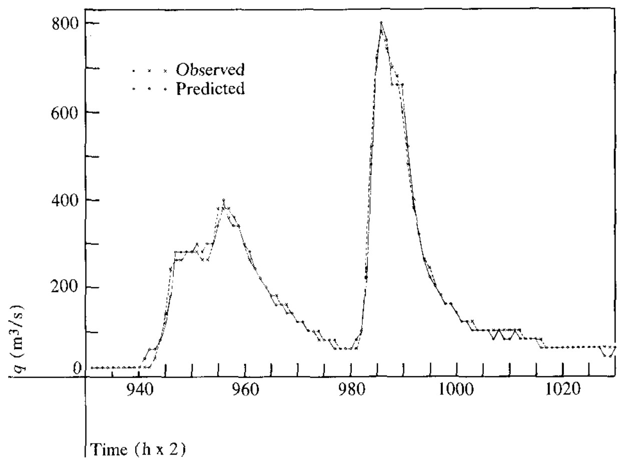

Figure 6 Measured and one step (two hours) ahead forecasts at Sasso d'Ombrone (discretized computer plot).

and

$$\hat{q}_{t+4\Delta t} = f(\mathbf{Z}_{t+\Delta t}, \hat{q}_{t+3\Delta t}, \hat{q}_{t+2\Delta t}, \hat{p}_{t+3\Delta t}, \hat{p}_{t+2\Delta t});$$
$$\hat{p}_{t+4\Delta t} = f(\mathbf{Z}_{t+\Delta t}, \hat{p}_{t+3\Delta t}, \hat{p}_{t+2\Delta t}). \tag{14}$$

Therefore, after the model is updated, $\hat{q}_{t+3\Delta t}$ is only a two step ahead forecast with a reduction of the variance of the errors of estimate, which as already stated can be expected to increase with the number of steps.

In any case, the variances and consequently the estimated standard deviations are very small compared to the actual discharge values of the flood events. Figure 6 shows an example of one step ahead predictions after stabilization of parameters, compared to the measured discharges for the largest storm event of the December 1964 record. The estimated standard deviation S_1 of the one step ahead prediction over the whole sample is

$$S_1 = 14 \text{ m}^3/\text{s},$$

while the high flows are measured in hundreds of m^3/s. Using the Kalman filter variance-covariance projection equation, we can determine that the comparable two step ahead prediction is

$$S_2 = 23 \text{ m}^3/\text{s},$$

while the three step ahead prediction yields

$$S_3 = 32 \text{ m}^3/\text{s}.$$

When compared to the values of the flood discharges, S_2 and S_3 are very small.

Figure 7 shows a series of forecasts for $1\Delta t$, $2\Delta t$, and $3\Delta t$ (Δt = two hours), plotted with observed flows at successive Δt time increments. Each plot represents an increment of $1\Delta t$ from the previous plot. Recalibration of model parameters is performed at each Δt. Forecast accu-

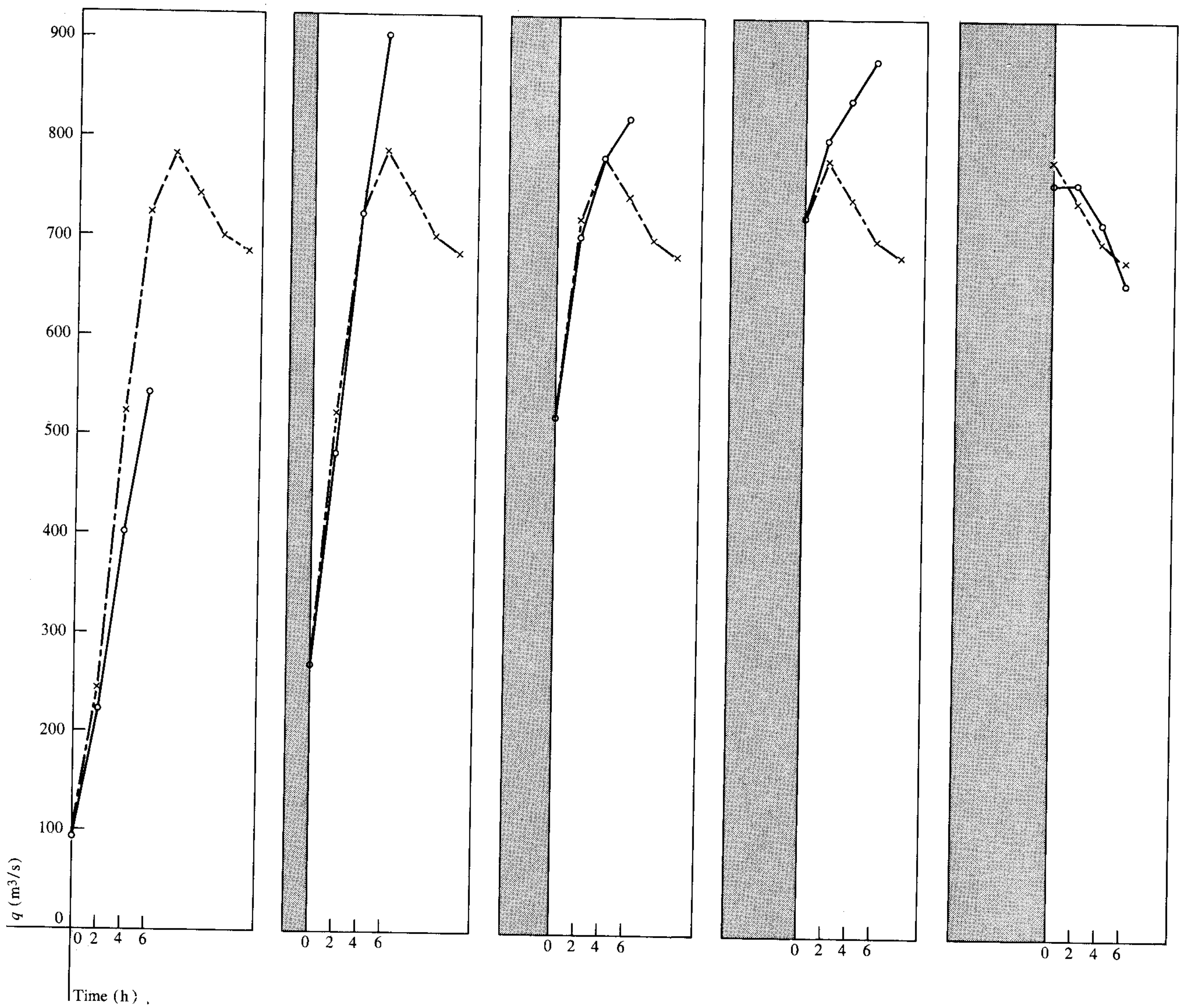

Figure 7 Forecasts for $1\Delta t$, $2\Delta t$, and $3\Delta t$ and observed flows at successive Δt time increments.

racy would be increased if the time of the cessation of precipitation had been predictable. Figure 7 may show that the forecasted values for two and three steps ahead lie outside the 95 percent confidence limit. This is due to the fact that the standard deviations were computed over the whole record of high and low flows, and the forecasts are relevant to the largest recorded flood.

Conclusions

As stated previously, the model implemented on a desk-top computer can be of great help to local flood-control authorities, enabling them to analyze the benefits and risks of alternative strategies and to make decisions in real time that may be close to optimal for the local situation. The model in fact not only forecasts the future values of discharges during storm events but also provides information about the accuracy of the forecasts. This information can be used both to evaluate the probability with which a warning level may be exceeded and to make a cost-benefit analysis of different possible courses of action.

References

1. L. Canali, "Un Metodo di Previsione delle Piene in Base alle Segnalazioni delle Piogge," *Giornale del Genio Civile* **6–7,** June–July (1969).
2. A. H. Thiessen, "Precipitation for Large Areas," *Mon. Weather Rev.* **39,** 1082 (1911).
3. T. A. Ciriani, U. Maione, and J. R. Wallis, *Mathematical Models for Surface Water Hydrology*, John Wiley & Sons, Inc., New York, 1977, p. 423.
4. "Intercomparison of Conceptual Models Used in Operational Hydrological Forecasting," *Operational Hydrology Report 7, WMO No. 429*, World Meteorological Organization, Geneva, 1975.

5. E. Todini and J. R. Wallis, "Using CLS for Continuous Daily or Longer Period Rainfall-Runoff Modeling," *Mathematical Models for Surface Water Hydrology*, John Wiley & Sons, Inc., New York, 1977, p. 433.
6. J. C. I. Dooge, "The Linear Theory of Hydrologic Systems," *Technical Bulletin No. 1468*, U.S. Department of Agriculture, U.S. Government Printing Office, Washington, DC, 1973.
7. E. Todini and J. R. Wallis, The Systems Approach to Rainfall-Runoff Modeling, in press, 1977.
8. E. Todini and D. Bouillot, "A Rainfall-Runoff Kalman Filter Model," *System Simulation in Water Resources*, G. C. Vansteenkiste, ed., North-Holland Publishing Co., Amsterdam, 1975.
9. G. E. P. Box and G. M. Jenkins, *Time Series, Analysis, Forecasting and Control*, Holden Day Publishing Co., Inc., San Francisco, 1970.
10. P. Young and P. Whitehead, "A Recursive Approach to Time-Series Analysis For Multivariable Systems," *Intl. J. Contr.* **25,** 457 (1977).
11. M. G. Kendall and A. Stuart, *The Advanced Theory of Statistics*, Vol. II, 2nd ed., Charles Griffin and Co. Ltd., London, 1967.
12. J. Johnston, *Econometric Methods*, McGraw-Hill Book Co., Inc., New York, 1963.
13. P. C. Young, "Recursive Approaches to Time Series Analysis," *Bull. Inst. Math. and Its Appl.* **10,** 209 (1974).
14. R. K. Mehra, "Identification and Estimation of the Error-in-Variables Model (EVM) in Structural Form," *RM-75-38*, International Institute for Applied System Analysis, Laxenburg, Austria, 1975.

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