SCHEDULING IN JOB-SHOP TYPE PRODUCTION

by

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February 1957

ABSTRACT

The Accessories Division of Thompson Products, Inc., is manufacturing hundreds of different types of assemblies including a variety of fuel pumps. The bulk of these products are manufactured in appropriate lot sizes to meet shipping commitments. As a part of a broad assignment to determine what role electronic data processors should play at Thompson Products, a study to improve this job-shop type production was initiated.

The study began with a survey of the existing production control system, then the development of a mathematical model for a job-shop type production scheduling system followed. This mathematical model allows the computation of start and completion dates for each operation on each part. It also establishes purchasing, labor, machine, and inventory requirements.

Accomplishments as of today, include (1) a new scheduling system utilizing the concept of manufacturing bands has been installed, (2) a parts classification system has been established which balances the cost of inventory carrying against the cost of data processing, and (3) a feasibility study for the utilization of electronic computers has been completed, and the most suitable and economically attractive equipment for installation selected.

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During the last few years, considerable effort has been expended in developing new scientific methods for the scheduling of manufacturing operations. This effort is well warranted, since the productivity of industrial organizations is intimately connected with the ability of scheduling production efficiently. In spite of the fact that a great deal of effort has been expended and important progress has been made in this field, it is still fair to say that no general theory of production scheduling has been developed so far. Most of the work applies to a restricted area of scheduling and does not have universal applicability. The paper we are to describe here is of the same nature. It has been successful in a particular type of production operation, and it is believed to have wide applicability. It does not have universal applicability, and it does not present a general theory of production scheduling.

The scheduling problem of the Accessories Division of Thompson Products, Inc., is a problem in scheduling of a job-lot type of production. There are hundreds of different types of articles manufactured and kept in inventory, and the principal problem is to determine what articles should be manufactured at what times, in what quantities, and on what machines. The problem is further complicated by the fact that many months of lead times must be allowed and sales requirements frequently change.

As the profit position of this industrial organization is seriously effected by the ability to schedule efficiently, it was natural to initiate a search for a better scheduling system. In particular, utilization of large-scale electronic computers, and the introduction of new scientific principles of scheduling were envisioned as leading to improvements.

In order to describe the work that has been conducted, one could take two alternative approaches. One could describe in detail the work as it has been conducted at Thompson Products, or one could present results in a generalized fashion. The first approach has the advantage that it can be very factual and specific. However, the second approach of describing the method in a generalized form has the advantage that it implicitly suggests the applicability to other production control problems. This is the reason that we select this second approach and describe the scheduling system developed in terms of the job-shop type scheduling problem faced by a hypothetical firm.

We begin this paper by a generalized statement of the problem of scheduling in job-shop type production. Then we describe the decision rules that have been developed to solve the problem of scheduling and, in particular, we describe the Time Assignment Scheduling System that has been introduced in Thompson Products. Finally, we briefly describe our current plans for the development of an optimum scheduling system based on the applications of the Monte Carlo method.

Statement of the Problem

A company is engaged in manufacturing hundreds of different types of assemblies. A highly simplified gozinto graph for one of these assemblies is shown in Figure 1.* For instance, Figure 1 shows that the particular finished product A_3 is assembled from four different articles, A_5, A_4, A_7, and A_{11}. Some of these articles are assemblies themselves, others are fabricated parts.

The articles are manufactured against sales orders that are considered firm. Some of the articles (e.g., finished articles) are manufactured on assembly lines; other articles are manufactured either on assembly lines or in a job-shop type operation. The articles are manufactured in various lot sizes. (There are only a few articles that are manufactured on a continuous line-flow type of operation, most of the articles are manufactured in lots, in a cyclic fashion.)

At the time we started our work, there was a wide-spread desire for improvements in production control. It was felt that parts were not manufactured in best quantities and not at the best times. There was a feeling --as so often is the case--that it should be possible to do better. Planning techniques were inadequate to predict machine load bottlenecks. Confidence in machine-prepared production control reports was limited, and decisions based on these reports were questioned. There was a need to make the production control reports accurate and timely, but more important there was a need for improved, explicit decision rules.

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*For a detailed description of the concept of gozinto graphs, see A. Vazsonyi, "The Use of Mathematics in Production and Inventory Control," Management Science 1, pp 70-85, 207-223 (1954-1955).

In order to appreciate the complexity of the production problem, let us consider Figure 2 where a historical record of a hypothetical production situation is described. The horizontal axis describes time in manufacturing days and hours. We distinguish between four labor classes, each of them referring to a particular machine or machine group. Ω^3_1, $\Omega^3_{1,2}$, $\Omega^3_{1,3}$ describes three successive operations of the third lot of A_1. It can be seen that the first operation on this particular lot is to be performed on the first group of machines. After this operation is completed, the semi-finished lot is transported to the third group of machines where the second operation is performed. Finally, the lot is transported to the second group of machines, where the third (or final) operation is performed. This means that at that point the third lot of A_1 is completed. Similarly, one can follow the production of the first lot of A_3. $\Omega^1_{3,1}$ describes the first operation on this lot; this is performed on the fourth group of machines. The second operation on this lot should be performed on machine group 3; however, because of interference, that is, because of the fact that this machine is busy manufacturing A_1 and A_2, we have to wait before the second operation $\Omega^1_{3,2}$ can be performed. $\Omega^1_{3,3}$ shows the third or final operation on this lot.

In many manufacturing firms it is customary to prepare charts in advance, similar to the one shown in Figure 2, and use them for planning purposes. One begins by setting up the shipping dates and then works "backwards." A complete chart for the future is prepared. Every time a certain operation is completed, the foreman consults the charts and determines what particular lot should be worked upon. If the lot is not available, he knows that the lot is late and he takes corrective measures.

In the specific case under discussion, it was considered impractical to follow this sort of planning scheme. There are hundreds of shippable items involved and thousands of parts manufactured and purchased. The possible number of combinations in this "jig-saw puzzle" are astronomical. Preparation of even a single chart of this type would be an exceedingly time-consuming job, but even then there would be no assurance that the first try is an efficient one. There is, however, a further difficulty.

Suppose we are looking at the plan this morning and trying to determine what operation we should perform. We observe that up to today we were on schedule and we have performed the three indicated operations according to our original chart shown in Figure 2. We recognize at this point that the second lot of A_2 has not arrived yet, and therefore operation $\Omega^2_{2,1}$ cannot be performed. Therefore, we perform (Figure 3) the second operation on the first lot of A_3, that is $\Omega^1_{3,2}$. We also perform operation $\Omega^3_{1,3}$ according to the old plan. After $\Omega^1_{3,2}$ is completed, the second lot of A_2 arrives and, consequently, the first operation $\Omega^2_{2,1}$ can be performed on machine 3. At this point, we recognize that our original chart is of no particular value, because things have changed so much that we have to develop a new chart. The event that a certain lot was not available is an "upset." The fact of the matter is that there are dozens of these "upsets" every day; machines break down, tools are not available, labor is not available, etc. This means, then, that quite often (possibly every day) a completely new chart would have to be developed. This is, in our particular case, an impossible job.

Furthermore, not only is it impractical to carry out the computations, but there is a conceptual difficulty. What is the point of carrying out this very elaborate scheduling computation, say for three months in advance, when we know that every day we will have to rework the whole schedule completely? Suppose every morning we could determine by some magic the "optimum" schedule. In what sense could such a plan be optimum if we have to change it tomorrow?

Earlier, we found it useful to look upon this problem in a different way.* Figure 4 shows the four production machines of our hypothetical problem. The available semifinished lots are conceived as forming a waiting line in front of each of the machines. $L^4_{3,2}$ refers to the fourth lot of A_3. The last index 2 denotes that the second operation was already performed. We placed this semifinished lot $L^4_{3,2}$ in front of machine 1, as the third operation on the lot is to be performed on machine 1. As shown in Figure 4, there are three lots waiting to get on the first machine, four lots to get on the second one, none for the third machine, and there are two lots waiting for the fourth machine. We assume that there is certain work being performed on each of the machines, with the possible exception of the third machine. Suppose now, that one of the lots that is being manufactured on one of the machines is completed. The foreman is faced with a decision problem: which of the semifinished lots waiting for his machine should he put on the machine?

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*A. Vazsonyi, "Production Control from the Point of View of Decision Theory," (Abstract), Management Science 1, pp 190, (January 1955).

We formulate our problem in production control, then, as the problem of developing decision rules that will instruct or aid the foreman in what to do.

Let us speculate for a moment on the types of decision rules one could conceivably have. Suppose there is a master priority list of all the parts to be manufactured, and the foreman is instructed to take the lot that has the highest priority on the list. This would certainly be a possible decision rule.

The trouble with this decision rule is that parts that are low on priority list would be pushed back more and more, and perhaps never would be completed. Subassemblies would not be available and the manufacture of some assemblies would stop. As new orders came into the shop, some of these lots would be manufactured first, and production would get out of balance. Very likely the whole production system would collapse.

To speculate more, let us assume that the foreman would have an instrument, like a roulette wheel, to determine by chance what part should be manufactured. This situation might lead to somewhat better production than the previous method, because as time goes on every lot would be eventually manufactured. One would expect, however, that inventories would grow very large.

We can readily see what is wrong with either of these decision rules: the shipping schedule is not taken into account as far as the sequence of manufacturing is concerned. Presumably, raw materials and production orders would be released in "accordance" with the shipping schedule, but that is all that relates to the shipping schedule. What we need is a decision rule that somehow takes into account the shipping schedule.

Development of Decision Rules

In fact, the people in production control did have some decision rules of this type. They had some sort of a "feel" that told them just how long it takes to get certain parts through the shop. The difficulty, however, was that this "feel" was quite uncertain, and it varied from one person to another. In our terminology today, we would say that the decision rules were not explicitly formulated and stabilized. What was needed first was not so much the development of optimum scheduling procedures as a stabilization of the unwritten decision rules.

The first step in the stabilization of the decision rules can be described with the aid of Figure 5. The horizontal scale is time, the vertical scale is the cumulative number of articles manufactured. On the right-hand side there is a line that denotes the shipping schedule, which tells how many articles should be shipped at what date. To the left of the shipping schedules shaded areas are shown, to be referred to as "manufacturing bands." Each of these bands refers to a particular article to be manufactured. The right-hand side edge of the band, labeled "outdate," refers to the cumulative number of articles that must be finished, the left-hand side, labeled "indate," refers to the cumulative number of articles that must be started. The actual manufacturing of the various lots of this particular article is conceived to be accomplished within the manufacturing band somehow, as shown in Figure 6. Indates and outdates for each lot can be computed, if the manufacturing bands are known. These indates and outdates will serve then as guides for the foreman to make decisions. Let us recognize, though that the concept of manufacturing bands somehow implies that there is a fixed "flow time" for each article.

These ideas so far are somewhat vague, and many questions can be asked. How seriously should the foreman take these indates and outdates? How should the width of the manufacturing band be determined? Does this whole scheme make sense?

In order to answer these questions, let us make our proposition more precise. Let us assume that there is a fixed width for each manufacturing band, and that these widths, to be called "make-spans" can be represented by a setback chart as shown in Figure 7. Suppose we set up the hypothesis that the plant indeed has been operating according to a scheme of this sort. Is there any way to verify this hypothesis?

We could examine the records of the company and study the history of each individual lot. Then we could make a statistical study and determine whether there is statistical significance to substantiate our hypothesis. Remember, however, that there are thousands of types of parts involved, and that such a statistical study, therefore, would require a great deal of data. In our particular corporation, such data were not available and we, therefore, decided to set up a simpler hypothesis.

We made the hypothesis that the make-span of each article depends only on the number of operations involved and on the total standard time required to manufacture the particular lot. A multiple regression analysis showed that the make-span in fact does not depend on the standard times, but correlates well with the number of operations. Figure 8 gives a hypothetical scatter diagram showing how the make-span is related to the number of operations.

On the basis of this correlation analysis, we have decided that it does make good sense to use these make-spans. We prepared a list of make-spans for each of the parts; we prepared set-back charts such as Figure 7, and we proceeded to install this production control system.

It will be of some interest here to describe some of the mathematical details of the scheduling procedure.

We denote by $\sigma_{i,k}$ the set-back of A_i in A_k. For instance, $\sigma_{8,3}$ is 90 days in Figure 7. We denote by s_i^m the number of articles A_i that must be shipped in the m'th production period. Finally, we denote by x_i^m the number of articles A_i that must be manufactured in the m'th production period to meet this shipping schedule. Then,

$$x_i^m = \sum_k T_{i,k} s_k^{m+\sigma_{i,k}} \qquad (1)$$

where $T_{i,k}$ denotes the total number of articles A_i required for the shippable item A_k. (In the particular example we had here, this number would be 1 or 0, depending on whether the particular article is required in the shippable item or not.)*

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*A more detailed description is given by H. T. Larson and A. Vazsonyi, "Data Processor Requirements in Production and Inventory Control," Proceedings of the Western Joint Computer Conference, Los Angeles, (March 1955).

In conjunction with our method of scheduling, we also developed a method of labor forecasting. One can readily see that once the make-span for every article is postulated there is a unique relation to predict labor loads. The total labor hours required in production period m on machine type n is given by

$$h_n^m = \sum_i \tau_{n,i} x_i^m, \tag{2}$$

where $\tau_{n,i}$ is the hours required to manufacture A_i on machine n, and h_n^m is the total hours required on machine n in period m.

The installation of this scheduling system required a great deal of systems and procedures work, the description of which lies beyond the purpose of this presentation. However, after the installation of this system it was recognized that in spite of the great improvements realized, and the large economic saving effected, there are still further improvements possible. This prompted us to continue our investigation and refine the scheduling system to a higher degree of precision.

Time Assignment Scheduling System

The scheduling system we have described so far, and which has been installed, specifies the start and completion dates for each lot of each part. This serves as an important guide but does not completely specify when each operation is to be performed. The system relies on a knowledge on the part of production personnel to fill in this gap and, in particular, utilizes decisions that must be based on judgment. In order to understand the problem better, let us consider now in detail how individual operations on each lot are to be performed.

In Figure 9, we show a detailed production plan for a particular lot. The two heavy dots represent start and completion dates. Each cross on the diagram represents a particular operation. For instance, it can be seen that the first operation takes two days. The second operation is performed on the third day, and the third operation is performed on the third day too. The fourth operation takes three days, as shown in Figure 9 by the three crosses. The rest of the diagram shows the schedule of the rest of the operations. In the scheduling system previously described, only the two heavy dots were specified and the production personnel, so to speak, maneuvered between these two end points. Now, we wish to specify in detail when each operation is to be performed.

In order to carry out this detailed scheduling system, it was necessary to develop rules which specify what type of production lot is allowed one day, two days, or three days, etc., of manufacturing time. In order to develop these rules, it was necessary to know the standard times required to manufacture a lot, and the set-up time required to get the machine ready. The time required for a particular lot to be transported from one machine to another is of great importance here too. We called this time the "transit time" and developed some general rules of what these transit times should be.

In addition, it was necessary to study the availability of manpower and many other factors that are important in production. On the basis of this investigation, we developed a classification system which specifies the elapsed time that must be allowed for each particular operation.

Starting with this classification system, it is possible to develop the type of schedules shown in Figure 9, and therefore, it is possible to specify start and completion dates for each lot and each operation.

The implementation of this Time Assignment Scheduling System required profound changes in the method of production control. To specify with this degree of detail, the schedule of each operation requires an intimate knowledge of all events occurring on the production floor. Consequently, it did not seem advisable to attempt to install this new scheduling system in the entire Division. It was thought better to design first a pilot installation. A relatively simple commodity was selected for the pilot installation, as it was felt that this smaller system could be monitored with relative ease, and without the expenditure of a great deal of manpower. However, care was taken that the commodity selected possessed all the significant features of the problem, so that the pilot installation could be considered as a true representation of the entire scheduling problem.

This pilot installation was only recently completed. We found from actual operating experience that our scheduling system was sound, and that only minor modifications were necessary in order to extend the work throughout the Accessories Division of Thompson Products. Currently, this extension is underway and it is too early yet to report on the outcome of the installation. However, on the basis of the pilot installation, it is estimated that direct labor expenses will decrease by 22% and in-process inventories will be reduced by 15%.

It is believed that the scheduling system described here, represents a very significant improvement in the operations of this firm. However, we do not believe in any way that this scheduling system is an optimum one. The development of optimum scheduling systems is an extremely difficult problem, and here we can report only on our plans. We are considering simulating the production scheduling problem with the Monte Carlo Method, on a large scale electronic digital computer. As our ideas are preliminary here, it will be perhaps best to describe the proposed approach through an example.

Simulation of Scheduling

Consider a very simple production scheduling problem where four parts, P, Q, R, and S, are to be manufactured on machines α, β, γ, and δ. The production requirements are shown in Figure 10. It can be seen, for instance, that 100 of assembly P is required by the end of the sixth week, 80 by the end of the ninth week, 120 by the end of the twelfth week, and 85 by the end of the fifteenth week. Assembly P requires as parts, Q, R, and S. As each assembly P requires two of Q, we have to have 200 Q's available when the first lot of assembly P is to be started. This means, as shown in the diagram, that 200 of part Q must be completed by the end of the third week. The schedule shown in Figure 10 can be interpreted in a similar fashion for the other parts.

The question arises whether this schedule is compatible with the machines available. In Figure 11, we show a schedule by two-hour intervals, which describes the production of all these assemblies and parts. The representation in Figure 11 is twofold. The lines headed by P^1, Q^1, R^1, S^1 show the history of the first lot of each part; the lines headed by α, β, γ, and δ show the history of each machine. (The two representations are somewhat redundant and must be in agreement.) We begin by producing the first lot of Q, i.e., Q^1. Figure 11 shows that Q^1 goes on machine β on day 1. Each symbol in Figure 11 denotes two hours of production, and so it can be seen that the first operation on lot Q^1 takes four hours. During the third quarter of day 1, Q^1 is in-transit to machine α. This is designated by the letter T. Then Q^1 goes on machine α for a six-hour period. As a comparison we see in the line headed α that Q^1 is, indeed, manufactured on machine α, as indicated by the letter Q^1. Then Q^1 is in-transit for two hours to go on machine γ where it stays for four hours. Then Q^1 is in-transit again, and becomes available as designated by the letter A. At the beginning of day 4, Q^1 is available for assembly. Manufacturing of P^1 on machine δ begins in the second quarter of day 4. This can be seen in the line headed by δ where the production of P^1 is listed. We also observe that according to the schedule we should have started P^1 on the fourth day, but we could not start it, since subassembly R^1 was in-transit. We list in the line headed by P^1 the letter D which shows that the start of P^1 is delayed.

The diagram in Figure 11 shows, then, a detailed description of the events that occur during the production of these parts. (The chart is to be continued for the subsequent lots of parts.) In order to prepare charts like Figure 11, it is necessary to consider all the important elements that enter into the production problem. For instance, we note that machine γ on day 5, during the first and second quarter, is under maintenance as denoted by the letter M. There are many other factors that must be included in a realistic situation, but here for the sake of simplicity, these other factors are omitted.

In order to carry out, in a systematic fashion, the preparation of such schedules, it is necessary to have decision rules which specify what to do in each instant. In particular, when different parts compete for the same machine, there is a need for a decision rule to indicate which part should go on the machine first. The simplest decision rule is to take the part first which arrives first. More elaborate decision rules which relate more intimately the production schedule to the delivery requirements have been suggested by various authors.*

In addition to the specification of these decision rules, it is necessary to know the statistics of each of the factors that enter into production. The statistical distribution of the time it takes to manufacture a part, is one of these factors. The statistical distribution of set-up time is necessary, too. We need to know the statistics of the time it takes for a part to be transported from one machine to another. Information on maintenance is required. However, when all this statistical information is available, and the decision rules are specified, then the scheduling process becomes automatic and can be performed with the aid of an electronic computer.

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*A. Vazsonyi, "Operations Research in Production Control--A Progress Report," Operations Research, Volume 4, No. 1, February 1956, pp 19-31.
R. T. Nelson, "Priority Function Methods for Job-Lot Scheduling," Management Sciences Research Project, Discussion Paper No. 51, University of California, Los Angeles, February 24, 1955.

We illustrate, tentatively, in Figure 12 by a block diagram of how the simulation process could be performed. The basic input is the shipping schedule as shown on the left-hand side. The next block shows the set-back structure that is to be employed in the computations. Then we need to store in the memory of the computer the operations sheets which describe on what machines the particular part is to be manufactured, and in what sequence. We need to store information on maintenance statistics. The particular Time Assignment Scheduling System to be employed, must be stored in the memory. We need to specify the particular decision rule to be employed, and finally we need to generate random numbers. This is the information from which the computer can prepare simulated production plans of the type shown in Figure 11. From each simulated production plan we obtain information about the effectiveness of the particular scheduling system employed. It is recognized here, that no single measure of effectiveness is available yet to evaluate the performance of a schedule. In Figure 12 we show three important measures that may be used. One measure is lateness or earliness of delivery. Another one is the inventory level, and the third one is machine utilization.

Let us inject here that it may be too early to say what these measures of effectiveness should be; the important thing is to recognize that whatever measure is accepted, this measure can be computed once simulated production plans are developed.

In summary, then, the simulation process runs as follows. A particular set-back structure is predicated and then many Monte Carlo runs are made with different shipping requirements. With the aid of specified measures of effectiveness, the particular system of scheduling is evaluated. Then, proposed improvements in the scheduling system are introduced, new runs on the computer are made, and these proposed improvements are evaluated.

One of the important problems we plan to study, is the problem of shortening lead-times. Instead of using the empirical set-back rules, (Figure 8), we plan to experiment with various proposals for shortening the lead times. With the aid of simulation, we will evaluate the validity of these new set-back structures, and will determine whether these set-back structures are practical in actual production.

In summary, then, we can say that our point of view of looking at the problem of production control as a problem in decision theory, has been fruitful. We have developed, installed, and tested, certain decision rules in a rather complex situation. Our study started with the purpose of introducing high speed electronic data processors, but a great deal of work had to be done before it could be specified which electronic computer should be employed. This is not surprising, as one can readily see that the decision processes of production must be clearly developed before computers can effectively be applied. We have learned that by combining electronic computers with thorough system studies, very significant improvements can be realized. There is every reason to believe that continuation of this work will lead to further important benefits to Thompson Products, Inc.

GOZINTO GRAPH

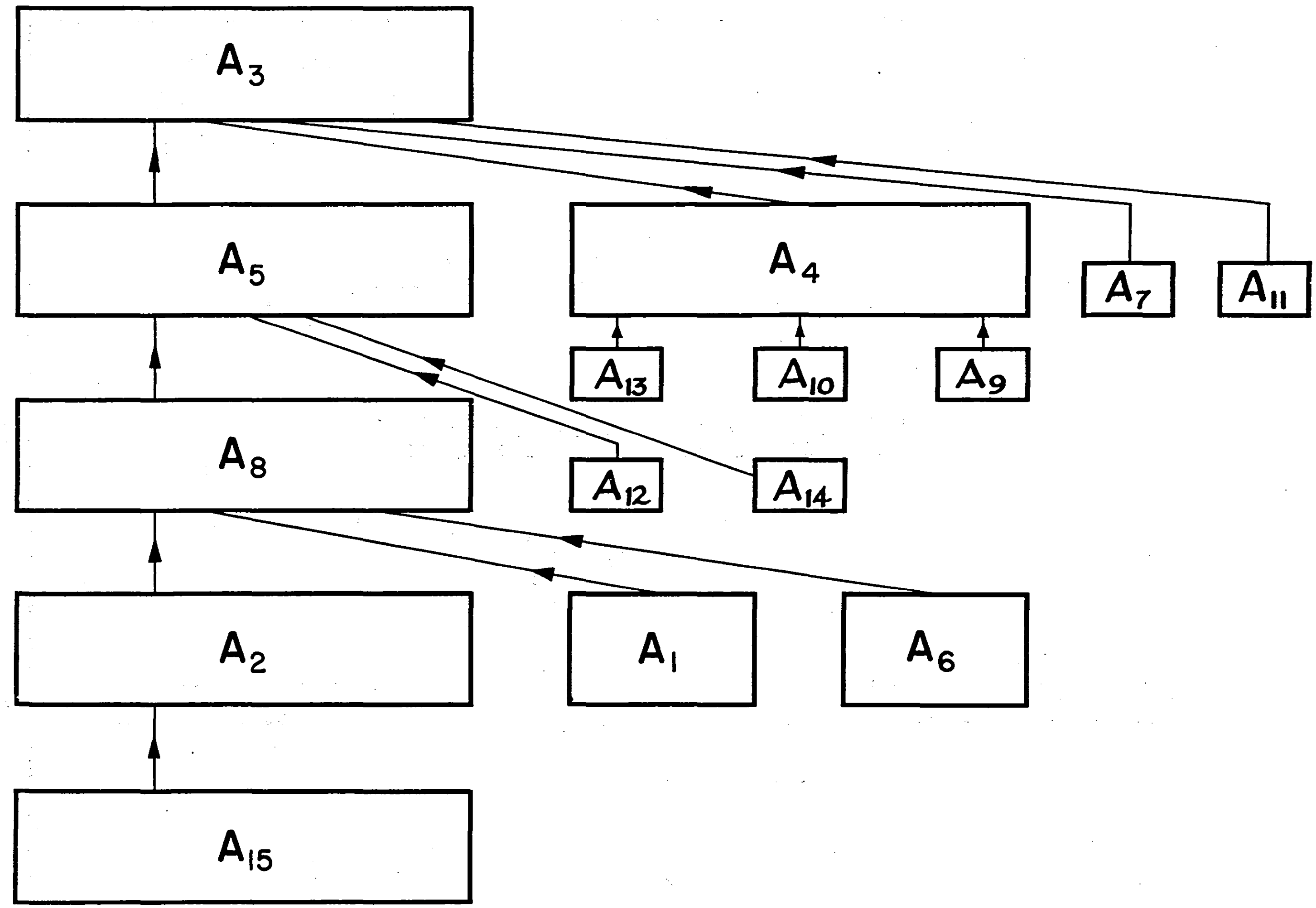


FIG. 1

SHOP LOADING

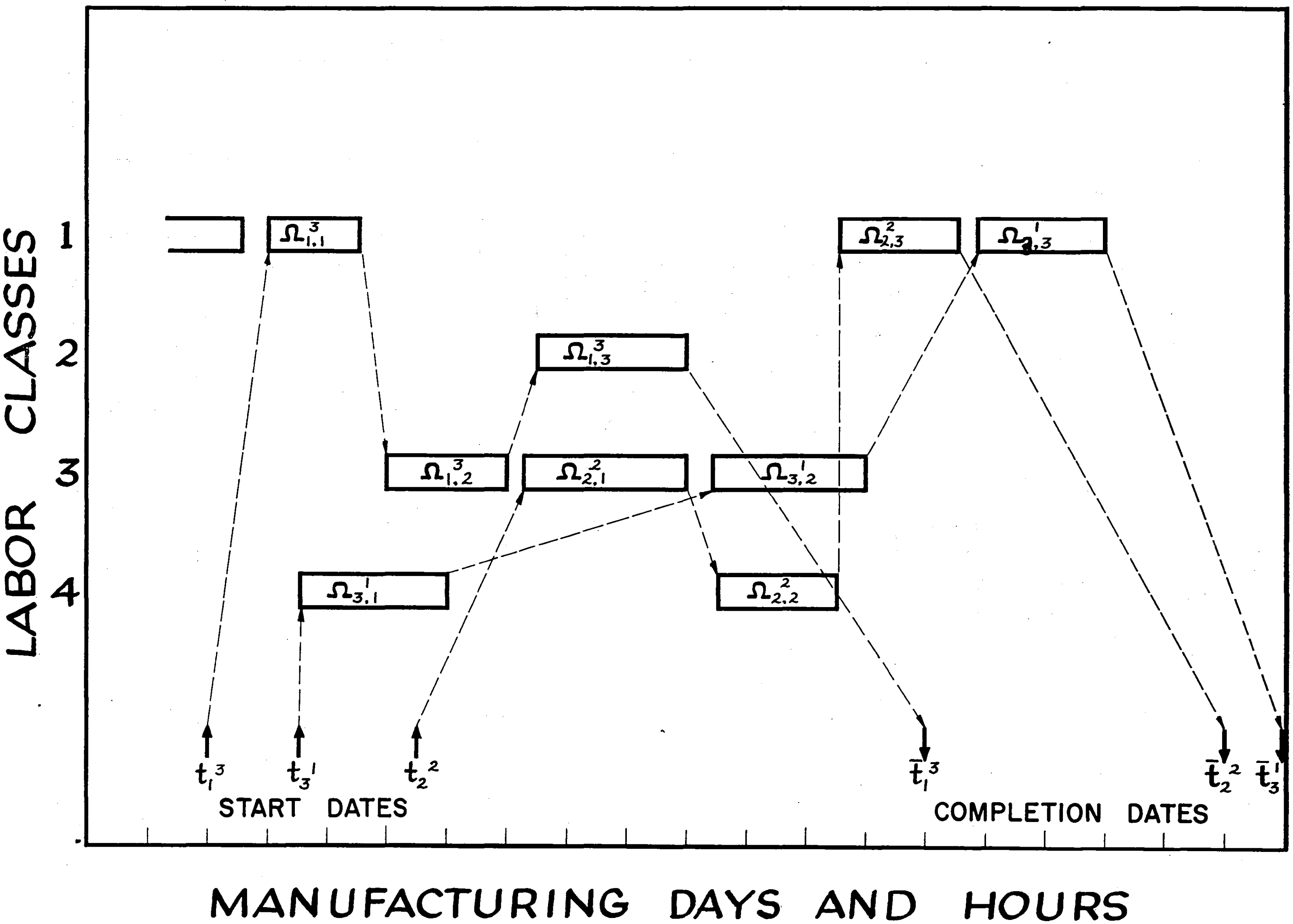
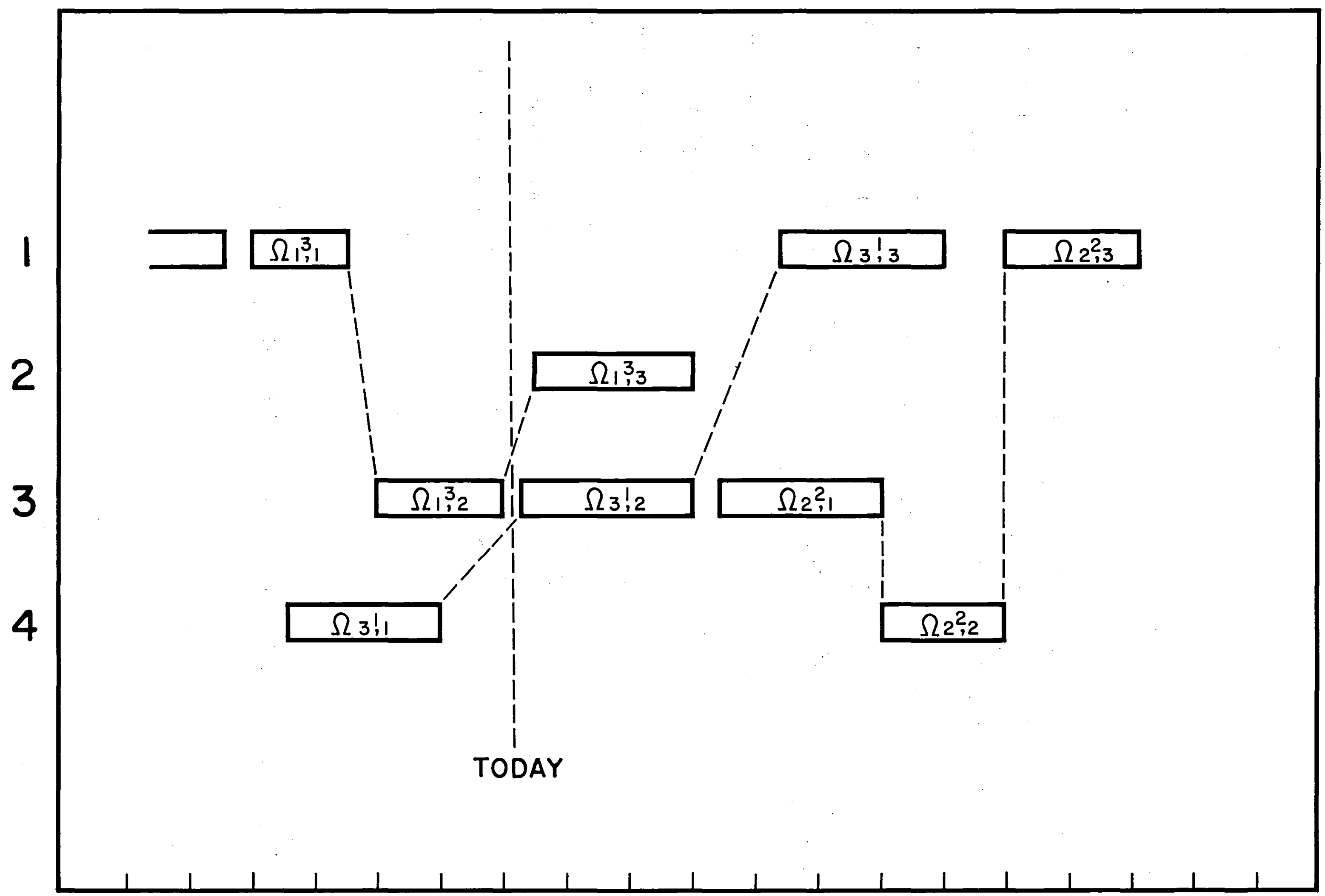


FIG. 2

SHOP LOADING



MODIFICATION OF SHOP LOADING

FIG. 3

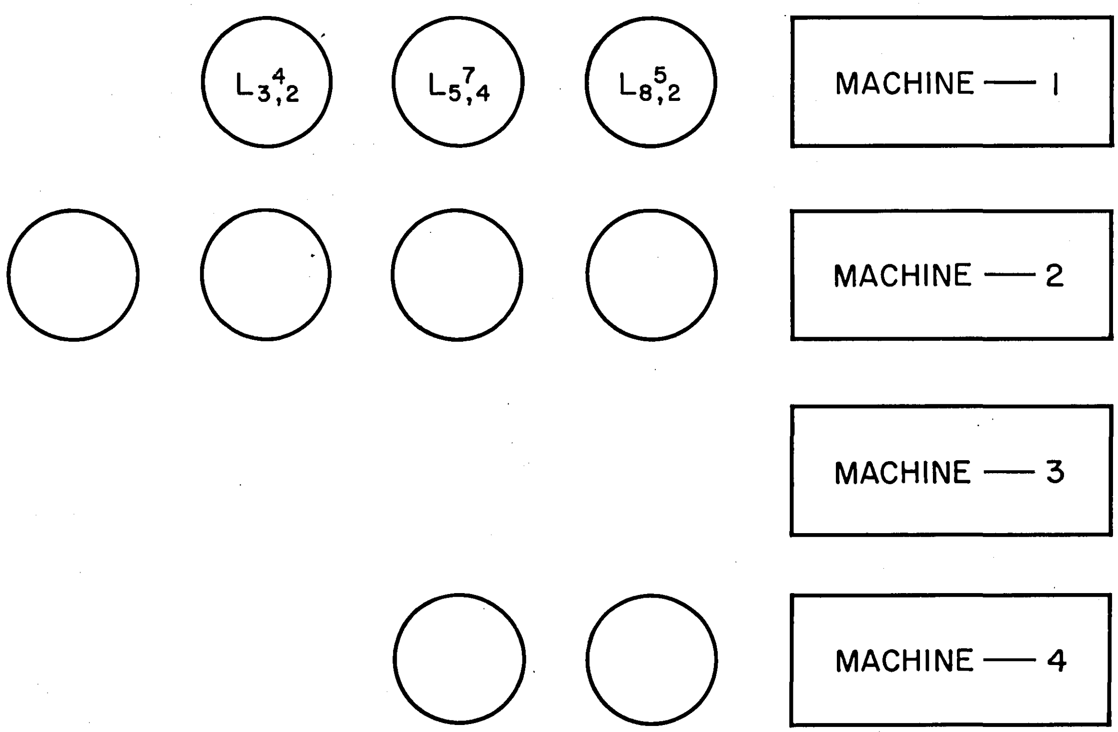


FIG.4 SHOP LOADING AS A PROBLEM IN WAITING LINES.

MANUFACTURING BANDS

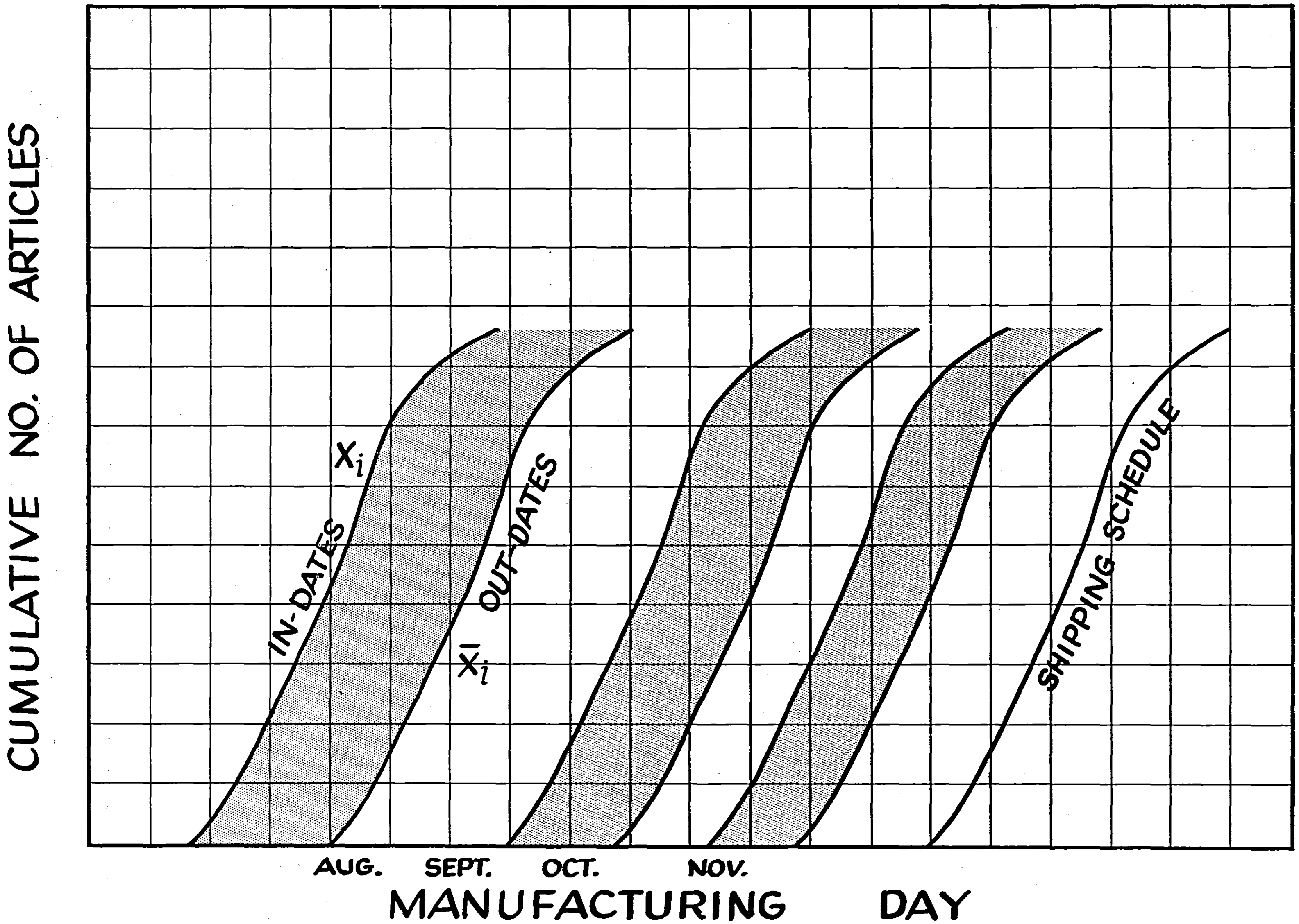


FIG. 5

MANUFACTURING BANDS

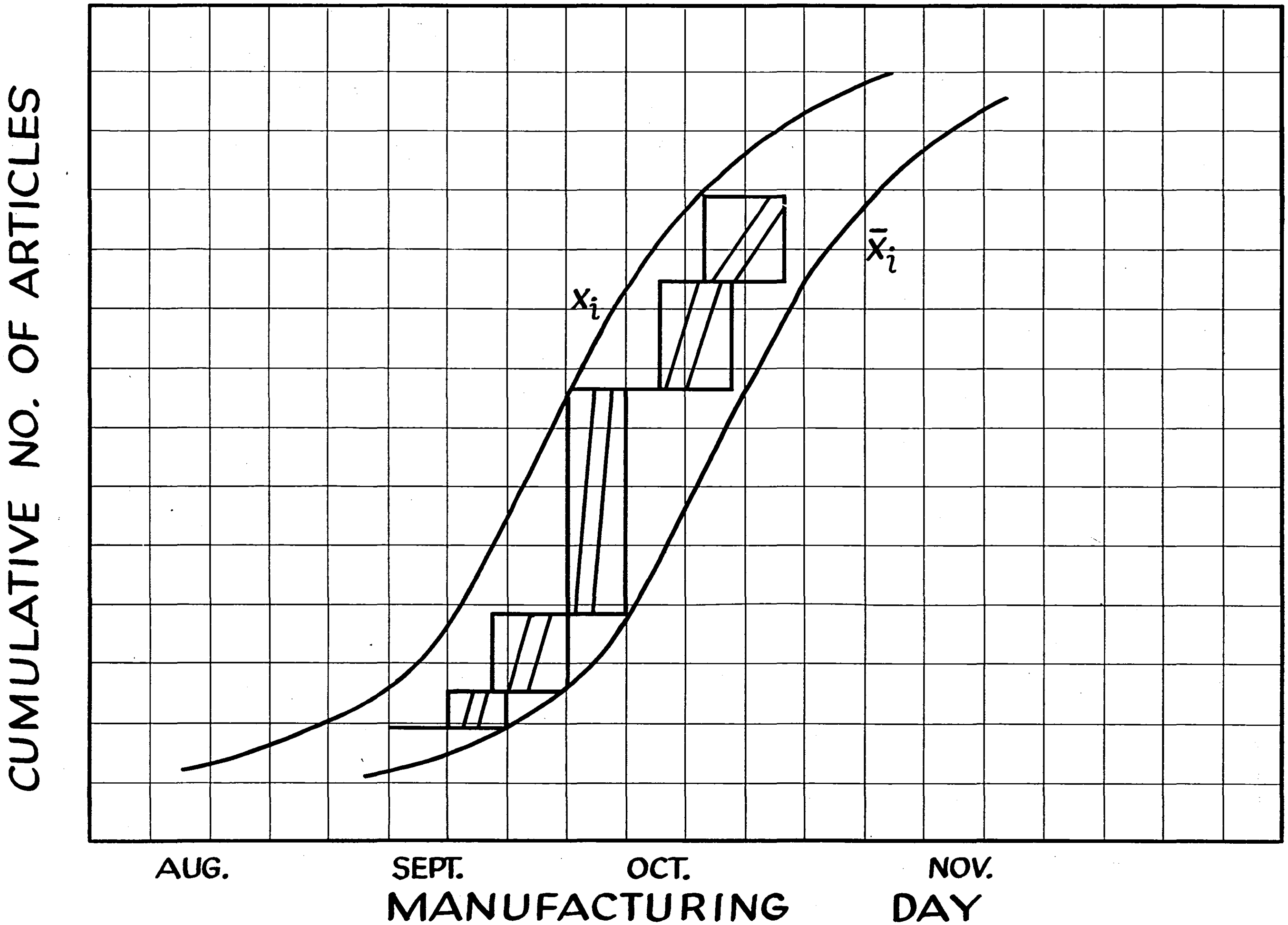


FIG. 6

SETBACK CHART

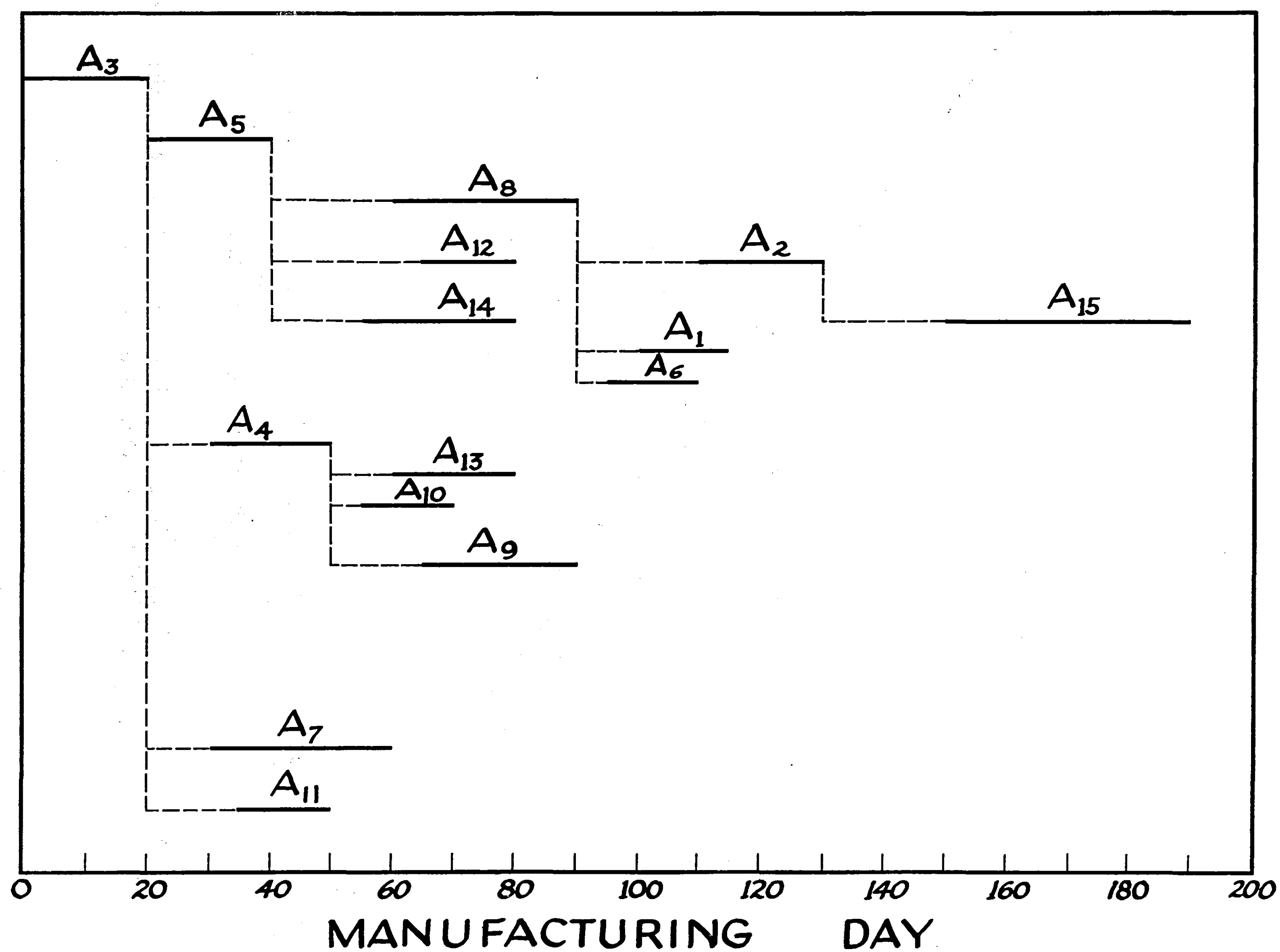


FIG. 7

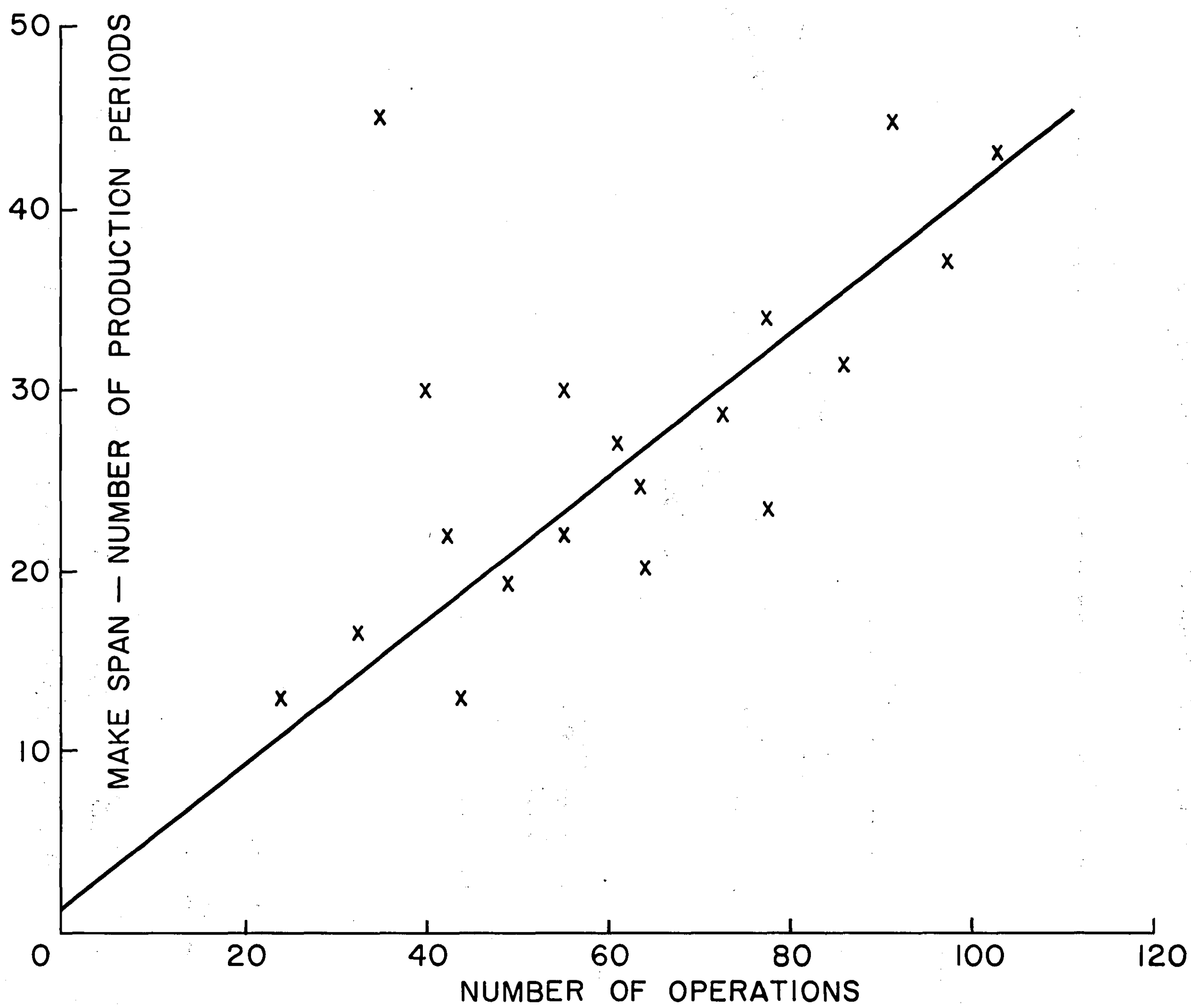


FIG. 8 STATISTICAL RELATIONSHIP BETWEEN NUMBER OF OPERATIONS AND MAKE SPAN.

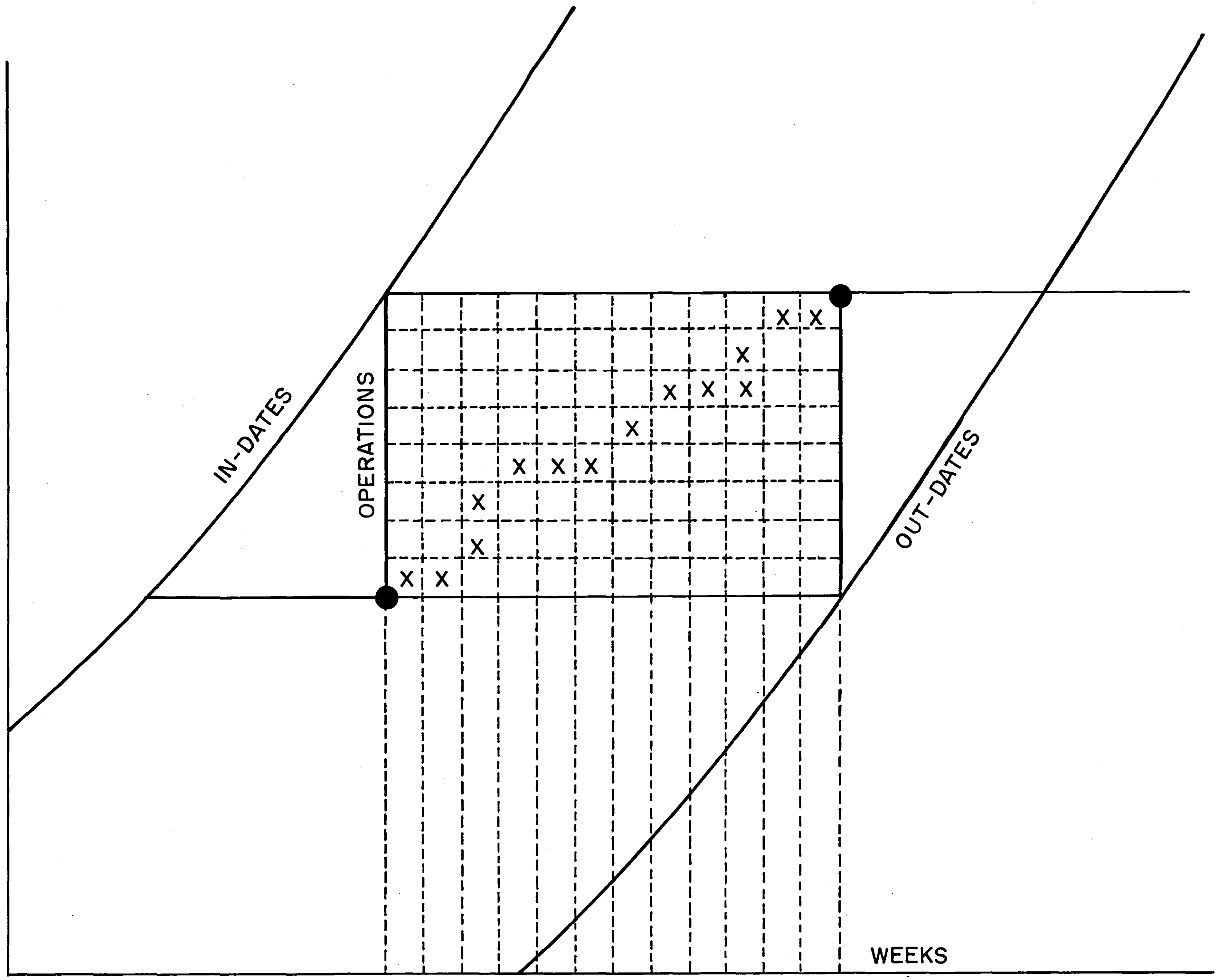


FIG. 9 TIME ASSIGNMENT SCHEDULING SYSTEM

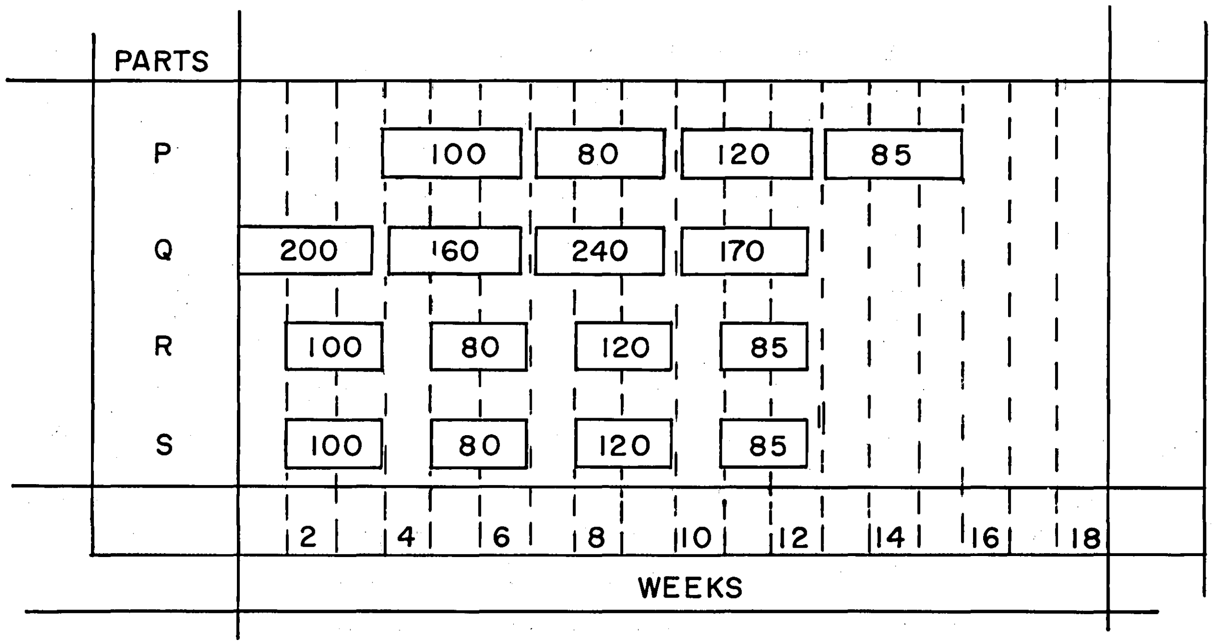


FIG. 10 GRAPHICAL REPRESENTATION OF SCHEDULES

		1	2	3	4	5	6	7	8	9
MACHINES	α	Q^1	Q^1 Q^1 R^1 S^1			R^2 R^2 Q^2 Q^2	Q^2 Q^2 S^2			
	β	Q^1 Q^1	Q^1	Q^1 S^1 S^1 R^1	Q^2 Q^2 Q^2		R^2 Q^2	S^2 S^2		
	γ		S^1 S^1	R^1		M M S^2 R^2	R^2			
	δ				P^1 P^1 P^1	P^1 P^1 P^1 P^1	P^1	P^2 P^2	P^2 P^2 P^2 P^2	P^2 P^2 P^2
ASSEMBLIES AND PARTS	Q^1	β β T α	α α T γ	γ T A A	A					
	R^1		W W α T	γ T W β	T					
	S^1		γ γ T α	T β β T	A					
	P^1				D δ δ δ	δ δ δ δ	δ T A A			
	Q^2				β β β T	T W α α	α α T β	T A A		
	R^2					α α T γ	γ T β T	A A A		
	S^2					W W γ T	W W α T	β β T		
	P^2							D D D δ	δ δ δ δ	δ δ δ T

FIG. 11 REPRESENTATION OF SIMULATED PRODCTION PLAN

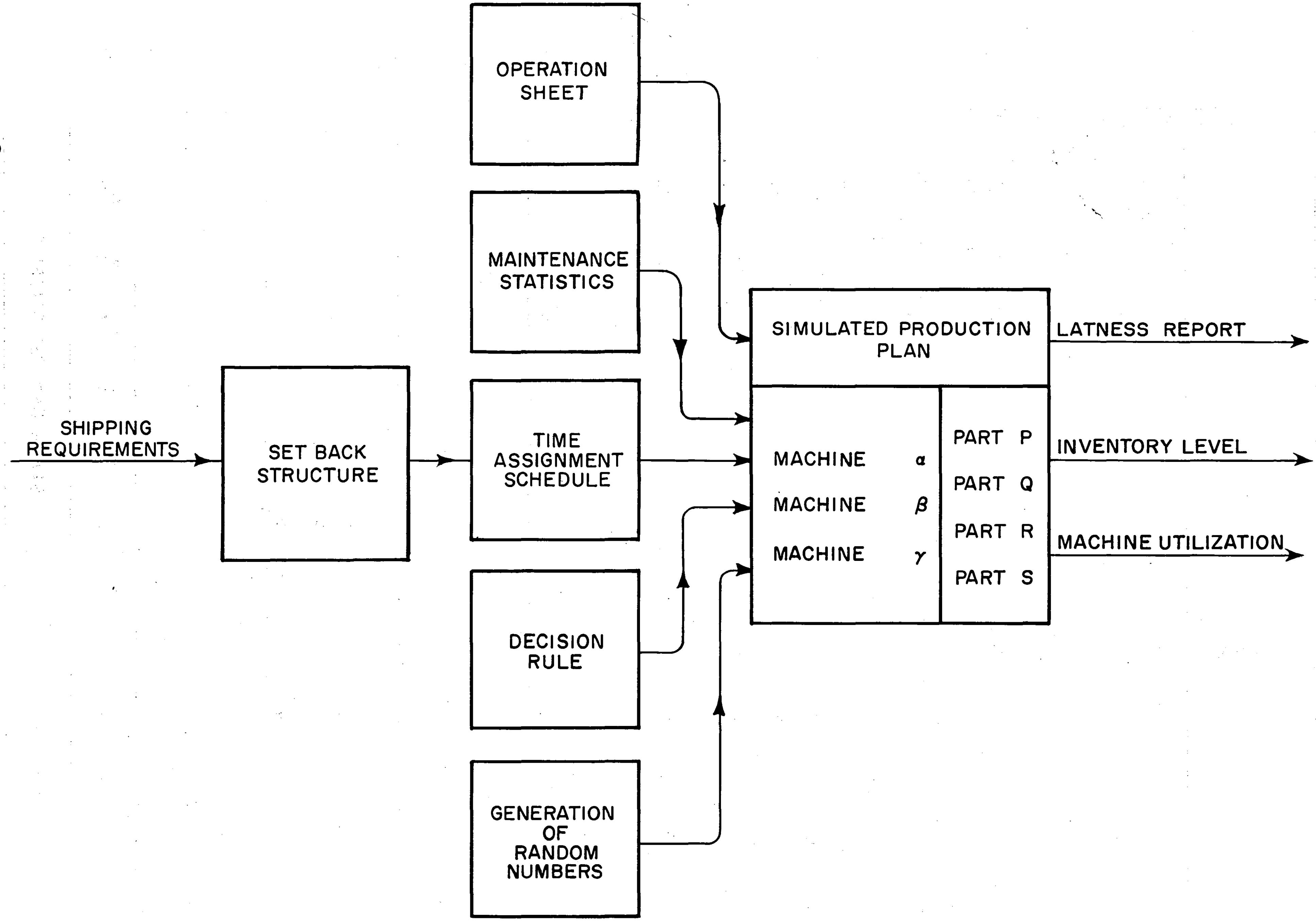


FIG. 12 BLOCK DIAGRAM OF SIMULATION