

UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE M 11 - 160

TITLE Matrix Multiplication (Closed) (DOI or SADOI)
 NUMBER OF WORDS 77 words
 TIME $[(1 \times k) (2d_A + 1) + M(k + j) (2d_B + 1) + (1 \times j) 2 k]$ ms.
 (See note 1.)
 TYPE Closed
 PURPOSE To multiply two matrices $A \cdot B = C$, retaining the product matrix C in the storage.
 TEMPORARY STORAGE 0, 1, b - k, b - k + 1, - - -, b - 1
 PARAMETERS S 3: 00 jF 00 bF
 j = number of columns in B
 b = location of the first element of B
 DESCRIPTION The routine is entered at word 0 from one of two entries:

		40 r			NO r	
(1)	p	50 p		(2)	p	50 p
		24 to routine				24 to routine

r = location of the first element of C

Entry (1) assumes neither $A = (a_{ik})$ nor $B = (b_{kj})$ is contained in the storage. Entry (2) assumes B is contained in the storage but A is not. In either case, before proceeding with the multiplication the entire matrix B must be contained in the storage, then matrix A is read in one row at a time--the (k + 1)st row replacing the k th row--forming C. The elements of a row of A immediately precede B, i.e., $a_{11} = N(b - 1)$, $a_{12} = N(b - 2)$, ... $a_{1k} = N(b - k)$. Hence the total number of locations required for matrix elements is $k + (k \times j) + (1 \times j)$.

Two checks are incorporated in this routine. One check tests to see that $-1 \leq c_{ij} < 1$. Whenever overflow occurs (i.e. $1 \leq c_{ij} < -1$) the computer will stop on an FF order in word 4LL. The other check tests to see that all rows of a matrix being read into the storage contain the same number of elements. If any two rows of a matrix contain a different number of elements the computer will stop on a FF order in word 29L.

METHOD OF USE

To avoid overflow, a_{ik} and b_{kj} must be scaled so that

$$|c_{ij}| = \sum_{k=1}^n |a_{ik} b_{kj}| < 1. \text{ This will be true if } |a_{ik}| < x/n \text{ and } |b_{kj}| < x/n$$

for $x^2 \leq n$.

Prepare Matrix Tape A by punching the elements (from left to right) of the first row of A followed by the second row, etc., as follows: $a_{00}, a_{01}, \dots, a_{0k}, N, a_{10}, a_{11}, \dots, a_{1k}, N, \dots, a_{i0}, a_{i1}, \dots, a_{ik}, N, J$. Each element must be preceded by a plus or minus (K or S) sign which can be followed by up to 11 decimal digits; the last element of each row must be followed by N; and the last element of the matrix must be followed by J in addition to N.

If B is to be read from tape, Matrix Tape B should be prepared in a similar manner. If B is contained in the storage, i.e., Entry (2) is used, the elements of B must be stored in the same sequence in which they would appear on tape, but the row and matrix ending symbols N and J should not be used.

When Entry (1) is used the computer will stop twice, first on a 24 order and then on a 20 order. At the first stop Matrix Tape B should be placed in the reader and at the second stop Matrix Tape A should be placed in the reader. After all stops computer operation should be resumed by moving the black switch.

NOTES

(1) A is an (i x k) matrix

B is a (k x j) matrix

d_A is the number of decimal digits in an element of matrix and likewise for d_B .

M = 1 if Entry (1) is used.

= 0 if Entry (2) is used.

DATE	<u>11/10/54</u>	Rt:	<u>3/5/59</u>
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APPROVED BY	<u>J. P. Nash</u>		

LOCATION	ORDER	NOTES	PAGE 1
0	00 K(M11) K5 F 42 76L		
1	10 20F 42 45L	Set address of c_{1j} Test for N or 4	
2	36 9L F5 25L		
3	42 25L L5 67L		
4	40 18L L5 65L	Prepare to read Tape A	
5	40 30L L5 63L		
6	40 21L L5 33L		
7	42 13L 00 20F	Set address of a_{11}	
8	46 13L 40 F		
9	81 4F L0 61L		
10	32 15L L4 61L		
11	J0 58L 10 4F	Form a_{1k} or b_{kj}	
12	75 F 00 4F		
13	L4 F 40 F		
14	50 F 7J 60L		
15	22 8L L0 58L		
16	32 22L L5 60L	Test for K(+)	
17	40 F L5 13L		

LOCATION	ORDER	NOTES	PAGE 2
18	[L4 59L] [42 19L]	-	
19	40 13L 41 F	-	Set address of a_{ik} or b_{kj}
20	F5 62L 40 62L		Count every row
21	[F5 24L] [40 24L]		Count first row
22	26 9L L0 58L		
23	36 25L L1 60L		Test for S(-)
24	26 17L 00 F		j counter
25	L0 58L 32 68L		Test for N
26	L5 22L 40 21L		
27	L5 62L L0 24L		
28	40 F L3 F	-	Test number of elements in each row
29	36 30L FF F		
30	[43 62L] [26 9L]		
31	L4 33L 00 20F		
32	46 68L 41 1F		
33	J0 1F S5 F		
34	50 F 74 F		
35	40 F 32 38L		

LOCATION	ORDER	NOTES
36	L5 1F 32 41L	
37	L4 F 36 41L	
38	26 42L L5 1F	
39	36 40L 22 41L	Form c_{ij} and test for overflow
40	L4 F 36 42L	
41	FF F L4 F	
42	40 1F L5 57L	
43	L4 34L 40 34L	
44	L0 68L 32 33L	
45	L5 1F 40 F	
46	F5 45L 40 45L	
47	F5 62L 40 62L	Has row i of C been completed?
48	L0 56L 32 51L	
49	L5 55L 46 34L	
50	L4 62L 42 34L	
51	22 32L L5 55L	Return to calculate $c_{i, j + 1}$
52	40 34L 43 62L	
53	22 6L 00 59L	Calculate $c_{i + 1, 1}$

LOCATION	ORDER	NOTES	PAGE 4
54	L4 F 40 F		
55	50 F 74 F		
56	26 17L 00 F		Constants
57	LL 4095F 00 F		
58	00 F 00 1F		
59	L5 3F 42 34L		
60	42 55L 42 33L		
61	L0 58L 42 13L		
62	42 54L 10 20F		
63	42 56L 42 57L		
64	L5 13L 00 20F		Interlude
65	46 54L 46 13 L		
66	46 34L 46 55L		
67	22 67L L5 53L		
68	22 1014F 26 59L		
59	26 1N 00 1F		
60	00 1F ON 3276F		
61	NN 3277F 00 F		1/10
61	00 10F		

LOCATION	ORDER		NOTES PAGE 5
62	26 17L		
	00 F		
63	F5 24L		
	40 24L		Constants
64	43 62L		
	26 9L		
65	43 62L		
	L1 24L		
66	L4 59L		
	42 19L		
67	L0 59L		
	42 19L		
68	50 F		
	43 24L		
69	20 2L		
	L5 54L		
70	40 13L		
	L5 66L		
71	40 18L		
	L5 63L		
72	40 21L		
	43 24L		Reset initial conditions and addresses
73	L5 22L		
	L4 61L		
74	42 25L		
	L5 64L		
75	40 30L		
	L5 55L		
76	40 34L		
	22 F		Link