

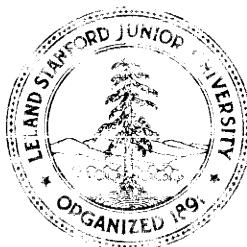
ON "PASCAL", CODE GENERATION, AND THE CDC 6000 COMPUTER

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Abstract:

"PASCAL" is a general purpose programming language with characteristics similar to ALGOL 60, but with an enriched set of program- and data structuring facilities. It has been implemented on the CDC 6000 computer. This paper discusses selected topics of code generation, in particular the selection of instruction sequences to represent simple operations on arithmetic, Boolean, and powerset operands. Methods to implement recursive procedures are briefly described, and it is hinted that the more sophisticated solutions are not necessarily also the best. The CDC 6000 architecture appears as a frequent source of pitfalls and nuisances, and its main trouble spots are scrutinized and discussed.

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On "PASCAL", Code Generation, and the CDC 6000 Computer

1. Introduction

This set of notes has a dual purpose. It is on the one hand directed to the user of the PASCAL compiler system who would like to gain some insight into the machine code which is generated for various basic operations. It is even recommended that he study these notes carefully, because their understanding may prevent him from certain pitfalls which are inherent in the use of the CDC 6000 computer [1].

On the other hand the notes may be of interest to compiler writers in general, because they point out some problems and dilemmas and our choices of solutions. It becomes apparent that the choice of the code to be generated is crucial for a good compiler system, and that it is far from trivial as is usually believed.

The true purpose of a higher-level language is that it allows a programmer to conceive his algorithms in terms of some convenient abstractions. For instance, he is given the opportunity to think in terms of familiar notions of numbers, of relations, and of repetitions, instead of having to express his program in terms of bitstrings, arithmetic instructions, and transfers of control. However, these abstractions are only truly useful, if he can assume that his implementation observes all the properties which are commonly attributed to these abstractions, or else if it automatically issues a warning. As an example, when dealing with numbers in a high-level language, one should like to assume all the common axioms of arithmetic to hold. Of course this is not possible, since computers can only represent finite ranges of values. So one expects to receive a warning, if an operation

has trespassed the limits imposed by the implementation and an operation generates a result not in accord with the rules governing the abstraction. So the system is expected to provide an error indication, e.g. if an overflow occurs in an addition, if a value is being assigned which lies outside the specified range of values of variables, or if an array index is used which lies outside the defined limits.

Unfortunately, such potential warnings require the execution of additional instructions, which in general is costly. As far as range checking is concerned, they can be requested to be generated by the compiler for run-time execution by enabling so-called options. (The A-option generates assignment range checks, the X-option index checks.) They are relatively costly, but may speed up the finding of logical mistakes a great deal.

As far as irregularities of the arithmetic are concerned, one has become used to receive these warning signals automatically from the hardware, particularly because they are easily generated by the hardware, whereas a solution to detect overflow by software is usually beyond any reasonably economical feasibility. Unfortunately, the CDC computer fails to satisfy even the most modest expectations in this respect, and the effort to provide a system with security in the above sense was therefore a series of constant frustrations. Equally disappointing are some of the "features" of its floating-point arithmetic instructions.

One can go only a relatively short distance in trying to correct mistakes of the hardware by means of software; otherwise a system becomes ridiculously inefficient and will not be used by conscientious programmers who are willing to take the peculiarities of a hardware into account and guarantee safety of their algorithms by analytical

rather than experimental means. And this would have been against the intentions of PASCAL. So all that can reasonably be done is to elucidate the shortcomings and limitations of the hardware that are still transparent through the "software cover", and to make the programmer fully aware of them. And this is the purpose of this note.

It concerns itself with the simple operations of integer and real arithmetic, with Boolean operations and with powersets. The reader is supposed to be familiar with the CDC COMPASS notation. The operands are usually assumed to have been brought into the X1 and X2 registers. (If they were loaded into other registers, a corresponding renumbering is necessary which is, however, irrelevant to the operation itself). Registers X1 - X5 are used as a stack for intermediate results, whereas X0 is used exclusively as local work register.

Section 6 deals with the topic of implementing recursive procedures and the addressing of local variables. Although the general techniques are well-known, analysis of possible solutions and their experimental comparison yielded some noteworthy results. It is shown that attempts to make full use of available hardware features such as base registers may not necessarily lead to an optimal performance. Again, the instruction set of the CDC computer is hardly optimal to implement mechanisms for recursive procedures. Conspicuously absent is a subroutine jump instruction which leaves the code invariant (reentrant).

2. Integer Arithmetic

Data of type integer or of subranges thereof are represented by fixed-point binary numbers. Addition and subtraction are represented by the

$IXi \quad Xj \pm Xk$

instructions. Other operations are implemented by short sequences of instructions, as outlined below.

2.1 Multiplication

Due to a recent change of the hardware, fixed-point multiplication can be performed by a single

$DX1 \quad X1*X2$

instruction. It should, however, be noted that this instruction is essentially a floating-point instruction, and yields incorrect answers for fixed-point operands with $|x| \geq 2^{48}$. This can be regarded as an overflow condition which is, alas, neither trapped nor indicated by the computer. A "safe code", checking against all imposed limits of operands and result, is quite elaborate and uneconomical by any standards, and was therefore not implemented.

If one of the operands is a constant C being representable as either

1. $c = 2^n$ $(2, 4, 8, 16 \dots)$
2. $c = 2^m + 2^n$ $(3, 5, 6, 9, 10, 12 \dots)$
3. $c = 2^m - 2^n$ $(7, 14, 15 \dots)$

$\left. \begin{array}{l} \\ \\ \end{array} \right\} m > n$

then the compiler generates the following code for the multiplication of X_1 by c :

```
1: LX1 n           multiply by  $2^n$ 
2,3: LX1 n
      BXO X1
      LX1 m-n
      IX1 X1 + X0
```

Again, overflow conditions are simply ignored. Case 3 yields only correct results, if $|X_1 * 2^m| < 2^{59}$.

2.2 Division (div)

Integer division is represented by the instruction sequence

```
PX1 X1
PX2 X2
NX2 X2 } pack
FX1 X1/X2 divide
UX1 B7,X1
LX1 B7,X1
BXC X0-X0
IX1 X1+X0 } suppress neg. zero
```

and suffers from the same basic shortcoming as multiplication: an operand $|x| \geq 2^{48}$ yields an incorrect result.

If the divisor is a constant $c = 2^n$, the compiler again produces an "optimized" code, performing division by shifting. Unfortunately, a single right shift instruction is unsatisfactory, because it may generate a "negative" zero as result. Negative zeroes, however, must not be allowed to occur, since comparisons may yield wrong answers if applied to them. Thus, the optimized division is implemented as

```
AX1 n      divide by  $2^n$ 
BX0 X0-X0  suppress
IX1 X1+X0  negative zero
```

Note that the unconditional generation and addition of a zero can be

accomplished with a code that is not only shorter than a conditional jump, such as

```
AX1  n
NZ   X1,L
SX1  B0
L   ...
```

but also avoids the insertion of padding instructions (NOPs) for word boundary alignment.

The D-option provides an additional security measure against division by zero. It causes the compiler to insert a

```
ZR  X2, error
```

jump instruction preceding every division instruction. (This applies to the Modulus operation as well.) It is particularly recommended in the case of integer division, where the actual divide instruction generates a "floating-point infinity" value, which is incorrectly treated by the subsequent conversion instructions and thereby represents a senseless result.

2.3 Modulus (mod)

The modulus or remainder operation is defined as

$$x \bmod y = x - (x \div y) * y$$

As it involves integer multiplication and division operations, it suffers again from the same deficiencies of the 6000 arithmetic. Its corresponding code is:

```
PXD  X1
PX6  X2
NX6  X6
FX6  X0/X6
UX6  B7,X6
LX6  B7,X6
DX6  X6*X2
IX1  X1-X6
```

2.4 Sign inversion

The use of one's complement representation for negative numbers makes again the most obvious choice of code

BX1 -X1

unsatisfactory, because it might generate a "negative zero". So we use

BX0 X0-X0
IX1 X0-X1

2.5 Comparisons

Since the computer does not offer a compare instruction, subtraction has to be used; this has primarily the disadvantage of generating wrong results in the case of overflow. The cases of testing for equality and inequality are handled correctly, because the one's complement addition generates an end-around-carry in the case of "negative overflow", thus maintaining a result indicating inequality. Note that the Boolean subtraction

BX1 X1-X2

cannot be used, because a comparison of x_1 and $x_2 = -x_1$ would yield a zero result, thus indicating equality.

Whereas equality testing is "safe" with the

IX1 X1-X2

instruction ignoring overflows, this is not the case for the tests of ordering $(x_1 < x_2)$ by subtraction and subsequent inspection of the sign bit. The reason is that if overflow occurs, i.e., $|x_1 - x_2| > 2^{59}$, then the sign bit will be the opposite of the true sign. This situation is quite hopeless, since overflow is in no simple way detectable on this machine. In order to obtain a (sign) bit representing the relation

$x < y$ for any values x, y , the following algorithm can be used:

1. Compare the signs of the two operands.
2. If they are different, then the result is obvious.
3. If they are equal, the subtraction $x-y$ can be performed without danger of overflow, and $x-y < 0$ is the result.

A minimal instruction sequence to perform these operations and avoiding the use of undesirable jump instructions is

```
BX0  X1-X2      compare sign bits
IX2  X1-X2
BX1  X0*X1      if unequal, choose sign of X1
BX2  -X0*X2     if equal, choose sign of X1-X2
BX1  X1+X2
```

Now the sign bit of $X1$ is 1, if $X1 < X2$, and 0 otherwise. Still, the effort to perform a faultless comparison is formidably cumbersome, and the PASCAL compiler does not generate it. The programmer is left with the responsibility to verify that for every comparison of x and y , $|x-y| < 2^{59}$.

2.6 Taking the absolute value (ABS)

The code used to take an absolute value is designed to avoid jump instructions, not only because they are long and slow, but because they usually introduce NOP instructions for alignment.

```
BX0  X1
AXC  59      generate 60 sign bits
BX1  X0-X1
```

2.7 Testing for even or odd (ODD)

Since one's complement representation is used for negative numbers, the least significant bit of the operand must be compared with its sign bit:

```
BX0  X1
LX0  59
BX1  X1-X0
```

This leaves the sign-bit of X1 equal to 1, if X1 was odd, and 0 otherwise.

The compiler "optimizes" in the case of ODD(x) with x being of a subrange type with only non-negative values. It then generates the single instruction

```
LX1  59
```

2.8 Summary

The foregoing explanations reveal that the absence of any overflow indication makes analytical verifications necessary that guarantee the non-occurrence of these conditions. An effective aid in experimental testing is the A-option, causing interval check instructions to be generated with every assignment to a variable that is declared to be of a subrange type. The A-option is activated by the "comment"

```
{$A+ ... }
```

and causes the code for an assignment to a variable

```
VAR V: a..b
```

to become:

SX7	*	location identification for error trap
SX1	a	
IX0	X6-X1	
SX1	b	
IX1	X1-X6	
BX0	X1+X0	
NG	X0,error	jump to error routine
SA6	V	

It should be noticed that unfortunately the attractive and shorter code sequence

```
SX7  *
SX0  X6-a
SX1  X6-b-1
BX0  -X1+X0
NG   X0, error
SA6  V
```

cannot be used, because the instructions

```
SXi  Xj ± K
```

perform an 18-bit arithmetic ignoring the leading 42 bits of the register X_j which -- of course -- is not in the spirit of a check.

This ignoring rather than checking of the leading bits in 18-bit arithmetic is the reason why the so-called "increment" instructions cannot be used by the PASCAL compiler, except in the following special circumstance: if a variable x is declared of a subrange whose limits are both less than 2^{17} in absolute value, then the assignment statement

```
x := x + k
```

is compiled as

```
SAL  x
SX6  Xl+k
SA6  x
```

3. Floating-point Arithmetic

The PASCAL compiler uses the complete set of F-instructions for arithmetic with values of type "real". Comparison is performed by subtraction due to the lack of a compare instruction. This is possible without handicap since the occurrence of overflow generates a signed "infinity" - value, but no immediate trap. Sign inversion is represented by

BX0	X0-X0	generate zero
IX1	X0-X1	

and the absolute value function by

BX0	X1
AX0	59
BX1	X0-X1

Arithmetic with the F-instruction possesses some peculiar properties which will briefly be reviewed, and has for instance the consequence that $x-y = 0$ does not necessarily imply $x = y$, if the difference is computed by an Finstruction. The trouble arises from the fact that F-arithmetic truncates without rounding, and F-addition truncates without post-normalization. Every addition is therefore compiled into two instructions:

FX1	X1 + X2	add/ subtract
NX1	X1	post-normalize

If the two values

a = 1720 40...00B	= 1.0
b = 1717 17...77B	= $1.0 \cdot 2^{-48}$

are compared by subtraction

FX0	X1-X2	a-b
-----	-------	-----

the result is

$$\begin{array}{r}
 1720 \ 40\dots \ 00 / 00 \ \dots \ 0 \\
 -1720 \ 37\dots \ 77 / 40 \ \dots \ 0 \\
 \hline
 1720 \ 00\dots \ 00 / 40 \ \dots \ 0
 \end{array}$$

where the slash marks the separation between the lower and the upper half of the 96-bit accumulator. The result is 0 although the two operands were different.

Notice that subtracting 0.5 from both a and b, and then computing their difference, yields

$$\begin{array}{rcl}
 a - 0.5 = 0.5 & : & 1717 \ 40\dots \ 00 \\
 b - 0.5 = 0.5 - 2^{-48} & : & 1716 \ 77\dots \ 76
 \end{array}$$

$$\begin{array}{r}
 1717 \ 40\dots \ 00 / 00 \ \dots \ 0 \\
 -1717 \ 37\dots \ 77 / 00 \ \dots \ 0 \\
 \hline
 1717 \ 00\dots \ 01 / 00 \ \dots \ 0
 \end{array}$$

i.e., a difference which is not zero. Thus the result does not only depend on the true result, but also on the values of the operands. This unpleasant property of the CDC F-arithmetic stems from the fact that automatic post-normalization is absent.

3.1 Rounding

It was at one time hoped that this defect could be avoided by letting the PASCAL compiler automatically generate R-instructions, which include a certain kind of rounding. However, R-arithmetic turned out to feature some even stranger properties, so that it was decided not to use R-instructions. In order to point these features out, a brief review over R-arithmetic is necessary:

The R-instructions differ from the F-instructions only insofar as a 1-bit is appended to normalized operands before the arithmetic operation is performed. Thus for instance the subtraction of $b = 1.0 \cdot 2^{-48}$ from $a = 1.0$ yields

$$\begin{array}{r}
 \begin{array}{r}
 1720 \quad 40 \dots \dots 00 \quad / \quad 40 \dots \dots 0
 \end{array} \\
 \begin{array}{r}
 \underline{-1720} \quad 37 \dots \dots 77 \quad / \quad 60 \dots \dots 0
 \end{array} \\
 \hline
 \begin{array}{r}
 1720 \quad 00 \dots \dots 00 \quad / \quad 60 \dots \dots 0
 \end{array}
 \end{array}
 \quad \left. \begin{array}{l} \downarrow \\ \text{1-bits appended} \end{array} \right\}$$

which of course is still zero.

The principal defect with "CDC-rounding", however, is that its effect is unpredictably either the addition of $1/2$ or $1/4$ in the last position, because rounding takes place before instead of after normalization (which must again be performed by a separate instruction). The following example illustrates this, which is shown on hand of a five-bit number representation:

$$\begin{array}{r}
 16 = \quad 10000 \quad / \quad 1 \quad \text{inserted} \\
 +17 = \quad 10001 \quad / \quad 1 \quad \text{round-bits} \\
 \hline
 33 = \quad 100010 \quad / \quad 0 \\
 \quad \quad \quad \underline{10001} \quad / \quad 0 = 34
 \end{array}$$

$$\begin{array}{r}
 31 = \quad 11111 \quad / \quad 1 \quad \text{inserted round-bit} \\
 +2 = \quad 00010 \quad / \quad 0 \\
 \hline
 33 = \quad 100001 \quad / \quad 1 \\
 \quad \quad \quad \underline{10000} \quad / \quad 1 = 32
 \end{array}$$

In the first case, the pre-rounding results in correct rounding of the not exactly representable 33 to 34, whereas in the second case pre-rounding has no effect.

The same phenomenon can be observed in the cases of multiplication and division. The following example again uses a five-bit number representation:

$$\begin{array}{r}
 \text{round-bit} \\
 \downarrow \\
 15 \times 12 = \underline{\underline{11110 / 1 \quad x \quad 11000}} \\
 \underline{\underline{11110 / 1}} \leftarrow \\
 + \underline{\underline{01111 / 01}} \leftarrow \\
 \underline{\underline{101101 / 11}} \\
 \underline{\underline{10110 / 111}} \quad \quad \quad = 176
 \end{array}$$

$$\begin{array}{r}
 \text{round-bit} \\
 \downarrow \\
 18 \times 10 = \underline{\underline{10010 / 1 \quad x \quad 10100}} \\
 \underline{\underline{10010 / 1}} \leftarrow \\
 + \underline{\underline{100 / 101}} \leftarrow \\
 \underline{\underline{10111 / 001}} \quad \quad \quad = 184
 \end{array}$$

In the first case, the rounding effect is nil, leaving the inexactly representable value 180 be an unrounded 176; in the latter case the rounding effect transforms 180 into the value 184. (Suitable adjustment of exponents is not shown here.)

A method introducing proper rounding instead of "CDC-rounding" relies on the use of the D-instruction set [2]. Whereas the F-instructions yield the high-order 48 bits of the 96-bit accumulator, the D-instructions yield the low-order 48 bits with a suitably adjusted exponent, thereby allowing access to a double precision result.

Notice that it is an ingeniously efficient method to compute a double precision result by

1. computing the DP-result and dispose of the low half (F-instruction), then
2. computing the same again and dispose of the high half (D-instruction).

This computer allows it to be done in no other way!

The PASCAL compiler will generate the following code for floating-point operations, depending on the choice of the R-option:

R-option	OFF	ON
$x \pm y$	$FX1 \quad X1 \pm X2$ $NX1 \quad X1$	$FX0 \quad X1 \pm X2$ $NX0 \quad X0$ $DX1 \quad X1 + X2$ $RX1 \quad X1 + X0$ $NX1 \quad X1$
$x * y$	$FX1 \quad X1 * X2$	$FX0 \quad X1 * X2$ $DX1 \quad X1 * X2$ $RX1 \quad X1 + X0$
x / y	$FX1 \quad X1 / X2$	$RX1 \quad X1 / X2$

Examples of addition / subtraction:

$$\begin{array}{r}
 1. \quad \begin{array}{r}
 1.0 \\
 1.0-2^{-48} \\
 - \hline
 \end{array} \quad \begin{array}{r}
 1720 \quad 40\dots \quad \dots 00 \quad / \quad 00\dots \quad \dots 0 \\
 1717 \quad 77\dots \quad \dots 77 \quad / \quad 00\dots \quad \dots 0 \\
 \hline
 1720 \quad 37\dots \quad \dots 77 \quad / \quad 40\dots \quad \dots 0
 \end{array} \\
 \begin{array}{r}
 1720 \quad 00\dots \quad \dots 00 \quad / \quad 40\dots \quad \dots 0 \\
 = \quad 1640 \quad 40\dots \quad \dots 00
 \end{array} \quad \text{after addition of high and low}
 \end{array}$$

$$2. \quad \text{Take } a = 1.0 \text{ and } b = 2^{-48}, \text{ then subtract } a - b :$$

F-subtraction yields

$$\begin{array}{r}
 a = \quad \begin{array}{r}
 1720 \quad 40\dots \quad \dots 00 \quad / \quad 00\dots \quad \dots 0 \\
 \hline
 b = \quad 1720 \quad 00\dots \quad \dots 00 \quad / \quad 40\dots \quad \dots 0
 \end{array} \\
 \begin{array}{r}
 1720 \quad 37\dots \quad \dots 77 \quad / \quad 40\dots \quad \dots 0
 \end{array}
 \end{array}$$

which, after normalization, is

$$\underline{1717 \quad 77\dots \quad \dots 76} = 1.0-2^{-47}$$

R-subtraction inserts a 1-bit after the slash in the first operand, and thus yields the result

$$\underline{1720 \quad 40\dots \quad \dots 00} = 1.0 \text{ exactly}$$

The combined use of F and D instructions yields the true result, because the normalization instruction left shifts the high order result to

1717 77... ...76

whereafter a "rounded" - addition is used to add the correction

+ 1717 00... ...01

yielding

1717 77... ...77 = $1.0 \cdot 2^{-48}$.

3.2 Conversion from fixed to floating-point (integer to real)

Wherever a real operand is permissible, PASCAL allows the specification of an operand of type integer as well. However, the compiler is then forced to generate the necessary representation conversion instructions, which are not only time-consuming, but potentially hazardous. It is therefore recommended to avoid "mixed-mode" arithmetic expressions wherever possible. The generated conversion instructions are

PX1 B0,X1 pack with zero exponent
NX1 B0,X1 normalize

The result of this conversion is wrong, whenever the integer operand in X1 is larger or equal to 2^{48} in absolute value, since the exponent bits are simply ignored by the P instruction. A test to verify that the operand is within bounds could be compiled as

BX0 X1
AX0 48
NZ X0, error

but is easily seen to be more costly than the conversion itself.

3.3 Conversion from floating to fixed-point (real to integer)

PASCAL does not provide for any implicit real to integer conversion.

However, the standard function `TRUNC(x)` allows to truncate the fractional part of a real number. The used code is:

```
UX1  B7,X1
LX1  B7,X1
BY0  XO-X0    } avoid
IX1  X1+X0    } negative zero
```

The result of this conversion is again wrong, if $|x| \geq 2^{48}$.

4. Boolean Operations

The standard type Boolean is defined in PASCAL as

```
type Boolean = (false, true)
```

Since the values of all scalar types are mapped onto the integers 0, 1, 2, ..., the values false and true are represented by the numbers 0 and 1 respectively.

The operations \wedge and \vee are implemented by the Boolean AND and OR instruction, namely

```
BX1 X1*X2 and
BX1 X1+X2
```

Negation is performed by

```
MXO 59
BX1 -X0-X1
```

If a relation has to be assigned to a Boolean variable, e.g.

```
b := x < y
```

then a sequence of instructions is necessary to obtain a 0 or 1 value. Again every effort is made to avoid the use of jumps. The following code is used in the above assignment; leaving a Boolean value in X1.

```
FX1 X1-X2 x-y
MXO 1
BX1 X0*X1 Extract sign bit
LX1 1 move it to correct position
```

Analogous code is generated for the relations $>$, \leq , and \geq . But unfortunately the equality relations cannot be reasonably implemented without a jump; in the assignment

```
b := x = y
```

the following instructions are generated:

```
FXO  X1-X2
BX1  X0-X0
NZ   X1,L
SX1  1
L ...
```

Boolean comparisons, although occurring rather infrequently, are treated as special cases, because a simpler and shorter code is applicable:

$p < q$	BX1	$-X1*X2$
$p \leq q$	BX1	$-X2*X1$
	MX0	59
	BX1	$-X0-X1$

} negation

$p \neq q$	BX1	$X1-X2$
------------	-----	---------

The remaining three relations are compiled analogously.

5. Powerset Operations

PASCAL 6000 restricts powerset types to be built only on base sets with less than 59 components. This allows a powerset value S to be represented by one "word", in which the i -th bit indicates the presence (1) or absence (0) of the element i in S .

5.1 Generation of the Singleton Set [i]

Assume that i is loaded into register $X1$, then

SB7	X1
SX1	1
LX1	B7,X1

Notice that the numbering of bits starts with 0 at the low order end.

This choice was made in order to be able to load powerset constants with small valued components (less than 18) by a single $SX1$ instruction.

5.2 Set Intersection, Union, and Difference

These three operations are implemented by a single instruction

intersection	BX1	X1*X2
union	BX1	X1+X2
difference	BX1	-X2*X1

5.3 Set Membership (in)

The relation $i \text{ in } S$ is implemented by shifting the bit representing i into the sign position which can be tested:

SB7	X1	i
AX1	B7,X2	S
LX1	59	

If the expression i is in the form of a constant c , then the compiler generates of course only the single instruction

LX1 59-c

5.4 Set Comparison

Sets can be compared for equality and inclusion. Equality is tested by a Boolean subtraction

BX0 X1-X2

and a subsequent zero test. Note that the peculiar property of the zero test to recognize a word with either 60 zero-bits or 60 one-bits as a zero is responsible for the restriction that powersets may contain at most 59 instead of 60 elements. If sets with 60 components were allowed, then a full set and an empty set would not be distinguishable by a single subtraction followed by a zero-test.

Inclusion expressed as $x \leq y$ and meaning $x \subseteq y$, is implemented by the single instruction

BX0 -X1*X2

which is followed by a zero-test instruction. The same instruction is used for the relation $x \geq y$, whereas strict inclusion ($x \subset y$) is not implemented.

Some Exercises Addressed to the CDC 6000 Expert

1. Is the following code to represent the function $\text{trunc}(X1)$ acceptable? If not, why?

BX0	$X0-X0$
PX0	$X0$
FX1	$X1+X0$
UX1	$B7, X1$
NZ	$B7$, overflow

2. Is the following code for $X1 \bmod X2$ acceptable? If so, prove it.

PX1	$X1$
PX2	$X2$
NX6	$X2$
FX6	$X1/X6$
BX0	$X0$
FX6	$X6+X0$
DX6	$X6*X2$
FX6	$X1-X6$
UX1	$X6$

3. Why can the instructions

BX0	$X1-X2$
ZR	$X0$, equal

not be used to represent a comparison $X1 = X2$? Prove that

IX0	$X1-X2$
ZR	$X0$, equal

always yields the correct action.

6. Implementation of Recursive Procedures

The language PASCAL has been carefully designed so that dynamic storage allocation is not required, with the following two exceptions:

1. Variables local to procedures may be allocated storage only when the procedure is called, and
2. Components of class variables are allocated storage by calling the standard procedure "alloc". An area of store is allocated to the entire class variable as soon as the procedure is called to which the class is local.

In this section we will briefly review the well-known techniques for handling recursive procedure calls and of allocating storage to their local quantities, and discuss the code selected to represent the procedure call mechanism.

Due to the first-in last-out nature of the hierarchy of activated procedures a stack may be used to allocate local variables. This is of great advantage, since storage retrieval is trivial in the case of stacks, resulting in low storage management overhead. We consider the set of local variables of each activated procedure as a record (often called "data segment") in the stack. Since their lengths may all be different, the most convenient method to thread the way back through such a stack is by constructing a chain of pointers linking the records. Every record then contains a "header" containing

1. the link to the previous record, and
2. the (frozen) program status (counter) of the calling procedure.

Variables are addressed relative to the origin of the record of which they are a part. The origin address is unknown at compile-time,

and must be determined at run-time. This can be done by descending through the link chain, until the desired record is reached. But how is the desired record recognized? The most straight-forward method which interprets the scope rules of an ALGOL block structure correctly is probably the following:

Method I:

1. Define the level of an object to be 1 greater than the level of the procedure to which it is local. The level of the main program is 0 .
2. Indicate the level of each record (equal to the level of its components) in its heading.
3. Whenever an object on level i has to be accessed, the record containing it is found by descending down the chain of links until the first occurrence of a level indicator with value i is found.

This accessing method has the obvious drawback of inefficiency (and of not being applicable in the case of parametric procedures). A slight modification, however, improves efficiency and generalizes to parametric procedures.

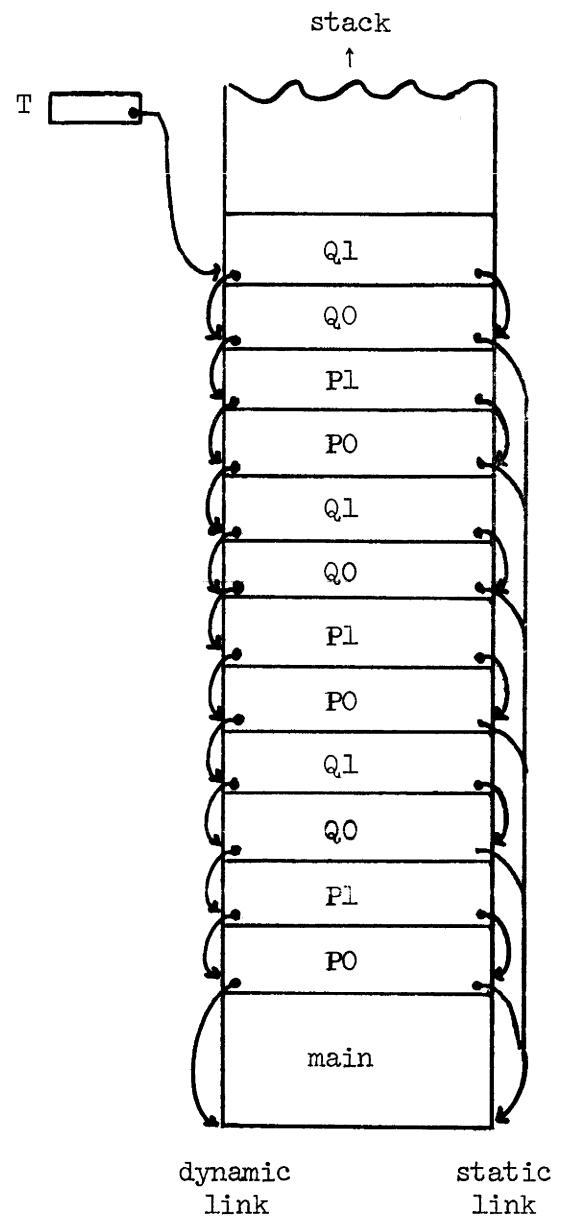
Method II:

Instead of indicating levels explicitly in the record headings, a second link chain is constructed connecting each record A with its static ancestor, i.e., with the record B of the procedure in which A was declared locally. In order to distinguish the two link chains, the former is called the "dynamic link" and the latter the "static link". An example of a state of computation is shown below for a given -- admittedly not very realistic -- program.

```

var v0;
procedure Q0 (procedure X);
    var wl;
    procedure Q1;
        var w2;
        begin w2 := wl+v0; X
        end;
        begin wl := v0; Q1
        end;
procedure P0;
    var vl;
    procedure P1;
        var v2;
        begin v2 := vl+v0; Q0(P0)
        end;
        begin vl := v0; P1
        end;
begin {main program} v0 := 0; P0
end.

```



Method III:

Although the use of a high-speed index register to represent the origin of the link chains improves access speed significantly, the process of descending down the static chain to the record (data segment) with the desired level is relatively time-consuming. An ingenious device to reduce access time was introduced by Dijkstra [4] and is now widely used in compilers for block-structured languages. The device

is an array of base addresses, called the *Display* D , which is at any time a copy of the static chain. If an object at level i is to be accessed, the origin address of its data segment is quickly obtained as D_i . The method is particularly attractive for computers with a set of high-speed index registers which can be used as the *Display*. The price for this increase in access speed -- apart from the reservation of registers -- is the setting and updating of the *Display* each time a procedure is called and terminated. To be more specific, the necessary actions are as follows:

1. if an actual procedure of level i is called, D_i has to be set;
2. if control is returned from a procedure at level i to one at level j , ($j \geq i$), $D_i \dots D_j$ have to be reassigned;
3. if a formal procedure at level i is called from a procedure at level j , $D_i \dots D_k$ have to be reassigned, where k is the level on which the static link emerging from the calling and the called procedures merge. Since k is not known at the time the procedure declaration is compiled, k can be chosen as zero without significant loss in efficiency.

This scheme was used in the implementation of PASCAL 6000. It is described in Reference 4. Registers $B_1 \dots B_5$ are used as the *Display*, B_5 is the origin of the link chains, and B_6 is the pointer to the top of the stack. The compiled instructions are the following:

Procedure call of P :

SX7	L	
EQ	P	
<hr/>		
L	SBj	B5
	SA1	B5]
	SB(j-1)	X1]
	
	SA1	X1]
	SBi	X1]

save return address
and jump

}

update the display, if $j \geq i$

Procedure entry:

P_F	SBi	X1	}	prolog, entry for calls of formal procedures
	SA1	X1]		
	SB(i-1)	X1]		
			
	SA1	X1]		update display
P_A	SA7	B5+2	}	save return address in header
	SX7	B(i-1)		save static link
	SA7	B6		
	SX7	B5		save dynamic link
	SA7	B6+1		
	SBi	B6	new display entry	
	SB5	B6	T	
	SB6	B6+L	top of stack, L = data segment length	

Procedure exit:

SB6	B5	}	reset top of stack
SA1	B5+1		reset T
SB5	X1		
SA1	B5+2		
SB7	X1		fetch return address and jump
JP	B7+0		

Notice that global variables in the main program are assigned absolute addresses. Since $B0 \equiv 0$, they can be considered as based on $B0$.

In the first half of 1971, Prof. C. A. R. Hoare and his collaborators modified and bootstrapped the PASCAL compiler for the ICL 1966 computer [6]. One of the more significant alterations concerned the elimination of the Display, due to the fact that the ICL computer has no set of index registers that are available for a Display, and since the use of a Display was not considered to be an advantage, in this case. During a visit of Prof. Hoare in July 1971, he suggested that maybe even with a register set available for the Display, the benefits gained should be investigated. His suggestion was certainly valid, since variables either global or local to the most recently called procedure could be accessed with the same speed even without a Display. Thus the gain from a Display is limited to faster access of objects at intermediate levels, while the price is the updating at every call regardless of whether such objects are accessed or not. A superficial look at the PASCAL compiler itself showed that accesses to such intermediate level objects were indeed relatively rare, and it was decided to generate a version that would not use a Display (Method II). This version still uses the address register B5 as origin of the link chains (and base address of the most local data segment) and B6 as pointer to the top of the stack. The generated code is:

Procedure call P

```
* X6 := base of environment of P
  SX7   L
  EQ    P
L    ...
```

Procedure entry:

SX0	B5	}	pack and store dynamic link and return address
LX0	18		
BX7	X7+X0		
SB5	B6		
SA7	B5+1		
SB6	B7+L		
* SA6	B5	stack pointer static link	

Procedure exit:

SA1	B5+1	}	fetch and unpack dynamic link and return address
SB6	B5		
SB7	X1		
LX1	42		
SB5	X1		
JP	B7+0		

Fetching an object x at level j from code at level i :

- 1) j = 0: SA1 B0+x
- 2) j = i: SA1 B5+x
- 3) 0 < j < i: SA1 B5
 SA1 X1 repeated i-j-1 times
 SA1 X1+x

A comparison of the codes generated by the two compilers shows that gains and losses of execution speed should be measured, but also those of code length. The shorter codes for procedure entry (2 - $2\frac{1}{4}$ words vs. 4 - 6 words), procedure exit (2 vs. 3 words), and procedure calls (no updating of display) are very attractive, particularly in a compiler where space is more on a premium than time. (It should be noted that the instructions marked with an asterisk can be omitted in the call or the entry code of procedures declared on the first level). Of course it must be kept in mind that the decision about which compiler is to be preferred depends not only on the weighting of space vs. time, but even more on the programs to be processed. But it is obvious that if the

majority of these programs rarely use nested procedure declarations, and often call procedures on the same level, then the compiler without Display is to be preferred. The compiler itself, although featuring nested procedure declarations, but seldom accessing intermediate level variables, belongs to this class. Comparisons of code generated by the two compiler versions produced the following results:

1. The efficiency of codes not using a Display is in the average slightly higher (the compiler itself runs about 1.5% faster).
2. The size of codes not using a Display is smaller (by about 4% measured on 25 sample programs, about 6% in the case of the compiler's code).
3. The compiler program itself is slightly less complex without Display.

This episode where a more sophisticated method was abandoned in favor of a simpler and more direct technique could well be added to the list of D. Knuth's examples of adverse influences of "computer science" on "computer usage" [5]. Their common characteristic is that improved methods are adopted without closer inspection of the nature and direction of the improvement, and without analysis of the circumstances to be improved. An interesting fact is that the Burroughs B5500 computer -- specifically designed for ALGOL implementation -- did contain exactly the two base registers required to efficiently address objects at levels 0 and 1. Unfortunately, addressing of intermediate level objects was impossible due to the software; this deficiency was justifiably criticized. The remedy adopted in the successor B6500 was, however, not a correction of the deficient software, but the inclusion of a full set of high-speed registers to serve as Display.

7. Summary of the Main Trouble Spots of the CDC 6000 Architecture

1. Use of one's complement arithmetic. In order to keep comparisons simple and efficient, the occurrence of negative zeroes must be prohibited. (Note that PL and NG test the sign bit only.) Various optimizations are more cumbersome and less effective, because negative zeroes must be suppressed by additional instructions. Some instructions are themselves unsafe against the generation of -0 !
2. No overflow check on fixed-point arithmetic. This lack is very serious and may cause wrong results in totally unexpected situations. Overflow check by software is prohibitive.
3. No compare instructions. The use of subtraction may cause wrong results, unless expensive precautions are taken.
4. Use of 48-bit multiplier and divider for fixed-point 60-bit numbers without warning of possible "overflow" of operands.
5. Floating-point addition and subtraction without automatic post-normalization.
6. Floating-point arithmetic with rounding of operands instead of rounding postnormalized results.
7. No subroutine jump instruction depositing the program counter P in an operand register, and no return jump loading P from a general operand register. This defect requires the use of 3 instructions each to jump and deposit a return address, and to retrieve it and return, whereas many other computers need only a single instruction for these purposes.

Conclusions

When considering these complaints, the reader should bear in mind that this computer's architecture was conceived in the very early 1960's. The CDC 6600 machine was a very advanced design for a special purpose: fast number crunching. The design relied heavily on the use of several arithmetic units working simultaneously ("in parallel"). Integer arithmetic was considered as almost dispensible, and overflow interrupts as undesirable, because of the impossibility to mirror the present state of the entire machine by a simple program counter and of resuming computation. The use of simultaneously operating units is apparently also made responsible for the otherwise incomprehensible absence of post-normalization, namely because the unit for floaint-point addition does not contain a left-shift circuitry. A few years later, the CDC 6400 (and 6500) computers were announced; they were to have the same instruction set as the 6600, but only one conventional integrated arithmetic-logical unit. Although the "reasons" for the absence of interrupts and post-normalization had vanished, these "features" were retained in the name of compatibility. It was apparently considered most important that pitfall loaded programs could be transported to the new machines at no extra cost. This policy of staying "upward compatible with all previous mistakes" was sternly maintained when the successor to the 6000 series was announced in 1971.

This attitude, which is by no means atypical among computer manufacturers, makes it doubtful whether any progress toward more reliable and more efficient computing will ever be achievable. It does not seem so, until the computer consumers' attitudes will no longer justify the present manufacturers' policies. They, in turn, will not

change before they are made aware of the hidden cost involved in using the present equipment. I am convinced that the cost incurred by the programmers having to discover bugs the hard way by reprogramming repeatedly, and having to reexecute programs many times until they were believed to be correct, is incomparably higher than the reduction in cost due to staying compatible with outdated architectures. The project to develop the PASCAL compiler for the CDC 6000 computer unfortunately provided ample support for this conviction.

Acknowledgments

I am grateful to W. Kahan for pointing out some additional problems with the CDC floating-point arithmetic as well as the method for obtaining correct rounding.

References

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```

005001 { $C+ T1: EXPRESSIONS AND ASSIGNMENTS }
005001   VAR I,J,K: INTEGER;
005004     X,Y,Z: REAL;
005007     N: 0..9999;
005010     P,Q: BOOLEAN;
005012   BEGIN { REAL ARITHMETIC }
005016     X := 1.0; Y := X + 3.14159; Z := X*Y + X/Y;
005105     X := X + (Y + (Z + (1.0 + X)));
005112     X := ABS(+Y); Y := SQR(X); Z := -X;
005117     {$R+ ROUNDED REAL ARITHMETIC}
005117     X := Y + Z; X := Y*Z; X := Y/Z;
005127     { INTEGER ARITHMETIC }
005127     I := 1; J := I + 100; K := I * J; K := I DIV J;
005140     K := (-J) MOD K; J := SQR(J);
005145     I := TRUNC(X); Z := I; X := I/J;
005154     { BOOLEAN ARITHMETIC }
005154     P := TRUE; Q := P ^ ~(Q v P);
005160     P := X = Y; P := I = J; Q := P = Q;
005171     P := X < Y; P := I < J; Q := P < Q;
005200     P := X ≤ Y; P := I ≤ J; Q := P ≤ Q;
005210     Q := ODD(I);
005212     { OPTIMIZATION OF INTEGER ARITHMETIC }
005212     I := I*8 + J*10;
005216     J := I DIV 8 - N DIV 2; K := I MOD 16;
005223     N := I + 100
005223 END .

```

005074	SA7	85+80	005103	SA3	80+005005
	SX7	85+80		FX2	X2/X3
	SA7	80+005000		FX1	X1+X2
005075	SB6	85+000001	005104	NX6	80,X1
	SA1	80+005225		SA6	80+005006
005076	BX6	X1		NO	
	SA6	80+005004	005105	SA1	80+005004
	NO			SA2	80+005005
005077	SA1	80+005004	005106	SA3	80+005006
	SA2	80+005226		SA4	80+005225
005100	FX1	X1+X2	005107	SA5	80+005004
	NX6	80,X1		FX4	X4+X5
	SA6	80+005005		NX4	80,X4
005101	SA1	80+005004	005110	FX3	X3+X4
	SA2	80+005005		NX3	80,X3
005102	FX1	X1*X2		FX2	X2+X3
	SA2	80+005004		NX2	80,X2
	NO		005111	FX1	X1+X2
				NX6	80,X1
				SA6	80+005004

005112	SA1	80+005005
	BX0	X1
	AX0	73
005113	BX6	X0-X1
	SA6	80+005004
	NO	
005114	SA1	80+005004
	FX6	X1*X1
	NO	
005115	SA6	80+005005
	SA1	80+005004
005116	BX0	X0-X0
	IX6	X0-X1
	SA6	80+005006
005117	SA1	80+005005
	SA2	80+005006
005120	FX0	X1+X2
	NX0	80,X0
	DX1	X1+X2
	RX1	X0+X1
005121	NX6	80,X1
	SA6	80+005004
	NO	
005122	SA1	80+005005
	SA2	80+005006
005123	FX0	X1*X2
	DX1	X1*X2
	RX6	X0+X1
	NO	
005124	SA6	80+005004
	SA1	80+005005
005125	SA2	80+005006
	RX6	X1/X2
	NO	
005126	SA6	80+005004
	SX6	80+000001
005127	SA6	80+005001

005130	SA1	80+005001
	SX0	80+000144
	IX6	X1+X0
	NO	
005131	SA6	80+005002
	SA1	80+005001
005132	SA2	80+005002
	DX1	X1*X2
	BX0	X0-X0
005133	IX6	X1+X0
	SA6	80+005003
	NO	
005134	SA1	80+005001
	SA2	80+005002
005135	PX2	80,X2
	NX2	80,X2
	PX1	80,X1
	FX1	X1/X2
005136	UX1	87,X1
	LX1	87,X1
	BX0	X0-X0
	IX6	X1+X0
005137	SA6	80+005003
	SA1	80+005002
005140	BX0	X0-X0
	IX1	X0-X1
	SA2	80+005003
005141	PX6	80,X2
	NX6	80,X6
	PX0	80,X1
	FX6	X0/XE
005142	UX6	87,X6
	LX6	87,X6
	DX6	X2*X6
	IX6	X1-X6
005143	SA6	80+005003
	SA1	80+005002
005144	DX6	X1*X1
	SA6	80+005002
	NO	

005145	SA1	80+005004	005163	NO
	UX1	87,X1		SA6 <u>80+005010</u>
	LX1	87,X1		SA1 <u>80+005001</u>
005146	BX0	X0-X0	005164	SA2 <u>80+005002</u>
	IX6	X1+X0		IX0 <u>X1-X2</u>
	SA6	<u>80+005001</u>		MX6 <u>00</u>
005147	SA1	80+005001	005165	NZ <u>X0,005166</u>
	BX6	X1		SX6 <u>80+000001</u>
	PX6	80,X6	005166	SA6 <u>80+005010</u>
005150	NX6	80,X6		SA1 <u>80+005010</u>
	SA6	<u>80+005006</u>	005167	SA2 <u>80+005011</u>
	NO			BX1 <u>X1-X2</u>
005151	SA1	80+005001		MX0 <u>73</u>
	SA2	<u>80+005002</u>	005170	BX6 <u>~X1-X0</u>
005152	PX2	80,X2		SA6 <u>80+005011</u>
	NX2	80,X2		NO
	PX1	80,X1	005171	SA1 <u>80+005004</u>
	NX1	80,X1		SA2 <u>80+005005</u>
005153	RX6	X1/X2	005172	FX1 <u>X1-X2</u>
	SA6	<u>80+005004</u>		MX0 <u>01</u>
	NO			BX6 <u>X0^X1</u>
005154	SX6	<u>80+000001</u>		LX6 <u>01</u>
	SA6	<u>80+005010</u>	005173	SA6 <u>80+005010</u>
005155	SA1	<u>80+005010</u>		SA1 <u>80+005001</u>
	SA2	<u>80+005011</u>	005174	SA2 <u>80+005002</u>
005156	SA3	<u>80+005010</u>		IX1 <u>X1-X2</u>
	BX2	X2^X3		MX0 <u>01</u>
	MX0	73	005175	BX6 <u>X0^X1</u>
005157	BX2	~X2-X0		LX6 <u>01</u>
	BX6	X1^X2		SA6 <u>80+005010</u>
	SA6	<u>80+005011</u>	005176	SA1 <u>80+005010</u>
005160	SA1	<u>80+005004</u>		SA2 <u>80+005011</u>
	SA2	<u>80+005005</u>	005177	BX6 <u>~X1^X2</u>
005161	IX0	X1-X2		SA6 <u>80+005011</u>
	MX6	00		NO
	NZ	X0,005163		
005162	SX6	<u>80+000001</u>		
	NO			

005200	SA1	80+005004	005216	SA1	80+005001
	SA2	80+005005		AX1	03
005201	FX1	X2-X1		BX0	X0-X0
	MX0	01		IX1	X1+X0
	BX6	~X1^X0		NO	
	LX6	01	005217	SA2	80+005007
005202	SA6	80+005010		AX2	01
	SA1	80+005001		IX6	X1-X2
005203	SA2	80+005002	005220	SA6	80+005002
	IX1	X2-X1		SA1	80+005001
	MX0	01	005221	BX0	X1
005204	BX6	~X1^X0		AX0	04
	LX6	01		LX0	04
	SA6	80+005010		IX6	X1-X0
005205	SA1	80+005010	005222	SA6	80+005003
	SA2	80+005011		SA1	80+005001
005206	BX1	~X2^X1	005223	SX6	X1+000144
	MX0	73		SA6	80+005007
	BX6	~X1-X0	005224	SA1	85+80
	NO			SB7	X1+80
005207	SA6	80+005011		JP	87+000000
	SA1	80+005001	005225	17204000000 000000000	
005210	BX0	X1	005226	17216220771740156064	
	LX0	73			
	BX1	X1-X0			
	MX0	01			
005211	BX6	X0^X1			
	LX6	01			
	SA6	80+005011			
005212	SA1	80+005001			
	LX1	03			
	NO				
005213	SA2	80+005002			
	LX2	01			
	BX0	X2			
005214	LX2	02			
	IX2	X2+X0			
	IX6	X1+X2			
	NO				
005215	SA6	80+005001			