This paper develops a queuing model that analyzes the capabilities of a low speed data channel for real-time data inputs. The model yields estimates of waiting, service, and overall transit times.

Low speed channel throughput can be increased by multiplexing low speed devices. It is assumed that the multiplexing operation employs common registers, the contents of which are saved at initiation and restored at completion of service for all outstanding requests. The model takes into account preemptive interference from high speed data channels.

Computer channel interference analysis

by W. Chang and D. J. Wong

As the state of the data processing art advances, the classical approaches to system design and analysis are becoming more and more inadequate. This paper is restricted in scope to data channel analysis, an important part of system design. The function of a data channel is to act as a communication link with external devices of diverse transmission rates.

The overlapping of processing and channel service provides one means of increasing the capability and efficiency of a computer. In this mode of operation, priority of service is normally given to the channels of highest transmission rates. Precautions must be taken that channels with low priorities are not slowed to the point of improper functioning. The capabilities of a channel, and its buffer requirements, can be assessed by evaluating the waiting-time distribution for character transmission.

If we let a high speed data (HSD) channel correspond to a selector channel and a low speed data (LSD) channel correspond to a multiplexor channel, the model to be discussed can represent the applicable models of IBM SYSTEM/360.[1]

Channel operation

There are three principle means of connecting remote terminals to a central processor. One method is to build an exchange that buffers the communication lines, assembles messages or message segments, and transmits data to or from the host computer.

Generally, the exchange is also provided with capabilities for polling message terminals and for performing many of the necessary housekeeping functions. Although an exchange relieves the host computer of the drudgery of line control, in many instances an exchange cannot be economically justified.

The second method is to tie the lines directly to the host computer. In this case, tasks that might have been done by the exchange are inherited by the processor. Forced to continually scan the lines, the processor is limited in its potential for other tasks.

The third method, which is analyzed within this paper, is a compromise between the previously described methods. The lines are connected to equipment that is capable of scanning lines, and of multiplexing and transmitting data at relatively high speeds. Devices for this method contain buffers for one or more, say c, characters per line. When c characters have been assembled, a request for service is initiated. Unless this request is honored by the host computer prior to the next service request from the same line, loss of data results. While data is being sent or received on a low speed channel, the processor must remain cognizant of the identity of the incoming traffic and assemble messages or message segments in proper locations. Assuming that special registers are not provided, the updating of control and status information requires the use of common registers. Since overlapped operation is almost universal in such systems, the contents of the registers must be stored prior to servicing a request and restored after completion of service.

An LSD channel is said to be *idle*, or *empty*, when no low speed data request is being serviced or waiting for service; the channel is otherwise said to be *busy*. Following this line of thought, low speed data service can be divided into two categories: (1) a request arriving while the channel is idle receives maximum service that stores registers, transfers data, updates registers and restores registers, and (2) a request arriving when the channel is busy receives minimum service (data transfer, updating). Thus, the registers can be stored at the arrival of the first request and restored after the last outstanding request has been serviced.

Another factor that influences the service time of the LSD channel is interference from HSD channels. An HSD channel provides a means of communication between the central processor and tapes, drums, disk files, and the like. Owing to its relatively high transmission rate, an HSD channel is designed to operate with one device at a time and the housekeeping required for multiplexing is eliminated. To realize the efficiency of simultaneous processing and transmission, the HSD channel is normally required to have its own counters and registers for buffering and control. When a device on the HSD channel requires service, it makes a request. Since the devices under consideration require prompt servicing, such requests are given highest priority by the central processor, and operations with lower priorities are interrupted and suspended until the request has been honored. Thus, LSD channel service is subject to interference created by HSD channel requirements.

The other influencing factor is the amount of traffic to be handled, since the probability of LSD requests waiting is a function of the traffic rate. Inasmuch as the communication lines operate asynchronously, it is reasonable to assume that requests arrive at random.

During the normal course of events, the central processor must perform many other necessary functions, such as polling, translation, header analysis, etc. All of these functions must be considered in the design of computer systems.

Channel interference is defined as the proportion of computer processing time required for control of channel operations. The low and high speed channels operate independently of each other; they interact only through their mutual use of the computer.

The physical situation is difficult to formulate. To avoid an overly complicated analytical model while still retaining a useful degree of accuracy, the following simplifications have been made:

- Combine the store and restore functions into one service-time increment.
- An LSD channel request is given maximum service only if all channels are idle at the time of arrival.

The error introduced by these simplifications is negligible. Because the service time of a high speed customer is in the neighborhood of a few microseconds, and the service periods are relatively far apart, the event that a low speed customer will find the LSD channel empty and the HSD channel busy turns out to have a small probability. Simulation results indicate that the mathematical model provides acceptable results.

Service time evaluation

To analyze channel operations, channel service time must first be evaluated. Service times for the HSD channels are relatively easy to find. Therefore, we confine attention to the LSD channel. (A summary of notation appears at the end of the paper.)

Let $H(x)$ be the service-time distribution of a character being transferred in the LSD channel without HSD channel interruptions. Define the function

$$\psi(s) = \int_0^\infty e^{-sx}\, dH\,(x).$$

Let $H^*(x)$ be the service-time distribution of a character in the channel with HSD channel interruptions. Define

$$\phi(s) = \int_0^\infty e^{-sx}\, dH^*\,(x).$$

To obtain a relation between $\psi(s)$ and $\phi(s)$, let ρ_1 be the proportion of time required by the central processing unit to serve one or more HSD channels, according to the system configuration. Since an HSD channel request may occur at any instant, and since ρ_1 is normally small, we can assume for analytical purposes that request arrivals are independent and distributed in accordance with a Poisson process. Let $H_1(x)$ be the service time distribution for the HSD channel, let a be the average time in microseconds required to serve a byte on the HSD channel, and let a_r be the following:

$$a_r = \int_0^\infty x^r \, dH_1(x).$$

Then the rate of service request due to the HSD channel operation is $\lambda_1 = \rho_1/a$.

Define

$$\psi_1(s) = \int_0^\infty e^{-sx} \, dH_1(x).$$

The busy period distribution $D_1(x)$ generated by the service request can be obtained by using standard queuing theory. Let

$$\gamma_1(s) = \int_0^\infty e^{-sx} \, dD_1(x).$$

Then $\gamma_1(s)$ can be obtained as

$$\gamma_1(s) = \psi_1\{s + \lambda_1[1 - \gamma_1(s)]\}. \tag{1}$$

For a detailed discussion, see Reference 2.

The service time distribution $H^*(x)$ can be obtained as follows. Let x be the service time which may be interrupted j times; each interruption lasting a busy period given by $D_1(x)$. Denote by $D_1^j(x)$ the jth iterated convolution of $D_1(x)$ with itself. Then, in the case of a stationary process

$$H^*(x) = \left[\sum_{j=0}^{\infty} \int_0^x e^{-\lambda_1 y} \frac{(\lambda_1 y)^j}{j!} \, dH(y) \right] * D_1^j(x),$$

where the righthand asterisk in the equation denotes convolution. Taking the Laplace transforms of both sides, we have

$$\phi(s) = \psi\{s + \lambda_1[1 - \gamma_1(s)]\}. \tag{2}$$

This expression was derived by Gaver,[3] and also by Avi-Itzhak and Naor.[4] Let

$$d = \int_0^\infty x \, dD_1(x) \qquad \text{and} \qquad d_r = \int_0^\infty x^r \, dD_1(x).$$

From (1) we obtain[5]

$$d = a/(1 - \rho_1) \qquad \text{and} \qquad d_2 = a_2/(1 - \rho_1)^3.$$

Now $H(x)$ takes two forms: a character that arrives while the LSD channel is busy requires $H_b(x)$, and while the LSD channel is empty, $H_e(x)$. Further define

$$\psi_b(s) = \int_0^\infty e^{-sx} \, dH_b(x);$$

$$\psi_e(s) = \int_0^\infty e^{-sx} \, dH_e(x).$$

Then, from (2), we can obtain $\phi_b(s)$ and $\phi_e(s)$.

$$\phi_b(s) = \psi_b\{s + \lambda_1[1 - \gamma_1(s)]\}; \tag{3}$$

$$\phi_e(s) = \psi_e\{s + \lambda_1[1 - \gamma_1(s)]\}. \tag{4}$$

Let $H_b^*(x)$ and $H_e^*(x)$ be the service time distribution when the LSD channel is busy or empty, respectively. The combined service time distribution with HSD channel interruptions is

$$H^*(x) = W(0)H_e^*(x) + [1 - W(0)]H_b^*(x)$$

and

$$\phi(s) = W(0)\phi_e(s) + [1 - W(0)]\phi_b(s),$$

where $W(0)$ is the probability that a character arriving at the LSD channel finds all channels idle. Let

$$h = \int_0^\infty x\, dH_b(x) \quad \text{and} \quad h_r = \int_0^\infty x^r\, dH_b(x);$$

$$k = \int_0^\infty x\, dH_e(x) \quad \text{and} \quad k_r = \int_0^\infty x^r\, dH_e(x);$$

$$b = \int_0^\infty x\, dH_b^*(x) \quad \text{and} \quad b_r = \int_0^\infty x^r\, dH_b^*(x);$$

$$e = \int_0^\infty x\, dH_e^*(x) \quad \text{and} \quad e_r = \int_0^\infty x^r\, dH_e^*(x).$$

From (3) and (4) we have[5]

$$b = h/(1 - \rho_1),$$

$$b_2 = h_2/(1 - \rho_1)^2 + h\lambda_1 a_2/(1 - \rho_1)^3,$$

$$e = k/(1 - \rho_1),$$

$$e_2 = k_2/(1 - \rho_1)^2 + k\lambda_1 a_2/(1 - \rho_1)^3.$$

The analytical model

The mathematical analysis of HSD channel operations is relatively simple. The applicable model, a priority queue model, has been solved by many authors and need not be discussed (see Reference 6, for example).

Because the presence of LSD channel operations does not affect the HSD channel, we shall concentrate our efforts on the analysis of the LSD channel. A single server with different service times due to the conditions of the queue has been thoroughly investigated by Welch.[7]

The mathematical model of the LSD channel, a special priority queuing model, requires development. The service time of a character that arrives randomly at the LSD channel is contingent upon the busy or idle status of the channel. The service period is also subject to the interruptions of HSD channels. The solution to this problem is described as follows. Let λ_2 be the input traffic density to the LSD channel. Let the input traffic to the channels be a Poisson process. The service times and the other symbols are previously defined. Further, let $U(z)$ denote the generating function of the queue size immediately after the end of service for a character of the LSD channel or a special class of HSD channel characters. The class of HSD channel characters of interest consists of those which terminate a busy period that originated when a character arrived on the HSD channel at a time when all channels were idle. $U(z)$ satisfies the following relationship:

$$\begin{aligned} U(z) = \{[U(z) - U(0)]/z\}\phi_b[\lambda_2(1 - z)] \\ + (\lambda_1/\lambda)U(0)\gamma_1[\lambda_2(1 - z)] \\ + (\lambda_2/\lambda)U(0)\phi_e[\lambda_2(1 - z)], \end{aligned} \tag{5}$$

where

$$\lambda = \lambda_1 + \lambda_2 \quad \text{and} \quad U(0) = U(z)|_{z=0}.$$

The Laplace transforms of the service time distributions of $H_b^*(x)$ and $H_e^*(x)$ are used because HSD channel interference prolongs the service period of a character in the LSD channel. Note that at the instant of service completion in the LSD channel, there can be no service request from an HSD channel, inasmuch as HSD channels are preemptive in nature. If, after completion of an LSD channel service, the first arrival is to the HSD channel, the arrival will initiate a busy period whose distribution has the Laplace transform $\gamma_1(s)$. From (5),

$$U(z) = \frac{U(0)\cdot z\{(\lambda_1/\lambda)\gamma_1[\lambda_2(1 - z)] + (\lambda_2/\lambda)\phi_e[\lambda_2(1 - z)]\} - \phi_b[\lambda_2(1 - z)]}{z - \phi_b[\lambda_2(1 - z)]}.$$

Since

$$U(1) = 1,$$

$$U(0) = (1 - \lambda_2 b)/[1 + (\lambda_1\lambda_2/\lambda)\, d + (\lambda_2^2/\lambda)e - \lambda_2 b]. \tag{6}$$

Let χ_n equal 1 if the nth departing character is from the LSD channel, and otherwise equal 0. Let ξ_n be the queue size of the channel at the departure point. Furthermore, let

$$G(z) = p\{\chi_n = 1\}E\{z^{\xi_n} \mid \chi_n = 1\}.$$

$G(z)$ can easily be obtained when $U(z)$ is known. The solution is

$$\begin{aligned} G(z) = \phi_b[\lambda_2(1 - z)][U(z) - U(0)]/z \\ + (\lambda_2/\lambda)U(0)\phi_e[\lambda_2(1 - z)]. \end{aligned}$$

Therefore

$$G(1) = 1 - (\lambda_1/\lambda)U(0).$$

Let $R(x)$ be the distribution of transit times across the LSD channel, where *transit time* is defined as waiting time plus service time. Denote by $\pi(s)$ the Laplace transform of $R(x)$. Then

$$G(z)/G(1) = \pi[\lambda_2(1 - z)]. \tag{7}$$

The number of characters present in the LSD channel immediately after completion of a transfer is equal to the number of LSD channel characters arriving during the transit time of the departing character. If $z = 1 - s/\lambda_2$, then

$$\begin{aligned}\pi(s) &= \frac{U(0)}{1 - (\lambda_1/\lambda)U(0)} \\ &\quad \cdot \frac{(\lambda_1/\lambda)\gamma_1(s)\phi_b(s) - \phi_b(s) + [1 - (s/\lambda_2)]\phi_e(s)(\lambda_2/\lambda)}{1 - (s/\lambda_2) - \phi_b(s)} \\ &= \frac{(1 - \lambda_2 b)[\lambda_1\gamma_1(s)\phi_b(s) - \lambda\phi_b(s) + \lambda_2\phi_e(s) - s\phi_e(s)]}{(1 + \lambda_2 e + \lambda_1 d - \lambda_2 b)[\lambda_2 - s - \lambda_2\phi_b(s)]}.\end{aligned} \tag{8}$$

From (6) and (8), $\pi(s)$ can be determined. The moments of the transit times can be obtained by differentiating (8) and setting $s = 0$. Thus,

$$\begin{aligned}t_1 = -\pi'(0) &= \frac{\lambda_1 b_2 + 2\lambda_1 bd + \lambda_1 d_2 - \lambda b_2 + \lambda_2 e_2 + 2e}{2(1 + \lambda_2 e + \lambda_1 d - \lambda_2 b)} \\ &\quad + \frac{\lambda_2 b_2}{2(1 - \lambda_2 b)} \\ t_2 = \pi''(0) &= \frac{\lambda_1 d_3 + \lambda_2 e_3 + 3\lambda_1 bd_2 - \lambda_2 b_3 + 3\lambda_1 b_2 d + 3e_2}{3(1 + \lambda_2 e + \lambda_1 d - \lambda_2 b)} \\ &\quad + \frac{\lambda_2 b_3}{3(1 - \lambda_2 b)} + \frac{\lambda_2 b_2}{1 - \lambda_2 b}\, t_1.\end{aligned}$$

Let $W(x)$ be the waiting time distribution of a character in the LSD channel. $W(0)$ is the probability that a character does not have to wait. Denote by $\Omega(s)$ the Laplace transform of $W(x)$. Since the transit time is the sum of the waiting time and the service time, we have

$$\pi(s) = W(0)\phi_e(s) + [\Omega(s) - W(0)]\phi_b(s)$$

or

$$\Omega(s) = \frac{\pi(s) - W(0)[\phi_e(s) - \phi_b(s)]}{\phi_b(s)}. \tag{9}$$

Substituting (8) into (9)

$$\begin{aligned}\Omega(s) &= \frac{[(1 - \lambda_2 b)/(1 + \lambda_2 e + \lambda_1 d - \lambda_2 b)]}{[\lambda_2 - s - \lambda_2\phi_b(s)]\phi_b(s)} \\ &\quad \cdot \{[\lambda_1\gamma_1(s)\phi_b(s) - \lambda\phi_b(s) + \lambda_2\phi_e(s) - s\phi_e(s)] \\ &\quad - W(0)\,[\lambda_2 - s - \lambda_2\phi_b(s)][\phi_e(s) - \phi_b(s)]\}.\end{aligned}$$

$W(0)$ can be determined as follows. From (7) and (8), the generating function of the queue size (number of characters to be served by the LSD channel) is

$$G(z)/G(1) = \frac{(1 - \lambda_2 b)/(1 + \lambda_2 e + \lambda_1 d - \lambda_2 b)}{\lambda_2 z - \lambda_2 \phi_b[\lambda_2(1 - z)]}$$
$$\cdot \{\lambda_1 \gamma_1[\lambda_2(1 - z)]\phi_b[\lambda_2(1 - z)]$$
$$- \lambda \phi_b[\lambda_2(1 - z)] + \lambda_2 z \phi_e[\lambda_2(1 - z)]\}.$$

Let p_0 be the probability of queue size equal zero. Then

$$p_0 = \frac{G(0)}{G(1)} = \frac{(1 - \lambda_2 b)[\lambda - \lambda_1 \gamma_1(\lambda_2)]}{(1 + \lambda_2 e + \lambda_1 d - \lambda_1 b)\lambda_2}.$$

$\gamma_1(\lambda_2)$ can be determined by equation (1). In particular, if $\psi_1(s) = e^{-as}$, we have

$$\gamma_1(s) = e^{-a[s + \lambda_1[1 - \gamma_1(s)]]}$$

and substituting λ_2 for s, we have

$$\gamma_1(\lambda_2) = e^{-a\{\lambda_2 + \lambda_1[1 - \gamma_1(\lambda_2)]\}},$$

which can be determined numerically. Note that $\gamma_1(\lambda_2)$ is a probability measure with a value larger than zero and less than 1.

Since the probability of a character that does not have to wait is equal to the probability of an empty queue at the channel, we have $W(0) = p_0$.

Summary

From the described model, one can compute average time, as well as the means and variances of service and transit times, for a multiplexed low speed data channel subject to high speed data channel interruptions. The arrival rates λ_1 and λ_2, and the service-time distributions $H_b(x)$, $H_e(x)$, and $H_1(x)$ are the computational inputs. The model has been useful in assessing the channel capabilities of various SYSTEM/360 configurations in the context of real-time applications.

ACKNOWLEDGMENT

The authors wish to thank J. P. Bricault, T. W. Gay, C. E. Skinner, and R. C. Soucy for many profitable discussions and A. Gayle and W. P. Margopoulos for encouragement and support on this project. We are also indebted to the IBM SYSTEMS JOURNAL referees for their guidance and helpful suggestions.

CITED REFERENCES AND FOOTNOTE

1. "The structure of SYSTEM/360," *IBM Systems Journal* **3**, 2 (1964).
2. L. Takács, *Introduction to the Theory of Queues*, (Oxford University Press, New York, 1962).
3. D. P. Gaver, Jr., "A waiting line with interrupted service, including priorities," *J. Roy. Statist. Soc.*, Ser. B, **24**, 73–90 (1962).
4. B. Avi-Itzhak and P. Naor, "Multi-purpose service stations in queueing theory," Proceedings of the Third IFORS Conference, Oslo, 1963.
5. Let m_n be the nth moment of distribution $A(x)$ and $\theta(s)$ be the Laplace transform of $A(x)$. Then

$$m_n = (-1)^n \left.\frac{d^n \theta(s)}{(ds)^n}\right|_{s=0}.$$

See T. L. Saaty, *Elements of Queueing Theory* (McGraw-Hill Book Company, Inc., New York, 1961) p. 59.
6. L. Takács, "Priority queues," *Operations Research* **12**, No. 1 (Jan.–Feb. 1964).
7. P. D. Welch, "On a generalized M/G/1 queueing process in which the first customer of each busy period receives exceptional service," *Operations Research* **12**, No. 5 (Sept.–Oct. 1964).

Summary of notation

$H(x)$	LSD channel service-time distribution (in absence of HSD interruptions)	
$H_b(x)$	$H(x)$ given that the LSD channel is busy	
$H_e(x)$	$H(x)$ given that the LSD channel is empty	
$H^*(x)$	LSD channel service-time distribution (with interruptions)	
$H^*_b(x)$	$H^*(x)$ given that the LSD channel is busy	
$H^*_e(x)$	$H^*(x)$ given that the LSD channel is empty	
$H_1(x)$	distribution of HSD channel service times	
$D_1(x)$	distribution of HSD channel busy periods	
$W(x)$	distribution of LSD channel waiting times	
$R(x)$	distribution of LSD channel transit times	
$\psi(s)$	Laplace transform of $H(x)$	
$\psi_b(s)$	Laplace transform of $H_b(x)$	
$\psi_e(s)$	Laplace transform of $H_e(x)$	
$\phi(s)$	Laplace transform of $H^*(x)$	
$\psi_1(s)$	Laplace transform of $H_1(x)$	
$\gamma_1(s)$	Laplace transform of $D_1(x)$	
$\Omega(s)$	Laplace transform of $W(x)$	
$\pi(s)$	Laplace transform of $R(x)$	
a_r	rth moment of $H_i(x)$	
h_r	rth moment of $H_b(x)$	subscript elided for 1st moment
k_r	rth moment of $H_e(x)$	subscript elided for 1st moment
b_r	rth moment of $H^*_b(x)$	subscript elided for 1st moment
e_r	rth moment of $H^*_e(x)$	subscript elided for 1st moment
d_r	rth moment of $D_1(x)$	
t_r	rth moment of $R(x)$	
$W(0)$	probability that an LSD arrival finds all channels idle	
p_0	probability that LSD queue size equals zero	
τ_0	arbitrary reference point on time scale	
ξ_n	queue size of LSD channel at n departures after τ_0	
χ_n	1 if the nth departing character is from the LSD channel, 0 otherwise	
λ_1	arrival rate at HSD channel	
λ_2	arrival rate at LSD channel	
ρ_1	proportion of processor unit time devoted to HSD channel operation	