APPROVED FOR PUBLIC RELEASE. CASE 06- 1104.


## PROJECT WHIRLWIND

Contract NSori60

REPORT R-192

A COINCIDENT-CURRENT MAGNETIC MEMORY UNIT

SEPTEMBER 8, 1950

## SERVOMECHANISMS LABORATORY

 MASSACHUSETTS INSTITUTE OF TECHNOLOGY
# APPROVED FOR PUBLIC RELEASE. CASE 06-1104. 

PROJECI WHIRLWIND

Roport R-192<br>A COINCIDENT-CURRENT MAGNETIC MENORY UNIT

Submitted to the
OFFICE OF NAVAL RESBARCH
Under Contract N5ori60
Project NR-048-097

Revort by<br>William N. Papian<br>SRRVONEGHATISMS LARORATORY MASSACHISETTS TNSTITUTE OF TNCUNOLOGY<br>Cambridgo 79, Massachuaetts<br>Project DIC 6315

September 8, 1950
(Thesis Dete August 31, 1950)

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

Report R-192

## POREWORD


#### Abstract

Because it presents information of general interest, this thesis report, which has had only limited distribution, is being issuad as a Project Whirlwind R-series report.

Any new data-storage development which suggests promise of increased reliability and decreased bulk is of immediate interest to the builders of a large scais digital computer such as Whirlwind. The use of rectangular-loop magnetic cores in a multi-dimensional storage scheme, as suggested by Jay W. Forrester, Director of Project Whirlwind, holds such promise. This investigation was, therefore, undertaken.

Mr. Forrester supervised the thesis work. Invaluable ald was received from members of the mathomatics, logical-design, electronicmcircuits, drafting, reports, and clerical sections of the organization. The author is also grateflil for the cooperation of Mr. John H. Crede and the Allegheny Ludlum Steel Corporation, who supplied the bulk of the metallic cores in the experimental work.


## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

Report R=192

## TABINE OF CONTLTESS

12ag
FOREWORD ..... 2
ABSRRAOS ..... 5
CHAPTYR I 工sTRODTOTIOM ..... 7
A. Background ..... 7
B. Basic Oparation of the Individual Core ..... $g$
C. Two and Three-Dimenaional Storage ..... 12
D. Attack on tho Problet ..... 17
CHAPTYR II COFE RUSPOMSE TMNES ..... 18
A. Fastors Govarning Response Times of Kagnetic Materials ..... 18
B. Bddy-Ourvent Shielding in Thin Tape ..... 19
2. The JEmaer Oaeo ..... 20
2. The Hop-Lineas Oase ..... 23
O. Ixperimental Rosulta and Comparisoms ..... 33
2. Slev Matalle Cores ..... 34
2. Fast Matallac Cores ..... 12
3. Ferritic Cores ..... 49

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104. ${ }^{4}$

Pese
CBAFIER III DEPORMATSON-REGANTION AND SIOMAL BATIOS ..... 52
A. Definitions and Requirements ..... 52
2. Information Retontion ..... 53
a. OML Retention ..... 54
b. 2KRMO-Retention ..... 54
2. Signel Retios ..... 55
3. Teat Setup ..... 57
C. Mxperimentel Remile ..... 62

1. Core kis 4382 ..... 62
2. Core Ferranic 3 hat $\operatorname{FHO}\left(b_{0} 0_{0}\right)$ ..... 66
3. Ferroxcube IV ..... 69
CHAPTER IV COMODOSIOK ..... 74
A. Rerponce Fimes ..... 74
4. Signal Ratios ..... 75
O. Core B-H Charnotorifetios ..... 75
D. Miscellaneous ConsSderations ..... 76
AH2Medix ..... 79
BIBLIOGRNHY ..... 83

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

Report R-192


#### Abstract

ABSglacis

4 mall. toroldel. ferrumagnotic core whose B-H oharscoteristicic is properly "rectengulat" in shape may be made to operate so that $2 t s$ flux polarlty reverses only when the sicht comblisation of two or three megnotiging coils are colncidentally exolbede she core ang then be ueed as a soIncident-curreat binery memory dovica uhioh might be asacabled.  seleotion within such a nystem vould be accomplished by moens of physical-1 ine multchiag along the two or thres space coordinesee.

The response times of rectengulans 100 p corse are fourd to vary over an extramely large range. Ton firnt approximation ${ }_{0}$ oddy-ourrent ahislding accounts for these reaponee timen, whiob range from tecthe of a eocond for some metelile cores to 2 ess Than a mionosecond for seme forriblc coren.

Information-retention ration and aloral ration axe defined and are used to assess the ability of a core to operase as a coineidont-durrent memory unit。 A test setup wich makes 1t posadble zo obtain these rabloe for digferent eefe of oparsting coaditions So devisod and used on a number of cores. Seleotol yomulis erp presanted and diecussed ralative to the perininent hysterenic-200p shape\%.


## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

Report R=192

The problen is bracketsd on the oze hand by a aetalli. 3
 sation and a 20 miserosecondo respones them, and on the other hand, by a Perritio oore (Ferremic 3ina Flog boo.) which has only Ies: alenal retion and a $1 / 2$-mixtozecond remponee time.

Farther developmesst worit chould bo aleed in two directionet townid improving matarial to reduce oldy currenta and inoroase hyotoresig-loop rootangularity, and tovard uncovering and solving the probleas associatod with asscabling large munbers of these ooros into a hiferspeed acsory syotem.

## APPROVED FOR PUBLIC RELEASE．CASE 06－1104．

GAFTYR
EHKRODUCSION


#### Abstract

A．Mackreound Data storage is ane of the sunationa in modern courgutaso oonsrol．and commanications syetout．It is a major suncisioa ta electrondc afgital computern，whose daesge have developad，to a eigificant dogree，around tho limitationa and potentialitioa of chosen atorage medsa．


The storage mediun now belige aneorporated Into the comptiter at Projeot hiritried sa s cathodenray type of electrootatlo storage taba．The daba are in the form of binnary digite viloh are stored ag spots of electric charge on a plane dielectric surface within the tube。A particular opot is＂aseotod＂by akning the cethode rayt two qpece ooordinates in the form or rertical and horisonsal defleotson voltagen determine the positson of the spot。 ＂Writing＂or＂roading＂the epot ie a socond atap accomplished by bringlag tho tube slements up to the proper voltege levels and turning on the cathode rey．
cais 领po of tube use enly two epace coordinatea in the selection gtug of the date storigo proosse．Tha ratension to threv or more space coordinatom vould rusult in s reluot on of both the boxag modium bulk ${ }^{2}$ and the mamber of subaivistome nlomg each coordizate ardis for a given storage capacity．

[^0]
# APPROVED FOR PUBLIC RELEASE. CASE 06-1104.。 

Report R-192


#### Abstract

geontion along a ooordinete axis ie datorininad. in tho cethociorray type of stcarage tube. by the leval of e cellaceson volesge. A more disorese doteryanation of coordinate pesitson in a storage modtum woul d be obtalnable by the selection of one of a mabor of physioal 1ines.

Esceain tovard Cevalopitent of a three-dimenstonal  celection proeens vae under way at Projeot Whisivind eariy in 1947. ${ }^{2}$ One of the media investigated vas the lov-presmare-ges elow-discharge tube. It, hovever, proved infeasible for $\bar{F}$ Fretion reasons.


York atarted agats in the spying of 2949. The developacht af anall ferromagnetic cores with reatangulaz hyeteresis loops reopened interest in the prohle. Is seuned posside that aitable cores could be ansetaled Into a thrcandmeasional sforage array.
 mechenian. ${ }^{1}$ thie theais ie part of that vorkf it is conoerned prinarily with the Indivicual oore.

The folloving eeotions of this chepter explain the olements of this threedimgnaional storage schese and indioats wome of the ceneral requirements on the core. The datalied discusel on of the Individan core starss in Chapter II。

## B. Benie Opuration of the Individna Core

Irleflyo a mail oere made of a "Mard" magnotio material ang be magnetised in ctis direation or the other, and ieft that way. This blestablifty, like that of a twosyonltion relay, eny be uaed

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

to express the two digita of tho binary syatem, zzino or OME. A mumber mey be stored, or written, by sending a current pulso throush a magnotizing coil on the core. Reveraing the pelarity of this ourrent reverses the cors's magnetization.

The binery number in the core, represented by the core's Rux direotion, may be enneed, or read, by observing the voltage induced in a sensing ooil when the mapnetizing coil carries a current pulae of fixed arbltrary polarity and magnetizing amplitude. Balativaly large sferal voltages will be induced if the core flux difection is reversed by the read pulae, mall ones if it is not. Thise baele achene is approximately the one investigated by the Computation Laboratory of Harvard Univeraity, ${ }^{3}$

A core with a safficiently rectangular hysteresis loop mey be unad in the folloring schene for utillzing line sorltohing along epace ooordinates in the selsotion etep of the storage procesa. Pigure $1(a)$ is a sohenatio representation of unch a core. Vindinga $A$ and $B$ are magnotizing, or selecting, coilag $S$ is a sensing coil. Assung that a hyatereais loop of the core is as shom in Pl gare 1 (D) and that, at the start, the oparating point is at the lower stable posision $-\mathrm{B}_{\mathrm{R}^{\circ}}$

The application of a megrotiziag force of amplitude $H_{H} / 2$ moves the operating point to $x_{0}$ resulting in a very mail change in flux denaity $B_{i}$ return to $H=O_{0}$, wish oocurs at the ond of the $H_{N} / 2$ pulse moves the opersting point to $-\mathrm{B}_{\mathrm{j}}$, a point not far removed from $-B_{R^{\circ}}$

(a) A TWO-CURRENT-COINCIDENCE MEMORY UNIT


# APPROVED FOR PUBLIC RELEASE. CASE 06-1104. 

The romalt for the explication and removal of the fuld In ia quite alfferents the operating point meves to $\%$, and then to "Bro The core reverces Its magnotisition upon appiloation of $H_{X^{i}}$ in the procest there is a large change of ilux density $3_{0}$ with a correapondingly Iatge prile indnoed in the stenal cofl.

If the currente $\mathbf{i}_{A}$ and $1_{B}$ are made eçal and of muoh an auplitude that they correapond to values of magnotising force equal to $H_{3} / 2$ each. then the magetigation of the core can be changed from
 dovelopmeat of a relatively large signal pulse vould, therefores depend on such coincidence. Reverifigs the ourrent direction revergea the procedure.
(It is apparcit that forme of noise and instability reanit from the minor hyoteresis loope travelled vhen mapnetising forees less than $\mathrm{H}_{\mathrm{y}}$ are applied and removed. These and other factors maice deairable a very high depree of hystereale-loop reotangularity and a relatively high ocorcivity in the core material. Guaelestatio operation has been assumed; obviouely, the frequency with which operations say ocour will be linited by losees, addy-currant shialding. end other factors. Later chapters discuss some of these problems in detal2.)

# APPROVED FOR PUBLIC RELEASE. CASE 06-1104. <br> Report R-19a 

 lemived from the ldoaldged loop drem in Figure 2. If the valuer of I for which the loug marna a shatp cornar into a new raction are celled
 the folleving bold trum

$$
\begin{align*}
& m_{n}>n_{2}  \tag{1}\\
& \frac{n_{1}}{2}<n_{2} .
\end{align*}
$$



$$
\begin{equation*}
2 x_{2}>x_{n}>x_{21} \tag{3}
\end{equation*}
$$

$$
o: H_{2}<2 H_{1}
$$

##  <br> If now, for axmaple, aine of thace cares are arranged


 1s the only core in the arrar mioh hat the tail magotising force $K_{K}$ inproseed. Cores $D_{0} y_{1}, C$ and I have $H_{H} / 2$ 4npreseced; the reat have so tepresaed sagontistag foree. The only core, thorefore whome magnotimation can be elpaificantiy affcoted is the one at the junction of the selceted lines. She outpot elimal ar. ba taken, artor
 comen.

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.



AN IDEALIZATION OF A B-H LOOP

FIG. 2

?
A TWO-DIMENSIONAL ARRAY OF CORES

## APPROVE


#### Abstract

Tha exteaneion to threo diaensione may bs accouplished ky sfacking two-dimensional arreys. 22ke the one of Pigure 3, in Dack of ench other, with respective $x$ and $y$ linen connected in comnon. In this arrangement the selection of innes $x_{l}$ and $J_{m}$ enorgizes a vertical plane and a horizontal plane in the array at shown in


 rigure 4.This resulta in the selection of a line $x_{l} y_{m}$ along wich all cores have full magnetizing force lmpressed. The rest of the cores in the $x_{l}$ and $y_{m}$ plames have only hallmagnotising force Lapressed; cores out of those planes have none。

All that is necesesty now is to have a thiry set of magnetiging (selecting) Wudings on the corns, connected together In each $z$ plane, and so wound, for each core, as to rosult in a magnetinting force oqual to $-H_{M} / 2$ for an applied ourrent of $-I_{N} / 2$.
 In magnetizing forces of oniy $\mathrm{H}_{4} / 2$ left on each core in the 11 mo $x_{l} J_{m^{\prime}}$ 0 except for the core at the junction of this line and the $z_{n}$ plane, core $x_{l} y_{m} z_{n}$ owich is the selsoted core。

Yor the Whisivind I computer, the a planes might well represent the 16 digits of the number on the number tus, vhile two 32mpesition ardtches control the $x$ and $y$ coordinates for selecting the desired register (or word) from the 102l-register storsge array.


A THREE-DIMENSIONAL SELECTION SCHEME

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104?

Report R-192
D. Attack on the Prohlen

At the start, and during some of the eanlier etages of the work, it becese apparent that two courses of attack on the problem vere posaible. One involved doing a great denl of detailed, rigorove research of a theoretical nature, and amasaing experimental data on some very iimited portion of the problem. the other involved keeping anaysis and date-taking st a minimun, and coneisted of a quick dovalopment progran desifned to demonstrate the practicability of the conoept and to five a murvey of avallable materiale。

The deoision aade leazed, in general, in the latter direotion. It was felt that a eraater oontribution to the art was to be made by obtaining an early, thouph reuph, assessment of the potentialities of magnetic oores for use in this three-dimensional storage soheme. Accordinply, the development of core-teeting equiparent to fill these noeds was pashed hard, as was the obtaining and prooessing of sample cores. Some timo had to be spent on analyais of the cddy-current shielding problen in order to proceed intelilgently with the development work, and tilis was dome.

The development wort culafated in the teet operation of two difforent cores which bracket the reapense-time and informationretention parts of the problem very wall. The realts Rive a rathet draatic demonstration of the feasibisity of the three-dimensional japnotienstorage echemn at least insofas as the individual units ara goncerned, and are considered to justify the oouree taken.

## 

## OHAPTER 12

COEE RUSPONSI TMMS

## A. Yactore Governing Recponge Timen of Mapnotic Material

The magnetic state of a mample of ferromegnetic material 10 A function of time as meamured from tha atart of some excitation. Factors entering into the function includo losses, aging and 21 ke phenemena, and eddy-current shieldingo

Lesses in the material, as well as in the driving ciroultry, affect the shape of a plot of applied magnesizing force $\left(H_{e}\right)$ vereve time. A discussion of the response of a magnotic cose as indioated by megnetizing force ( $H$ ) or Mux density (B) at points In the material wey postulate a given frinction and coneecuentiyo may aldestop the question of losses. This is done here。 $H_{e}$ is applied es a "rectanpular" prise with a rise time less than $1 / 2$ microseoondf for most of the material s cested this may bs considered a step function.

Longs and short-term aging phenomena, "residual 10 sses". ${ }^{4}$ and dimeneional resonances ${ }^{5}$ are ignored. It is asmened that these ese not first order effects daring the time intervais used, and that the magnetis moment at a point in a material responds immediately to a change in $H$ at that point. ${ }^{6}$ Experimental results indicate that these are reamonable assumptions for a first approximation to the behavior of the cores tasted.

##  <br> Report R-192

## B. yddy-Ourrent Shielding in thin Tape

When eddy-ourrent ehielding in the only algrificent feotor In the reaponee time of a magnetic material, the fommalas governing H and $B$ at points in the material, for a step-function of externally applied $H_{0}$ mey be derived from Kaxoll's Kquationg. (Ses Appendix.) If displacement currents may be disregarded and "semi-infinite-plate" cocentry is assumed for the material (both of which may be done for a. thin metallic tape or ribbon) one remultant set of formulas ias

$$
\begin{gather*}
\frac{\partial^{2} H}{\partial x^{2}}-\sigma \frac{\partial B}{\partial t}=0, \text { where } \sigma \text { is the conductivity }  \tag{4}\\
\text { of the material; }
\end{gather*}
$$

$$
\begin{equation*}
B=S(H) \text {. } \tag{5}
\end{equation*}
$$

When B is a multh-valued and non-linear function of H , the problem of solving the equations is begond present-day techniques. If the multivaluednese of the functions may be dieregerded (as in the oace where half of one partscular loop in the B-H plane is beinc travelled) the solution is more easily accomplished. The eipple-valued linear case is discuseed first, followed by the single-valued non-linear eace.

Theoretioal discuseion is oonfined to the thin-tape chape because all of the presently useable metallic corse are wound of thin tape, the ferritic cores which have aquare or reotangular arose sections have extrenely hich resistivitios and rather low pormenbilipies so that oddy currents are alwost nepligible in thean

## APPROVED

and finally, the qualitative concegts, which derive easily for thin tapes hold in a general way for all shapee.

## 1. The Linear Oase

When $\mathrm{f}(\mathrm{H})_{0}$ above, is not only aingle-valued but also Insear, ve may define a constant, called the permeablittyo $n$ N $\mathrm{B} / \mathrm{H}$, and the colution for H becemes (see Appendix for derivation):

$$
\begin{gather*}
\frac{A}{H_{0}}=1-\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{2}{n} \sin \frac{n \pi}{a} \cdot n^{2} \frac{1}{7} \text {. for odd } n_{0}  \tag{6}\\
\tau \equiv \frac{n^{2} \cdot n}{\pi^{2}} .
\end{gather*}
$$

Wheres a is the thickness of the ribben, or tapes $x$ is meacured from the surface of the tape into the material; $\mathrm{F}_{\mathrm{e}}$ is the magitude of the appiled atep-ifmetion of Ef the ingerial yas uhmarnotised at the otarto

Figare 5 chove $\mathrm{H} / \mathrm{H}_{\text {, }}$ plotted vextes $\mathrm{t} / \tau$ for four values of $\mathrm{x}_{0}$ It Is Interesting to note that the curves leave the origin with soro alope, with the trivial exoeption of the curve for $x=0$.

In Figure 6 , $1 / H_{\text {e }}$ is plotted vergua $x$ for varlous values of $t / T$. Since ve agmine that the Rux density, $B_{0}$ Is I Inearly proportional to $H_{0}$ these curves show the growth of $B / B_{\text {a }}$ a funotion of time throuqhout the material. The fiux $\beta_{0}$ is propertional to

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104

APPROVED. FOR PUBLIC RELEASE $:$ CASE 06-1104.









CALCULATED $\left(\frac{H}{H_{e}}\right.$ vs $\left.x\right)$ vs $\frac{t}{\tau}$ I WNP 120,128

## APPRQVED

$\int B d x, 0 r$ the aree under each curve．A plot of those areac veraus e／＇$\tau$ is shom in Figure 7a；this plot masy also be arrived at by proparly velghting and arming the curven in Figure 5．Note that near the oricin the offeots of the ontaide portions of the material（xaell $x$ valuos） predominate，and the curve reaches its mextaun elope for extreasly amall values of $1 / 7$ en that．for all practical purposes．it may bo assumed to leave the origin with meximum（bat finite）alope．The alope is plotied in Yigure 7bi it is iinearly proportional to $\mathrm{dp} / \mathrm{dt}$ 。 so that dt chowe the shape of the voltage pulse， 0 ．which appears across a ansing coil wound on the core．The pulse has its maxiau amplitude ato or Fery near．the origin and falle off from there on outo the pulse amplitadc is needigible by the tise $\%=7 \%_{0}$

## 2．Tho Non－Linear Oage

When the fanction $f(f)$ 。 9 。（ 2 ），is singie－ralued but not ifnear，equatione（4）and（5）combine to givet

$$
\begin{equation*}
\frac{\partial^{2} H}{\partial x^{2}}-\sigma \frac{\partial g(H)}{\partial t}=0 \text {. } \tag{B}
\end{equation*}
$$

The function $f(H)$ is commonly described by draving the pertinent portion of the pertinent hystereais loop for the material． Such a ourve is mhown in Figury 8 for one of the more atisfactory experimental cores．It is obriously hifhly nop－linear and not easily desoribed by any eimple analytical expression．This is unfortunates， for equation（8）becomee separable when $f(K)$ is a reasonably sisple pover of H ，wach as $\mathrm{H}^{1 / 3}$ ．

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

$$
\begin{aligned}
& \text { (b) CALCULATED } \\
& \mathrm{K}_{\mathrm{d} \mathrm{\theta}}^{\mathrm{dt}} \mathrm{vs} \frac{1}{\mathrm{t}}
\end{aligned}
$$



## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

 .... 12,000
$B=f(H)$ for
RE \#MTS 4382


$-2000$
-4000
-6000


## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

Report R-192

The problea, however, does 1 and itself to a numerical anolyals. One vale started using the disference equation.

$$
\begin{equation*}
I(H)_{x_{0}} t+\Delta \frac{t}{0}=f(H)_{x_{0} t}+\frac{\Delta t}{\sigma(\Delta x)^{2}}\left(H_{x \vee \Delta x_{0} t}-2 H_{x_{0} t}+H_{x-\Delta x_{0} t}\right)_{0} \tag{9}
\end{equation*}
$$

which follows from equation (4) in a etraightforvard menner. The fanction $f(H)$ vas approximated by three stralght ilnes fitted to a curve like that of Yigure $\mathrm{g}_{\mathrm{a}}$ and celculations made on a deak calculator. The work proved to be very tedious and time-consumingo and was abandoned before any conclus.ve reants wore reached. The problem wll be prepared instead for solution by the Whirivind I computer. and the resulte will be avaliable shortly after this thesis is written. The need for a oulak and rouph qualitative solution ramained, and the folloving approximations augested themeelves.

Asgue that the material starte from a steady-state oondition at the point $-\mathrm{B}_{\mathrm{R}}$ ond, for an appliad magnetising step function of amplitude $\mathrm{H}_{0}$. follows along an idealized path in the B-H plane as dran in Figure 9a。 411 of the material reaches the point $H_{2}$ immediately and then enters the high-permeability repion. Since ve are only interested in flux changes. or the transient part of the solution the problem mey be simplified eliftily by changing the initial conditions to $K=B=0$, and considering that the path travelled is the one dram in Figure 9bo
 approximately, to the portion of each curre that is to the rient of the

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.



## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

Report R-192
intersection of the curve and the saturation- $2 \operatorname{ing} \mathrm{H}_{0}=\frac{\mathrm{H}_{2}-\mathrm{H}_{2}}{\mathrm{H}_{2}}$. All of the oross section above and to the lert of that interesction is ansumed differenth/ to be completely saturated. Since the dimerentei panaeablility is very mach higher (by a fector $10^{5}$ in many practical cases) for the rogion below the saturation line than for the region above, only the squares belew that ilne are counted as contributing toward the growth of the
 sotal area under the saturation-1ine。

Also, the lack of any changing flux to the loft of the curve and saturation-ine intersection means that the effective thickness. $a_{0}$ of the matertal has been rednced accordingly, to $a_{n}$. sinoe the aquara of the thickness enters into the quantity $t / \tau$ in the innear analysis. one approximation includes reducing the successive increments of t/", wy the factor $\left(\frac{a_{2}}{2}\right)^{2}$. The remitant sets of curves of $\frac{\phi}{\beta_{0}}$ and $x \frac{d \phi}{d t}$ are drave in Figures 10 and 11.

Hurther speculation on this spproximation bringe up the point that the offective thickness of the neaterial is reduced by saturation auly insofar at contribution to the changing flux is concerned, but not ineofar as the orosesectional area avallable to the flow of eddy ourrents is concerned. This would sesm to call for reducing muosseive $\frac{8}{F}$ inorements bo the factor $\left(\frac{h_{h}}{\frac{a}{a}}\right)$ rather than by the square of that ratio. The quantities $\frac{f}{f}$ and $x \frac{d}{d t}$ have been plotted for this second approximation 13 Figures 12 and 13. Ay will be seen later in this chapter (see regures 17. 15, 201. 208 \& the shapes of the $\frac{d y}{d \delta}$ ourves, for oither


## APPROVED FOR PUBLIC RELEASE. CASE 06-1104



A-48007 $F_{3}$ APPROVEDFOR PUBLICRELEASE. CASE 06-1104.


## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.



## APPROVED FOR PUBLIC RELEASE. CASE 06-1104. 33

Report R-192
approximation, bear a fair reseablance to the output pul ses obtained exporimentally.

Parther theoretical judgement regarding the value and validity of the approxdaations requires deeper analyod a particularly the reanlts of the numerical solutiong at the compater will perform them.

## C. Experimental Hecults and Comparisonis

Kxparimental doternination of the atatic magnotic arate of a core is not couvenientily obtained. A dynamic ploture of the changes oocurring in a core is easily recorded, however, as a fandly of curves ahowing the rate of change of flux, $\frac{\text { d }}{d \delta}$, vergus time, to for applied step functions of H at varging amplitudes, $\mathrm{H}_{9}$ o These ourves are, in fiact, the roltage pulses induced in a vinding on the core, and man be vieved on a cathodo ray oscillosoope or aynohsoscope. Yrom suoh femilies, more particular pieces of data say be axtracted for ploting other curves; e.f., reaponse times (puleo lengthe) verana $\mathrm{H}_{0}$ mplistudes.
$H^{0}$, incidentally, is taken to be the applied magnetomotive force divided by the mean circumference of the toroidal core. The ratio of outer to inner dimeters wae close enough to 1 to varrant the aasumption that ateadr-state $H_{0}$ vas constant throughout each metallic core.

For cores with an extremely wide range of response times, and chare quantitapive comparisons can be made betweon difforent thicknesses of the aame core material, the data is often presented in the form of

## APPROVED.FOR PUBLIC RELEASE. CASE 06-1104. ${ }^{3}$


#### Abstract

  corad and the fexritic cores, fevailse of pus ge shapes are shom aul other data extracted vhere necossary.


Testing wat done almost exalusively on cores which have rectangul ar hysterasis 2oops.

## 1. Slow Matallic Cores

The tape-vcrund Daltamax cores, as supplied by Allegheny Ludiun Stael Corp, or its mbeldiaryo Arnold Ingineering Corpoo are very alow cores. largely because of their phonomenally high maxdmum efferetiel pernaabilityo $\mu \mathrm{di}$. From Figare $2.4, ~ \mu \mathrm{di}$ 。 which is givon by the alope of the asturacion hysteresis loop as it crosses the $H$ axds. may be obtatned as approsimatoly $1.3 \times 10^{6}$ gnassa/carsiod. Using eq. (7), for 2mal thick taps, vith a resiativity of about $35 \times 10^{-8}$ ohm-meters, the oaloulated value for one "time gonstant", $T_{0}$ is 1.2 miliseconds. Jrom Figure 70 , the time to the end of the $\frac{d f}{d f}$ pulse for the innear case should be about $7 T \approx 8.4$ millisooends.

Tor the sctral saturable material (see Yigure 14) the above racooning indicates that as the amplitude of the appised $H_{e}$ approaches $H_{e} \approx 0.075$ oersted, the time, $t$, to the ond of the outpat pul se should approach 8 m illisaconds. The curve drawn for the experimont ally taken daps (see curve marked 2-nil Daltamex on Fipure 15) extrapolaten to beyond 200 milliseconge for $H_{e}=0.075$ oersted. The correlation hers is bad cue In part to unavoidable inscouraoies in the data, and the fact that



## APPR OVPE

interlaminar conduction may not be negligible on these eerly samales. It is also likely that assumptions mentioned in the third paragraph of this chapter are poorly fulfilled in many cases.

The fourth curve in Pigure 15 is for a 2mill Deltamax core which was processed so as to raise its coercive force, $H_{c}$, to about 0.23 oersted. (See Figure 16). It also has a lower maximum differantial permeabilityi $u_{\text {dii }} \approx 0.7 \times 10^{6}$ gamsses/oersted, about half that for regular Deitamax. As $\mathrm{H}_{e} \rightarrow \mathrm{H}_{2} \approx 0.3$ oersted, the response time of this core should approach about 4 milliseconds; it appeers to approach about 100 milliseconds, giving as bad a correlation with linear theory as the other Deltamax cores above.

Corralation with the non-linear epproximation of Figures 11 and 13. for $H_{f} / H_{e}=H_{2} / H_{e}=0.1$ is fair. The response times of the three cores, expressed as $t / \tau$, where $\tau$ is calculated from the physical characteristics of each mesterial using eq. ( $\gamma$ ), average to 0.18 for the above $\mathrm{H}_{2} / \mathrm{H}_{\mathrm{e}}$ ratio. The same pulses from Figures 11 and 13 end at $t / \tau$ values of 0.11 and 0.20 respectively. The correlation gets poorer as $H_{2} / H_{e}$ gets amaller.

Relative comparisons made between the experimentally taken curves of different cores yield good results. According to the linear theory, $\tau$ goes as the square of the tape thickness. For a given $H{ }^{\circ}$ therefore, the three regular Deltamax curves of Figure 15 ehould be spaced from each other, along the $t$ axis, by a factor of $2^{2}$, or 4 .

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104



This doee not hold well for the curves through most of the range shown but does begin to hold better as the carves come in toverd $\mathrm{H}_{2}$, whore the B-H paths approsch aloser to 1 inearity. For exmaples at $H_{0}=0.2$ oerated, $t$ is abont 10 milliseconds for the 2 mill curve and about 3.5 mi211seconds for the l-mill curve. This factor of nbout 3 decreases to 1 ess than 2 for highor $\mathrm{H}_{\text {- }}$ values, and apponrs to inarease (possibly to 4) for lover $H_{0}$ values.

The curve for sore MIS 4450 may be compared with the 2 minil ousve for Deltamax by comparing reaponse times, $t$, for amall equai Inerements of H above respective $\mathrm{H}_{2}$ valuee. The 2 -nill Dal tamax 0 , for $H_{0}=H_{2}+0.125=0.2$ oersted hat at of 10 milliseconds; core MTS $4450_{0}$ for $\mathrm{H}_{-}=\mathrm{K}_{2}+0.125=0.425$ oersted hae at of about 5 ailliseconde. Shis factor of 218 very close to the ratis of the fis valune for the two aateriale. Gnce again, comparison involving iinear theory is good for low $\mathrm{H}_{6}$ velues, and poor for h1gh ones where departura from innearity is extreme。

Figures 17 and 18 are photographs of soope traces showing the crovth of output pulses as a function of $\mathrm{H}_{\mathrm{c}}$ amplitudes for a $\mathrm{m}_{\mathrm{m}} \mathrm{mil}$ Delteanax core and core MTS 4450. There is some qualitative resemblance between the ehapee of the predioted pul ees of Yigures 11 and 13 and these photographs.


เшит
$180 \mu \mathrm{sec}$

TWO FAMILIES OF OUTPUT PULSES (FOR VARIOUS AMPLITUDES OF APPLIED $H_{e}$ ) FOR A 2-MIL DELTAMAX CORE

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.



TWO FAMILIES OF OUTPUT PULSES
(FOR VARIOUS AMPLITUDES OF APPLIED
$\left.\mathrm{H}_{\mathrm{e}}\right)$ FOR CORE MTS 4450

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

Report R-192

## 2. Yeat Kotallic Cores

It is fmportiant to recall, at this point, that when a core is need in the coincident-current scheme it is mwitched by maknotizing
 data in this seetion extend appreciably beyond $2_{2}$, but emphasis is often laid on the rogion up to that point.

The cores discussed in the previous section are not suitable for use as coinoldent-ourrent memory units because their response times. for $H_{0}$ less than $\mathrm{ZH}_{2}$, are of the order of milliseconds. If eddy-ourrent shieldingo as discussed, is the important time-consuming affect, then the equivalent time constant. $\tau_{\text {o }}$ of the aaterial is a useful oriterion and may be reduced, in a metallic core by reducing tape thichness, by inoraasing material resistivity, or by reducing permeability. Unfortunately, it is not practioally feasible at present to reduce thicknesses to mah below l-mil or to inarease reaistivities very much above 50 or 100 miorohmen in the rectangulan-loop metallic materiale. Cores are available, however, with rabstantially lover differential

highly-rectanpular hysteresis loop, for core KTS 4382, whose maximun differential permeability is about $0.125 \times 10^{6}$ gaseses/oersted. shis value is lower than that for Delteseax by a factor of about 0.1.

When the coineident-current initation, $H_{0_{\text {max }}}<2 \mathrm{H}_{2}$, 10 oonaidered it becomes apparent that oore MTS 4382 hould be faeter

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.



FIG. 19

## APPRQVED $\mathrm{D}_{1} \sqrt{2}$ OR PUBLIC RELEASE. CASE 06-1104.

than a l-mil Deltamax core by get another factor uhen it is used in the proposed scheme. Jrow the hysteresis 2oop of Figure 19:

$$
\begin{aligned}
& \mathrm{H}_{\max } \leqq \mathrm{H}_{2} \approx 2.3 \text { oersteds, } \\
& \mathrm{H}_{8} \approx H_{2} \approx 1.6 \text { oersteds, and } \\
& \mathrm{H}_{0} / \mathrm{H}_{\mathrm{m}_{\text {max }}} \approx 1.6 / 2.3 \approx 0.50
\end{aligned}
$$

Wherees, for Deltemaxs

$$
{ }^{\mathrm{H}_{0} / \mathrm{E}_{\text {max }}} \approx 0.075 / 0.08 \approx 0.94^{\circ}
$$

A glanoe at the thsoretical approximations to the grouth of flux with time for various valuee of $\mathrm{H}_{\mathrm{o}} / \mathrm{H}_{0}$ (Figures 20-13) indiaates that steady atate is remohed much sooner at lover ruluea of $\mathrm{H}_{\mathrm{o}} / \mathrm{H}_{0}$. A vory rough extrapoiation on rigure 10 indicates that steady ataito is resched at $t / \tau \approx 0.6$ for $H_{s} / H_{0}=0.5$, whereas for $H_{f} / H_{0}=0.94$ eseady state is probably not reached until $s / \tau>5$.

Wo would expect, therefore, that core NTS 4382 would be about ten times as fast as lemil Deltamax under linear conditionso and perhape one hundred times as fast when ueed under non-linear conditions for $H_{\text {e }}$ values in the vicinity of $\boldsymbol{z}_{1}$ per rempective core. From the expncimental l-mil curve of Yigure 15 , the reaponce time for $H_{0}=0.0 \beta$ oersted $\mathrm{is}_{5}$ of the order of tene of milliseoonde. From Pigare 20 , for $H_{0}=\hat{2}_{0} 3$ onrsteds. the experimental reaponse time for core MTS 4382 is about 0.7 milliseoonds. The factor is of the oxder of a hundred, as expected.

APPROWED FOR PUBLIG RELEASE. CASE 06-1104.


## APPRQVED FOR PUBLIC RELEASE. CASE 06-1104.

Yigure 21 ahow photographs of a 1 andily of output pulses for this core. There is grod resemblanoe in ganeral shape to the curves of Figures 11 and 13.

All of the previous calculations and comparisons have been based on the asmumption that a material travelled along its so-called saturation hystereais 100 p and that, therefore, the function $f(H)$, was ainele-valued, and values of $\mu_{d s}$ could be taken from tho we hoops as thown in Figure 14, 16, and 19. Beomace of the need to maintain ainglevaluedness shen attempting to obtain responso-time correlations between theory and experiment, mont of the data was taken for a material travelling along its saturation loop. This vae done by applying anymetrical magnetiging forces an mhow on the amall aketches an Figares 15 and 20 , where $H_{\text {g }}$ was kept constant at a value sufficient se saturate the cors.

Aatually. cores will not be worked on saturation loops in the proposed soheme, and it is important to see how reaponse times change when syanotrical expitation is applied and varied. Data taken under these conditions for core NTS 4382 ars plotted as the daehed line in Figure 20 , and are photogrephically illustrated in Figurs 22. The racaction in response times is very large. The same type of data use taken for Deltmax coren, and is plotted separately in Figure 23. Reaponse times are lowered again but not as drasticallyo

As is indicated by additional evidence in the next ohapter. the reduction is due largely to the reduotion in Hif ae inner 2oops are travelled in the B-lf plane. There is al so some reduction is

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.


(a)

(b) $20 \mu \mathrm{sec}$

TWO FAMILIES OF OUTPUT PULSES (FOR FIVE He AMPLITUDES) FOR CORE MTS 4382 UNDER ASYMMETRICAL EXCITATION. NOTE RESEMBLANCE BETWEEN (b) AND THEORETICAL APPROXIMATION TO dø/dt IN FIGURE II.


س $20 \mu \mathrm{sec}$

> A FAMILY OF OUTPUT PULSES (FOR FOUR He AMPLITUDES) FOR CORE MTS 4382 UNDER SYMMETRICAL EXCITATION. NOTE LARGE REDUCTION IN RESPONSE TIMES COMPARED WITH ABOVE.
$\|_{\|}^{\|} \quad \mathrm{t}$ vs $\mathrm{H}_{\mathrm{e}}$

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

retained firy density and in concaiverforee values, which would also act to reduce response time for fixed hevalues。

Ralative correlations of the above type exiat between the already-mentioned metallic cores and others tested. Two which fall mader this section's classification are MSS 6464 and MTS 4370 , both propared by Allegheny Ladium Steel Corp.

## 3. Ferritic Cores

Cores made of the magnetic forrites promise a majer reduction 1a reaponse times, mainly dae to their high resistivities, and secondayily dae so their low permabilities, low saturation fixx densities, and high ooeroivities。

The following values indioate hev difforent the oharacteristica of ferrisio oores nay be from metalilo ones. They are fer Perramio $3^{4}$ had a product of the Ganoral Carrmics and Steatite Corpo the comparisona is made against regulax Deltanax and is quite axtreme.
Material Deltanas Freronio 34ed

| Max. Aifferential permeability. | $2.3 \times 10^{6}$ | about $10^{3}$ |
| :---: | :---: | :---: |
| $\mu_{\text {di }}$ \% gansees/oarsted |  |  |
| Conerive force, $\mathrm{K}_{\mathrm{o}}$ ( oucsteds | 0.06 | 3.5 |
| Saturation Rlux denaity, $B_{g}$ foulsans | $1.6 \times 10^{4}$ | $2.3 \times 10^{3}$ |
| Volume reniptivity at room temp. $\rho_{0}$ ohen | under $10^{-4}$ | $10^{9}$ |

## APPROVERED FORPOR P-192

Figurs 24a, belov, is a photogreph of scopn traoes mhowing the outpat polses from a mall Ferranic $3^{4}-A$ F209 tosoidna core wh a neas-square arosensection of about 0 .2-inch on edge. The appised $H_{e}$ was agmanetrical, so that the pulase ahom resulted from roturning along a path very close to the saturation loop.

Tigare 24b ehow the reduction in response times reanlting from aymetrical excitation, as discuseed for metallic cores.

Figure 24 c , when compared with 71gure $24 a_{0}$ shows the reduction in responas times reaulting from reduoing the radial bhickness of a Forranio 3 hed Fiog toroid to about $\frac{1}{2}$ ita original Talu*。

In the thrse figures a vary reduoed renge of responso-tine Talues is evidant. This rechution is largsly dus to the now finite riss time of the $\mathrm{H}_{\mathrm{p}}$ function; the no longer negigible riae time offactivaly pute a hower linit on the response timee of the ferritic cores for $\mathrm{H}_{9} \gg \mathrm{H}_{2}$. The fact that the foroide have ralatively large radial thicknesses also contributes to this offect, and reduces the aocuracy of any remponse-time correlations.

The reaietivity of the Yorrinic core is about $10^{23}$ times that of Deltamax. It is mufficiently hieh so that maorosoopio eddy currents have been reduced to a neplighble value and response times are of the same order of magritude as the rise and fall times of the H, pelses and of tho assosiated circultry. It is also possible that dimplaceasht durrante and readdual lossen ars signifioant
limiting factor: in this area,

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.


(a) FAMILY OF OUTPUT PULSES (FOR THREE He AMPLITUDES) FOR FERRAMIC 34-A FIO9 UNDER ASYMMETRICAL EXCITATION.

(b) FAMILY OF OUTPUT PULSES (FOR THREE He AMPLITUDES) FOR FERRAMIC 34-A FIO9 UNDER SYMMETRICAL EXCITATION.

(c) FAMILY OF OUTPUT PULSES (FOR FOUR $H_{e} A M P L I T U D E S$ ) FOR FERRAMIC 34-A FIO9; WITH RADIAL THICKNESS HALVED, UNDER ASYMMETRICAL EXCITATION.

# APPROVED FOR PUBLIC RELEASE. CASE 06-1104. <br> Report R-192 

GHAPTER 121


The operation of a magnetic core ae doweribed in Socition I of Chapter I raisea special problens vith respect to stabslity of operation and what would commonly be called elgnal-tomoise ratios. In the first eection of this chapter, these problems are discuesed, and important ratios and oriteris are set up and defined. There followe a brief look at the teat setup used for obtaining the experimental remite. These remults, presented in the form of photopraphs and numerical values, are then discussed and rolated to hysteresis-loop chapes.

## A. Definitions and Reguirements

Nuch of the disouteion below revolves around the voltage outpput pulses from a core for difforent situations and different coresinformation ntatee. The meagure of these output pulees may be taken as their areas, peak amplitudes, or amplitudes at chosen points in time. Unless othervise atated the pulse areas are taken to be the quantities under discussion. Polarities are often diarogarded; it is aseumed that, in the final aystem, the sensing device may be equally senaitive to signal of either polarity.

Aeforence motuld be made to Figures? ? (Chepter 1) throughout thle eectiop.

# APPROVED FOR PUBLIC RELEASE. CASE 06-1104. <br> Report R=192 

## 2. Inforyation Rotentson

This phrese is used to reser to the absitity of a core to maintain a R1ux direction and magnitude, repreasmbing a zino or a OMI, in the reoc of exostation by pon-gelooting ( $H_{N} / 2$ ) purson. Fleure 1b shows hov, after putting the core into state $-B_{R}$ with a sull $H_{M}$ pulse, a pulee of $\mathrm{H}_{\mathrm{N}} / 2$ moves the core to a new state, $-\mathrm{B}_{\mathrm{N}}$. Additional applications of $\mathrm{F}_{\mathrm{n}} / 2$ pulaes tend to run the stase of the oore further up along the $B$ ands, disturbing the information in the oore, or perhaps, destroying that information. An intuitive inference to be dram from Pigureibis that disturbance of the state of a core is reduced when the portion of the B-H loop between $H=-H_{H} / 2$ and $\mathrm{H}=+_{\mathrm{H}}^{2} / 2$ approaches the horizontel. The degree to which that portion of the loop is horizontal is one of the meesures of what is called 200 p rectangularity.

The atate of a core is manifosted by the 81 ze and chape of the voltage pulse out of a sonsing coil in reaponse to the read pules of H . Change in amplitude or ares of this output pulse as a function of the number of intervening nonsselecting ( $H_{4} / 2$ ) poises 1s. thea, an indsoation of the sensitivity of a core to disturbance.

If a OME as written by a $-\mathrm{H}_{\mathrm{M}}$ mite-oris puise, is at
 then raading may be accomplimhed by applying a *H/ read palse, and the remult is a lexge putput voltage puies for a ofre and a mall output puiep for a arirp.


#### Abstract

Hon-selecting read or write-2KRO pulsas will tend to sun a ONE poation up along the 3 axds than reducing the response to a road pule later; they will not, however, elgnipicantly affect the 2 anco position. On the other hand, non-solecting write-ONE pul ses will tend to sun a $2 \times R D$ position down along the $B$ axis thus increasing the response to a read pulse later; they will not, however, elgnisicently affect the orrs position. a. Orx-Ratention


An "undisturbed ONE" is defined as the information etate of a core after a ozy has been written and before any other pulses of H have been applied. A "disturbed $\mathrm{ONF}^{\prime \prime}$ is the state of a core when a very large number of nop-aelecting ( $H_{4} / 2$ ) road or writewis pulses have followed the write-oas pulse.

The "Casi-retention ratio" is defined as the ratio of output elgnale for the two cases:
$\frac{\text { D1 oturbed ONS }}{\text { Undisturbed ONE }} \equiv$ onk retention ratio, $R_{2}$.
I approsches 1 in value for the ideal oase and is
less than 1 in practice.
b. ZERO-Rotention

An "undisturbed $x$ ERO" is derined as the information
atate of a core after a $2 \mathbb{2} R \mathrm{O}$ has been wattien and before any other pulees of 1 have been applied. A "disturbed 2 zR0" is the state of

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

a core uhen a very large number of non-seleating ( $\mathrm{NH}_{4} / 2$ ) w 2 ite-0r2 pul ses have followed the write-bxito pulse. The wirc-retention ratio Ss defined as the ratio of output atgnals for the two cases:
$\frac{\text { Df aturbed 2ERO }}{\text { Undisturbed Z2RO }} \equiv$ 2ERO-retention ratio. $R_{0}$
It approaches in in value for the ideal cass and is greater than 1 in practice.

## 2. S1mal Bation

A number of aignal ratios may be made from the abovementioned four basic quantities. the most meaningful one is the "disturbed sifnal ratio", which is dafined as the ratio of outpot algnals for a disturbed OKE and a disturbed 2 ZRO , as follownt
> $\frac{\text { DI aturbed ONK }}{\text { D1 sturbed } \angle \mathrm{ERO}} \equiv$ Dioturbedonignal ratio。 $S_{D}$

Sinco disturbance generally rednces the output signal from a core with a ORE, and raisen the output signal from a core with a $4 B R O$, the $S_{D}$ ratio is a really critical eriterion of a core's performance. $S_{D}$ approachea infinity in the ideal oase, and epproaches or falle below 1 for the unestiafactory case. In practice it chould be much greater than 1 if reasonable discrimination between the binery digite is to be obtained.

## APPRQVED．


#### Abstract

Another type of voltage output fros a core occurg when 2t receives a non－selecting $\left(H_{M} / 2\right)$ pul ee。 This gignal 1t，1．2ks noise，undesirable；the word noise is reserved，however，for efgnal．e which are more comeniy described by that name，such as those resulting from inductive or capacitive pick up，atc．The firot－ mentioned signals will be called non－selecting（abbroviated Ms） signal 8 。


The largest N－S alfnal from a core should occur in response to the first nomesecting $\left(H_{K} / 2\right)$ pul se of a polarity which tends to reduce the magnitude of the retained flux．If the core holds a ONS this would be a non－selecting read or write－A．HO pules $\left(+H_{H} / 2\right)$ 。

The ratio of a disturbecoar eignal to an H－S sipnal is defined as followst
$\frac{\text { Digturbed ONS }}{\mathrm{H} 8 \text { sienal }}=\mathrm{N}-\delta$ signal ratio， $\mathrm{S}_{\mathrm{ns}}$ ．

This ratio，1ike $S_{D}$ ，approsches infinity ideally，should be much ereater than 1 for satisfectory operstiong and is an ifportant criterion of coincident－current core performance．

The garden variety of noise is not considered here；it will be one of the mejor probleas involvad in the development of assemblies of these cores into largessale memories．

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.T

 Report R-192
## 8. Toat Sotup

A singe e-pulse. or puch-benton. type of arrangamont wes isrgt used in the experimental work. It was abondoned in favor of sn automatic test setup wich can put a repetitive pattern of $H$ pulses on a core to nimulate, approximately, some of the conditione under which the core might operate in practice. This setup makes it very eacy to adjust the significant $H$ amplitudes during operation and to see and record the output pulses from a teat core for different operating modes.

Previous conalderations lead directly to the pulse pattarns deaired for the testing. These are shown in Figure 25. The sequence of events for each mode of operation $a_{0} b_{0} c_{0}$ or $d_{0}$ may be followad with the ald of Figure ib(Chapter I).

The lock diagrati of Yigare 26 and the circuit schematic for the Core-2enting Pul se Amplifier (rigure 27) show the equipment arrangement and eircuitry used to prodnce the above pulse patterne of magnotizing force.

The setup is very 2 exexble and is made up largely of block of project tent equipment. ${ }^{7}$ Repetition rate may be varied by varying the free-running frequency of the hiph-frequency multivibrator in the MV Frequency Divider. All pul ses are equal in duration; the duration may be varied by adjusting the delay in time between the pulse into end the pulse out of 0 \& D (Gate and Dalay) Unit 2. Pulse quplitudes gre adjuated by front papel controls on the Core-Testing Pulse Amplifier. The modee of oparation and the munber of inserted nonselecting pulses are deternined by the condition of the two ewitches.


CORE-TESTING PULSE PATTERNS OF H



APPROVED FOR PUBLIC RELEASE. CASE 06-1104.


CORE-TESTING PULSE
AMPLIFIER

FIG.

## APPRQVED.

SW 1 and SW 2, and the dividing ratio of HI - to 20 m 2requency pron aes out of the HiV Yrequency Divider, at desailed below.

With the sync Inplet of the aynchroscope connected to recsive the gate output of 0 a $D$ Unit $i_{0}$ as shown, it aweepe once for overy pal ee, repardless of operatine mode. All outpat pal ses start, therefores at the sane point on the scope soreen (a point detesnined by the delay time of the pul oe out of 0 D Unit 1), and the riuned traces are superimposed. Scope loads may be connected to the sensing coll on a core for Viewing output pulaes, or aerose the $50 \Omega, 20 w_{0}$ series resiator which is common to the two magnetizing windings, for viewing the current pulses.

To produce operating mode a (rigure 25), the MV Yrequency Divider (Figare 26) 2e adjueied to divide by $2, s 0$ that the repetstion rate for "10" frequency pulses is hali that for "HI": mode bie produced by dividing by any integer larger than 2. Opening evitech SW 2 and dividing by 2 reanta in mode o. Mode dnay be obtained by opeaing SW 1 and dividing by an integer greater than 2; the pattern wil, however, by upaide down and must be interpreted that way uniess the trouble If taken to reverse either the magnetiging ooil Ieads or the scope 1 eade as vas done in obtainine some of the disturbed-sero photographe of the next section.

## 0. Exprimental Rornits

Yew coree were found which could operate eucceasinily in the coincidept-cuxrent soheme. Of the metallic ones. ooro MSS 4382 (first mentioned in the previous chapter) vae the beat on all counta.

## APPRO耳ED F FigR PUBLIC RELEASE. CASE 06-1104. ${ }^{62}$

## 2. Coro M'S 4382 .

The teat resulta for this core are oummarized in the photographs of Pigure 28 and the table of Figure 29.

Yigure 28 is worth a considerable anount of study. The three colums of photographs are for three arbitrary values of $H_{H H^{\circ}}$ the center column was taken with the magnitude of $H_{n}$ close to the optimum valueg the two side columne show the changes ocourring for amaller and larger values. In the top row are shown the voltage drops across the series resistor comon to both magnetizing coils: the scale is calibrated in oorsteds since the voltage drop, multiplied by a known constant. Rives the napnetiging force, $\mathrm{H}_{3}$ these pletures, al thouph taken for mode $b_{0}$ indicate the magnitudes of $H_{n}$ and $H_{N} / 2$ for the entire oolumin.

The next four row show the voltape pulses appenring on the core's sensing, oo1l as the core rencts to the pulses of H .

Some jitter in the KV Yrequency Divider between division by 2 and division by 3 made it possible to see the undisturbednows output trace and the once-disturbech-ORB output trace simplaneousily. This is labeled mode ab and is shown in the second row of the figure. The three positive traces (starting with the largest ono) represent outputs rrom an undisturbed ONX, a once-disturbed ONE, and one non-selecting read pulee, respectively. The negative izace results from the writeONE act.

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.



## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

The third row thowa bede bo the heavy center trace resuite from a large number of consecutivo non-selecting road pulsos; tha finar trace juat above it is from the very first of those. The positive outaide trace ropresents the disturbedonis output, and the nogative trace, a beforo, is from the write-0aris act.

Row 4 , mode $c_{0}$ show only one trace for the undisturbed LERO.

Mode $\mathrm{d}_{0}$ row 50 howe three traces. The negative trace represents the disturbed CERO; the heavy positive trace results from the nonsalecting write-0nif pulses; the faint positive trace results Pron the first one of the nonselecting write-ONX pulses.

The firat table in Figure 29 hove approximate infomation retention and sipnal ratios as obtained from the photographe. The results for columen 1 and 2 ( $H_{p} \leq$ optimum) are most accoptable. Increasing $H_{i n}$ to the optimus value reante mainly in a reduced response time and hieher signal output levale. The inarease to column 3 ( $\mathrm{H}_{\mathrm{M}}>$ optimum) reduces the response time and raisee output 1 evele again. but it almo results in drastically reduced aignal ratioso particuieriy $S_{D}$.

Reference to the hyateresis loop (Figure 19) for this core ahows that the $H_{H} / 2$ value ( 1.5 oersteds) of column 3 exceeds the velue ( 1.2 oersteds) of Figare 29, and is about equal to the $H_{c}$ of the material. In column 2 , howover, $H_{H} / 2 \approx 2.2 \leqq H_{1}$, giving eatiafactory operation.

APPROVED FOR PUBLICRELEASE. CASE 06-1104.

| RATIO | OUTPUT SIGNAL | METALLIC CORE MTS 4382 |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $H_{M}<$ <br> OPTIMUM | $H_{M} \approx$ OPTIMUM | $\mathrm{H}_{\mathrm{M}}>$ OPTIMUM |
|  | AREA <br> RATIO | 0.95 | 0.95 | 0.50 |
|  | AMPLITUDES AT $5 \mu \mathrm{sec}$ | 0.97 | 0.97 | 0.63 |
| $\mathrm{R}_{0}$ | AREA | 1.5 | 1.7 | 18 |
|  | $5 \mu \mathrm{sec}$ | 1.0 | 1.0 | $\rightarrow \infty$ |
| $S_{D}$ | AREA | 10 | 13 | 0.80 |
|  | $5 \mu \mathrm{sec}$ | $\rightarrow \infty$ | $\rightarrow \infty$ | 0.70 |
| $S_{n s}$ | AREA | 12 | 16 | 8 |
|  | $5 \mu \mathrm{sec}$ | $\rightarrow \infty$ | $\rightarrow \infty$ | $\rightarrow \infty$ |
| RESPONSE time |  | $\begin{gathered} 25 \\ \mu \mathrm{sec} \end{gathered}$ | $\begin{gathered} 20 \\ \mu \mathrm{sec} \end{gathered}$ | $\begin{gathered} 15 \\ \mu \mathrm{sec} \end{gathered}$ |



INFORMATION RETENTION AND SIGNAL RATIOS FOR THE BEST METALLIC AND FERRITIC CORES

## APPROVEDEOR PUBLIC RELEASE. CASE 06-1104.

Compaxison of Figure 19 with the family of toll charagtorlatice
 vell as H excuraions are reducod, until some point is reached below which the loops collapee almost entirely. The prodominnat characteriatios of the loop collapse are the large reductions in retainnd flux deneity, $B_{R}$. These reductions are ereater, percentage-wise, than the concomitant
 decreass soo, the overall effeot is to madnt-in a loop ahape on wich the oofneldant-ourrent oriterlon $H_{0} / 2<H_{1}$ holds until the loops bepin to collapse antirely. Aiso held vall it the flatnese of the near-horizontal portions of the loope, thus retaining famerable Nos algasl ration ( $\mathrm{S}_{\mathrm{n}}$ ).

## 2. Coze Ferranic ikna Mo9 (boc.)

Thete is the beat farsitic core tested and has reasonably aocegtabie retention and algnal ratios plus an axtrumely thort reaponse tiee. It is, however, rather sansitive to the $H_{N}$ mplitudes applied, and allous leas deviation frow the opt isava $H_{M}$ than doee tho netallic cors previoualy discusened.

Fipare 31 has photographe of the mapnetizinc pulses and the outpput pulses for the three important modes of operation. Scope leads vers reversed when taking the photos of the H pulsss and the node d output palese, so that all pertinent traces appear as poaitive ones. Othervias interpretation is the sume as for Figare 28.

(a) FAMILY OF B-H CHARACTLRISTICS FOR METALLIC CORE MTS 4382


FERRAMIC 34, TYPE A
(b) SATURATION HYSTERESIS LOOP FOR

FERRITIC CORE FERRAMIC 34 A AS
PUBLISHED BY THE MANUFACTURER

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

A-360GF-1108


## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

The romstant 1mportant retion are tabalated in $7 i$ gare 39 nost to those for core Mis 4332. The Pervitic core has a rempence thay about one-forsinth that for the motaliso coro, but it has appreciably poorar retention and aignol ration.

The response tian improvement is largely due to the terrific reduction in eddy currente. The relatively poor signat ration are tue largely to a lower degree of hyateresi-loop reotangularity; a oopy of a hysteresis loop for Ferremic 3hai is ahom in 7rgare 30b.

Both the response-time valuet and the mignal ratios at an aptimin point in time could probably be improved somewhat by deorsasing the rise times of the pulase of H .

## 3. Ferroxqube iv

an Interenting denonatration of the efvect if difforent degrese of hyateresie-loop reckanpularity upon a core's performanoe way be had by rineting a negatively magnetostrictive oore with a compreseion drawstrap and checking, its performance, and its B-K characteristice under varying dogrees of compression.

Thie vas done with a toroidal emple of Ferroxoube IV as aupplied by the Korth American Ph1lips Co. The photographs of Figure 32 how that core's performance when under a good apount of compression and when under nons. The emergence of a fatr degree of Loop reotangunarity under oora compresaion may be seen in the upper 3wo photograyhti.


## APPROVED FOR PUBLIC RELEASE. CASE 06-1104. ${ }^{\text {¹ }}$

Roport R-192

Along with the appearanco of 200p rectangularity under compression ts the abditty of the core to perforn, with roesonango retoation and signal ratios. in the coiresdentocurrent scheme. Tho ocre, then under no comprossion, has completely waccaptable ratiose as show in the second colum of the figure.

The remponse time of the core increases by a factor of about 10 when under compression. This is very roughiy equal to the differentinl
inersase in the madimum finewrint permeability ( $\mu_{\mathrm{d} 1}$ ) with compression, as approxmated frow the middla loops of the two B-R oherneteristios shown. This is further evidence in support of some of the dincuseion in the precedine chapter.

Lifeselize photographs of the cores are dieplayed on the next two pares. Figure 33 ghows pictures of the fallowing: oore KTS 4450 , cased and wound (top 2sft): a mall-aize, cased sample oore which bas charnotaristioa like MTi 4382 ( Bop conter): the Porroxoubs IV oore with compression drawstrap and windings (top right); a regular Daltamax core, aased and wound (botBon left); a l-mil Daltamax core out of ita case (bottom right). Figure 34 shows cores
 are wound and mounted 0 a male plug of the Corensentige Pule Amplifier ast used during the tests.

A-36010 APPROVED FOR PUBLIC RELEASE. CASE 06-1104.



LIFE-SIZE PHOTOGRAPHS OF GORES MTS 4382 AND
FERRAMIC 34A FIO9(b.o.) AS MOUNTED ON CORE TESTING PLUGS READY FOR USE IN THE TEST SETUP

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

QHaEMER IV
OONOUSIOM

The precoding two ohepters have dwelt on two phases of tho problem of operating magnetic cores in a coincidentmourrent menozy scheme which would be adaptable to a ine-sultching type of selection along two or three apace coordinatea.


#### Abstract

A. Regponse Times

The best response times presently attainable run to about 20 slaressoonde for metallic coree and about $1 / 2$ alarosecond for ferritic cores. The latter tise is aore than low enouph by the hief-spasd-mesory standards of hirlvind 1 . Some deorease in remponse times is posaible if the radial thicieness of the ferritic sorolds is reduceds further improvesent mey be attainable by ohanging material charaoteristics ach as the volume reaistivity.

The 20 -microseconds response time of the best metallic cort is too grant for hiphespeed memory applioution in Whirluind $I_{0}$ but is probably reasonable for intormediate applications and for slower computers. Very significent decreases in this core's response time are possibla by reducing tape thickness (the Allegheny Ladlus Steel Corp. In worting soward $1 /$ hmin teqe - this hould reduce response timse to well below 5 microseconds), inoreasing material resintivity, and reducing the retained flux deneity.


## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.


#### Abstract

E. 83. grel Rayios

Core MS 43E2 hae probably se good a sot at snformasions setantion and angnal ratioe a ulil be neaded in a colncidenforrent soheme mich Involves no mose then threo currontn. The area fation are extranely good, and the optimon-point-1n-tise railos are practically perfect. Thess are all the reault of a Boll character1â1c which combines sharp corners, abrupt eaturation, and the fact thet $H_{1}$ and $\mathrm{H}_{2}$ are alose to each other yet ralatively far from tho origin wo that is is enay to fixd an $H_{M}$ which fulfalle the oriterion. $H_{2}<2 H_{1}$. $\xrightarrow[1]{H} \rightarrow \operatorname{Ha}_{6}$


## C. Gore B-A Characterietion

The operation of a core as a coinoident-current menory unis requiren a hich degree of hysteresie-loop rectanpularity. It al so requires that the hysteresis loops of the core collapse toward the origin. for deoreacing excuraione of $H_{0}$ in a certain way.

Future data on mapnetic cores for this type of application should be In the form of familles of Bh curven, or hysteresis loops, taken for aymetrical excitation valuee mich rence from zero to near saturation. with emphasis on the region about $\mathrm{H}_{0}$. A family of auch ourres. supplenented by physical data regarding dimensiong, resietivityo Ourie temperature, etc., will convey about all the information zpeeded in order to assess the core's alitability. There in quite a xeamblance bepween such data and more oommonly seen vacuus-tube data.

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

D. 4tpoollangous Considorotions

There aro some pheses of the problem wheh have besn castrod or only 11 ghti.y mentloned so fer.

One of these is the fact that readine the $\operatorname{coro}^{\circ} \mathrm{B}$ information is the eame as vriting a saro into it. The core, in other words, in cleared upon being read. This is not a major impediment to the schens ${ }^{\circ}$ s usefulness in a large-scale computer; it is not too difficult to rewalte information extracted from a memory register. thougt should be given, hoverer, to meang of oircunvanting this problen.

The energy irreveraibly put into a core during the witching process is oulte aipnificant. The required pover may be a problem to supply and to diselpate as heat from the corss.

The average power into oore uTS 4382 during the 20 microseconds ewitching time is of the order of 10 vatts. If the core is operated at a $50 \%$ duty oyele (a revereal every 40 alaroseconds) it has to diseipate about 5 vatts without experiencing a dengerous teaperature rise. Since this core is falriy large (aee Figure 34) and has a high Ourie temperature, it can probably operate under these conditions fairly weil, and the proble may only be that of removing the combined heat cencrated in an asseably of such coret. Onder the same duty cyele of 50\% (a reveraal avery miorosecond) core Ferramic 3 han 7109 (b. O.) mey heve to dissipate about 10 watte. This oore is already rather mall. and it has a relatively low Ouris temperature $\left(280^{\circ} \mathrm{C}\right)$. Unless epseial prepautions are taken, this core's temperature may rise, during operstion, to p point where its hysteresis-loop shape ip adversely affected.

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.


#### Abstract

Tho onext into tho core is in the form of a current pulas  pulave for a lange mubar of corea may be a aerious probleng parbicularly If the current magnitudes are kept high in order to reduce the number of turns needed on each megnetizing or selecting vinding.


The aignal-ratio problea may be acute in a large assembly of cores. Whether undesired signals (NoS algnale and cero a) will add or cancel each other will depend on the sensing cirouit arrangement so The tising of the selecting pulees also onters into whether or not NoS signals bacome objectionabie.

Thore is a large, thin spike at the bepinning of esch output pulse from those cores which have reaponse timen longer than about 5 microseconds. This apike may also be saman an the start of each $x \frac{d f}{d i}$ pules on the eheoratical approxisations of Plgures 21 and 130 were the H pulses are coneldered to be atep functions. Where t pulses have riae tieen which are negligible fractiona of the core reapozse time, the apikes appear. It is obvious, then that the actual shape of these magnetizing pulses is a oritioal factor which needs to be deterained for each situation; and the spikes may de reduced as deaired at the expense of some alipht increases in remponse timer.

Inductive pickup may beoome a real headache and the proper use of unilatoral impedance olements, oareful relative placement of cores, and other precankions may be necessary.

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

Report $\mathrm{R}-192$

Thess and other miscellaneous considerations require further study and experiment. Particularly needed is the ers perimental operation of a few cores in a two-dimensional pilot assembly. This work will begin at Project Whirlwind in the near fatare.


WNP: ap

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

## APPERNDIX

DERIVATION: $H$ inside a thin ribbon as a function of $x$ and $t$.


Maxwell's equations:

$$
\nabla \times \overrightarrow{\mathbf{B}}=-\frac{\partial \vec{B}}{\partial t}, \quad \nabla \times \vec{H}=\vec{i}+\frac{\partial \vec{D}}{\partial t}
$$

Due to the assumed geometry, and disregarding displacement currents:

$$
\begin{aligned}
& \mathbf{z}_{\mathbf{z}}=B_{x}=B_{y}=\frac{\partial \overrightarrow{\mathbf{E}}}{\partial z}=\frac{\partial \mathbf{E}_{x}}{\partial y}=0, \\
& H_{x}=H_{y}=1_{z}=\frac{\partial \vec{D}}{\partial z}=\frac{\partial \vec{H}}{\partial z}=\frac{\partial H_{z}}{\partial y}=0,
\end{aligned}
$$

leaving:

$$
\frac{\partial \mathbb{Z}}{\partial x}=-\frac{\partial B_{z}}{\partial t} \quad \text { and } \quad \frac{\partial H_{z}}{\partial x}=-1 y^{*}
$$

$$
\text { Substituting } \quad E_{y}=\frac{1}{\sigma} t
$$

gives:

$$
\frac{\partial^{i} y}{\partial x}=-\sigma \frac{\partial B_{z}}{\partial t} \text { and } \frac{\partial H_{z}}{\partial x}=-i
$$

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

Differentiating the seconi equation $w / r$ to $s$, manipulating, combining, and dropping subocripts results in:

$$
\begin{equation*}
\frac{\partial^{2} H}{\partial x^{2}}-\sigma \frac{\partial B}{\partial t}=0 \tag{1}
\end{equation*}
$$

Wher 3 is a singlevalued, linear, function of $H$, we may define a constant, called the perneability, ns:

$$
\mu \equiv \frac{B}{H},
$$

in which case eq. 1 becomes:

$$
\begin{equation*}
\frac{\partial^{2} H}{\partial x^{2}}-\sigma \mu \frac{\partial H}{\partial t}=0 \tag{2}
\end{equation*}
$$

The solution for tin equation follows a similar one that starts on pare 189 of reference 8.

Boundary and in'tial conditions are:

$$
H(0, t)=H(a, t)=H_{e} ; H(x, 0)=0_{0}
$$

Say that:

$$
\begin{aligned}
& \quad \exists=H_{S}+H_{T}=\text { Steady state sol' } n+\text { transient sol'n. } \\
& H_{S} \text { satisfies one particular sol'n of eq. } 2:
\end{aligned}
$$

$$
H_{S}=H_{e}
$$

To find the transient part of the sol'n tre set up new boundary conditions:

$$
\begin{align*}
& H_{T}(0, t)=H(0, t)-H_{S}=H_{e}-H_{e}=0  \tag{3}\\
& H_{T}(a, t)=H(a, t)-H_{S}=H_{e}-H_{e}=0 \tag{4}
\end{align*}
$$

Another particular sol'n of eq. 2 is:

$$
\begin{equation*}
H=\left(c_{3} \sin b x+c_{4} \cos b x\right) e^{-\frac{b^{2} t}{\sigma \mu}} \tag{5}
\end{equation*}
$$

Substituting eq. 3 into eq. 5:

$$
\begin{align*}
& H_{T}(0, t)=\left(c_{3} \sin 0+q_{4} \cos 0\right) e^{-\frac{b^{2} t}{\sigma \mu}}=0 \\
& \therefore c_{4}=0, \text { and } \\
& H_{T}=c_{3} \sin b x e^{-\frac{b^{2} t}{\sigma \mu}} \tag{6}
\end{align*}
$$

Substituting eqi 4 into eq. 6:

$$
\begin{aligned}
& H_{T}(a, t)=c_{3} \sin b a e^{-\frac{b^{2} t}{\sigma \mu}}=0 \\
& \therefore \sin b a=0 \text {, and } b=\frac{\text { nII }}{a} .
\end{aligned}
$$

so that:

$$
\begin{equation*}
H_{T}=\sum_{n=0}^{\infty} c_{n} \sin \frac{n \pi}{a} \pi e^{-\frac{n^{2} \pi^{2} t}{\sigma a^{2} \mu}} \tag{7}
\end{equation*}
$$

New initial conditions are:

$$
H_{T}(x, 0)=H(x, 0)-H_{S}=0-H_{0}=\cdots H_{0} .
$$

Substituting these into eq. 7:

$$
\begin{aligned}
& H_{T}(x, 0)=\sum_{n=0}^{\infty} c_{n} \text { sin } \frac{n \pi}{a} x=-H_{\theta} \\
& \therefore c_{n}=-\frac{2 H_{e}}{n \pi}(1-\cos n \pi)=-\frac{4 H_{e}}{n \pi}, \text { for odd } n .
\end{aligned}
$$

So that:

$$
\begin{equation*}
H_{T}=-\sum_{n=1}^{\infty} \frac{4 H_{e}}{n \pi} \sin \frac{n \pi x}{a} e^{-\frac{n^{2} n^{2} t}{a^{2} \sigma^{\mu}}} \tag{8}
\end{equation*}
$$

And:

$$
\begin{equation*}
H=H_{0}\left(1-\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n \pi x}{a} \cdot \frac{-\frac{n^{2} n^{2} t}{a^{2} \sigma_{\mu}}}{e} \text {, odd } n\right. \text {. } \tag{9}
\end{equation*}
$$

## APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

Fuport R-192


2. Joy W. Porreater, "D1gital informationain Threo Dimensiono Ueing Magnetio Cores, "Projoct hhitiwind Report R-187. (May 16. 1950), M.I.T. Servomechailena Laborabory.

2. Jay W. Yorrester, "Data Storege in Three Dimensions," Projeot Whirivind Kemorandum H-70, (April 29. 1947), M.I.T. Servomechan! ems Laborabory.
3. Harvard Univeraity Computation Laboratory, Erogress Ioports 206 (August 1948 - Hovenber 1949). Investigations for Desien of Difotal Calculating Hachinery. (particularly Wo. 2, part IV).
4. J. L. Snoak. Hen Dovelopmente in Yerromacretic haterial so (Now York: Elsevier Publighing $\mathrm{Co}_{0}$. 1947).
5. Broakman, Dowing, and Steneck, "Dimeasional mefecte Reaulting from a Hiph Dieleotrio Conatant Pound in a Yorromagnetic Ferrite " The Physical Reviev. 77.1 (Jan. 1950).
6. $H_{0} M_{0} B_{0}$ sorth, "Magnetiem," Eeviev of Modern Phygicg, 19 (Jan. 1947).
7. $\mathrm{H}_{0} \mathrm{~A}_{\text {. Rethbone, "Specifications for Standard Test Zquipment," }}$ Project mirlvind Report R-243. (Jan. 18, 1949), M.I.T. serromechani emi Laboratory.
B. P. Frankiin, Differential liquation for kiectrical knpineors (New Yoxk: John Wiley \& Sons, Inc., 1933).

[^0]:    1．Suparsoripta rerer to numbored stang In tho blbisography．

