DIGITAL COMPUTER LABORATORY
GRADUATE COLLEGE
UNIVERSITY OF ILLINOIS

HANDBOOK FOR STATISTICAL USERS OF ILLIAC

by

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ATTENTION

An error was made in not numbering pages in this booklet. A few moments on the part of the user will take care of this oversight with use of this Table of Contents as a guide.

The purpose of this pamphlet is to present, in a consistent form, the Statistical Library Routines available to Digital Computer Laboratory users on Illiac.

The condensed routine writeups will give, when available, information on:

1. General purpose of routine.
2. Operator's Instructions (This portion may be copied on Operator's Instruction sheet submitted with production problem). See (Procedure for submitting a production problem).
3. The preparation of tapes to be used.
4. Estimated time for running problem.
5. FF stops (usually indicative of an error) if any. (This should also be copied on Operator's Instruction sheet).
6. Routines, if any, that will utilize the output with little or no change on output tape.
7. Comments that, it is hoped, will facilitate the use of these routines.

In order to gain consistency throughout, the following nomenclature will be used.

R-Matrix: An R-matrix is a correlation matrix obtained generally from programs labeled K-2, K-8, or K-9. It can be obtained in two forms: the lower triangular or in rectangular form. IMPORTANT: If the intention is to use the R-matrix for further Illiac use e.g., factor analysis, it must be in triangular form.

ILLIAC Row: Most Statistical Routines use a form of Illiac readin that places an imaginary decimal point immediately following the sign. This should be held in mind when scaling. For example, K-8 output is scaled 10^{-1}. The output tape may be ---- +1000 -0293 +0026. This would be interpreted as +1; -.293; +.026.

It is possible to place a group of numbers on Illiac tape such that your teletype printup will be in columnar +029 or row, +029 -032 +046, form. (Or
-032
+046
conceivably, scattered over the paper +029 -032 +046). To Illiac, these numbers are considered as a row of numbers when it is followed by an N, J, F, or L.

5th-Hole Character: At times a fifth-hole character (i.e., carriage return-line feed; space; one-hole delay) is an integral part of a parameter or data tape. When such a character is required it will be so indicated, and must be included. If no indication is made, these fifth-hole characters can be included or may be excluded without resultant harm.

+ or K; - or S: Unless specifically indicated, a + and - may be punched instead of the letters K and S (e.g., a parameter requiring a no. S may be punched as no. -).

n and v : Much statistical work involves the use of a set of observations. The set may be people, observations or any group which contains for each unit in

that group a similar number of measurements or observations for each sample. These are also called variables or tests. Unless otherwise noted in this pamphlet the measurements, regardless of what they are, will be referred to as v or variables and the group which provides the measurements will be referred to as n or sample size. In general, this will take the form of a rectangular matrix with n rows, each row consisting of v variables. At times the term f factors will be used. This consists of f columns with v variables in each column.

d_o and d_i: In specifying d_i (decimal digit input or the number of digits in each element input), leading zeros as well as trailing zeros are to be counted. A number +00300 would have an i value of 5. This holds true for d_o (decimal digit output or the number of digits in each element output). Most of these routines are scaled 10^{-1}. This means that a leading zero will be punched out. Therefore, if you wish 3 significant figures in your output, it is a safe thing to ask for a d_o value of 4. When exceptions to this arise, the program writeup will indicate them.

Output Read Back in ILLIAC: Many times you may wish to use Illiac output for input in a different problem at a different time. Be sure to check the output tape to be sure extraneous titles, headings, etc., are not present. Generally not doing this will cause Illiac to hangup. Either remove the offending part, or mark the tape where the operator should begin reading the tape. When practicable in the following descriptions, such output (with headings, etc.) will be indicated.

Factor: A column or row of v (variables) signed fractions.

R-Matrix: Any correlation matrix of v (variables) by v signed fractions.

Matrix: An array of numbers placed in an n row, in column form. Usually denoted by A_{ij}, where i = number of rows and j = number of columns.

Output: Results received from Illiac.

Input: Information on tape fed into Illiac.

Triangular Matrix Form: The lower left hand side or upper right hand side of a symmetric matrix.

Square Matrix Form: An array of elements in matrix form where the number of rows and columns are equal.

Routine: Any tape that has been written to solve a problem on Illiac.

Master Tape: Any tape that has been written to solve a problem on Illiac.

Program Tape: Any tape that has been written to solve a problem on Illiac.

Variable: A single measurement that can vary in value for each time measured.

Sample Size: The number of times an identical set of variables are measured.

Order of Matrix: A square matrix of n rows and n columns has an order n.

Columnar Form Printout: Results printed as a column on the teletype printer.

Row Form Printout: Results printed as a row on the teletype printer.

Matrix Form: Results printed out as n rows and m columns in a rectangular form.

Parameters: Used to limit Illiac programs to your particular problem.

Signed Numbers: Integers or fractions preceded by a + or -.

d_o: The number of digits (does not include sign) in each element of output - include all zeros.

d_i: The number of digits (does not include sign) in each element of input - include all zeros.

A_{ij}: Used to denote a matrix with i rows and j columns.

Black Stop: Any 24, 20, 34, 30 order. Illiac stops on these + on a black switch up causes Illiac to resume.

Black Down: Called "black disable", "black ignore", "stop disable" - causes Illiac to ignore or bypass all black stops.

Terminating Symbol: Any symbol (usually N, J, F, L) which ends a row or column of numbers and/or a matrix.

Element: A signed fraction or integer, usually a_{ij} in a matrix.

Observations: Sample size, a set of variables measured over a group n of people.

Tests: Variables or measurements.

5th-hole Delay: An Illiac punch which has one hole in the fifth position. Does not show up on teletype printer.

5th-hole Character: Any Illiac character other than 0 thru L that contains a fifth-hole punch.

Sexadecimal Character: The Illiac characters 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, K, S, N, J, F, L.

Character: Any column of key punched holes on Illiac tape.

Diagonal Element: In a matrix of n rows, n columns the elements a_{11}, a_{22}, ..., a_{nn}, i.e., element at intersection of row 1, column 1; row 2, column 2, etc.

Cumulative: Summed or accumulated values over calculations.

Machine Error: Error on Illiac caused by machine malfunction.

Programmer Error: Error on Illiac caused by user.

Absolute Value: Any number, positive or minus, that is assigned a positive value.

Symmetric Matrix: A matrix A_{ij} whose a_{ij}^{th} element = a_{ji} element.

Transformation or Transform Matrix: A matrix x_{ji} which transforms a matrix A_{ik} into matrix B_{jk}.

Rectangular Matrix: A matrix of n rows and m columns.

Vector: One row or column of a matrix.

Fixed Number: Any number where the decimal point is fixed by user.

Operator's Instructions: Instructions filled out on an Illiac "Program Instruction" sheet to enable operator to run problem.

Sums of Rows and Columns: Σr and Σc. A value obtained by summing elements across rows or down columns.

Matrix Transpose: Take a matrix A_{ij} and interchange rows and columns to get A_{ji}.

Scatter Plot: A graphic plotting of two columns of numbers, one plotted as X the other as Y on an XY axis.

OF Stop: A stop on Illiac that usually designates the end of a machine run.

Identity Matrix: A matrix with ones in the diagonal elements and zeros for all other elements.

Scaler Matrix: A matrix with the same constant in each diagonal element and zeros in all other elements.

$$A_{ij}\pm B_{ij} \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \pm \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} a_{11}\pm b_{11} & a_{12}\pm b_{12} & a_{13}\pm b_{13} \\ a_{21}\pm b_{21} & a_{22}\pm b_{22} & a_{23}\pm b_{23} \\ a_{31}\pm b_{31} & a_{32}\pm b_{32} & a_{33}\pm b_{33} \end{vmatrix}$$

Frequency: The number of times a number or event occurs.

Non-singular Matrix: A matrix whose determinant value = 0.

Capacity of Program: The highest values of data size that can be run on any one program. Do not exceed.

Sums of Squares of Rows and Columns: Σr^2 and Σc^2. Each element in a row or column is squared. The squared elements are then summed over rows or columns.

$$A_{ij}xB_{jk} \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \times \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} =$$

$$\begin{vmatrix} (a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31})(a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32})(a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33}) \\ (a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31})(a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32})(a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33}) \\ (a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31})(a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32})(a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{23}) \end{vmatrix}$$

FF00XX Stops: These stops will indicate two types of errors to the user. They are: (1) a machine error or; (2) an error on the part of the user, i.e., data tape error, parameter error, or the structure of the data causes an arithmetic error. Should such an error occur, the operator will note the error on the "Operator's Instruction Sheet" and return it to you. The type of error is designated by the XX and/or location of the FF stop. It is requested that you list, when given, on your "Operator's Instruction Sheet" the FF stops given in the following writeups. Also include a description, when given, of the errors indicated by these stops.

Time Estimates: All production problems submitted must have time estimates. These are used by the computer operator's in scheduling production. Since an underestimate of time will result in your problem being returned before it reaches completion, it is to your advantage to see that the time estimate is great enough to cover the running of your problem. Time estimates are given for routines when possible. Many times these are, at best, estimates.

Procedure on Submitting Production: Following is a step by step procedure for using Illiac facilities:

1. Fill out and submit to the Illiac operator a "Problem Specification" sheet (this will be returned in a few days, and you will be assigned a problem number for use on Illiac). Obtain the number from the operator.
2. Place data on tape according to the format required for the routine you are using.
3. Place on tape any parameters that are required.
4. Prepare the master routine for your problem. There are three libraries which contain copies of master routines. One is the Statistical Library (denoted by KSL); the Illiac Active Library (denoted by an alphabetic letter and number such as K-8, M-12, etc.) and the Illiac Auxiliary Library. The Illiac teletype operator can help you locate these. The tape in the library must be duplicated by you. This duplicated routine is then submitted with your production problem.
5. Fill out a "Operator's Instructions" sheet for the Illiac. All blank spaces are important----fill them in. It is on these sheets that is placed the sections in these writeups called "Operator's Instructions" and "FF Stops".
6. The operator will at this point give you the necessary information on: how to turn in your problem; when you can possibly expect your results; and where you can pick up your results.

Black Switch, White Switch: On Illiac are two switches used by the operator in running a program. Generally, unless told differently on your "Operator's Instructions", the operator will use the white switch to read in the master and will use the black switch from then on. There are times you need to give further

instructions to the operator. If the writeup herein includes instructions to run black down or to white switch up and down to read in a new tape, this instruction is important, and must be included on your instructions. The operator's are not mind readers. Therefore, include any unusual instructions if you wish efficient service. If no instruction is given, the operator will assume you wish to use the black switch once your master tape is in.

Data Input: Many times a program will not read in all of the data tape. This occurs when output from one program is read back into Illiac as input on another program and only part of the output is needed for input. It is important that you indicate this to the operator. Such indication might be

(3) Data tape (will not all read in) 24---

NOTE: AT TIMES, ERRORS ARE FOUND IN THE DESCRIPTIONS OF ILLIAC PROGRAMS. SHOULD YOU, IN USING ANY OF THE FOLLOWING ROUTINES, FIND ERRORS, PLEASE INDICATE THEM TO THE SECRETARY IN THE TELETYPE ROOM SO THAT SHE MAY MAKE THE NECESSARY CORRECTIONS.

KSL 1.11

PRINCIPAL AXIS FACTORS

PURPOSE

This routine will extract the largest K factors from a correlation matrix (K-2, K-8, K-9 output in triangular form) by an iterative procedure suggested by Hottelling. The routine will fail in the exceptional circumstance that two or more of the largest K eigenvalues are of the same size. The number of variables in the correlation matrix must be < 111. The output is in the form of a Title (must be eliminated for further runs of output on Illiac) the factors in columnar form, signed, scaled 10^{-1}, an N at end of each column. A variance table giving Factor number, latent root or eigenvalue, percent of variance extracted for each factor, and the cumulative percentage of variance extracted. This table is followed by the residuals left in communalities and a triangular matrix of residual variance left in original correlation matrix.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape	241NO
2. Parameter tape	241KF
3. Communality tape	241K8
4. Data tape (black switch down)	24180
5. (Center black switch during residual output)	34122
6. Black switch up	241NO (if no residuals desired, stop at this point)

At this point a new parameter tape, communality tape and data may be read in.

PREPARATION OF TAPES

1. Master - Duplicate library copy KSL 1.11
2. Parameter - F space d_0 space d_r space

where F = number of factors desired

d_0 = number of decimal places in output

d_r = number of decimal places in residual output. (This may be set to 0 if no residual matrix is desired).

3. Communality tape - this consists of a column of signed elements scaled 10^{-1} and terminated by an N. The elements may be either +1 or trial values less than +1. For values less than 1, a zero will precede each value.

4. Data tape - this usually will be K-8 output. It consists of a triangular matrix, each element scaled 10^{-1} and signed. If the matrix is punched by hand it consists of the upper half of a symmetric matrix by columns or the lower half by rows.

TIME ESTIMATE

No time estimate given----varies greatly.

For example, in a 5 factor, 14 variable problem the following times were noted.

F1 - 25 seconds; F2 - 25 seconds; F3 - 85 seconds; F4 - 30 seconds; F5 - 45 seconds----30 seconds to punch out 4 decimal residual matrix.

Two 50 variable 2 factor problems were run. Their times were:

1) First factor - 1 minute 6 seconds; second factor - 1 minute 40 seconds and 3 minutes to punch out residual matrix.
2) First factor - 50 seconds; second factor - 3 minutes 44 seconds - 3 minutes to punch out residual matrix.

FF STOPS

None listed

OUTPUT

The output consists of a Title; the factors printed in columnar form, scaled 10^{-1}, each column terminated by an N; a variance table; an N terminated column of communality residuals; and a triangular (lower half) matrix of residuals. Before using the factors for a run in Illiac, be sure the title has been removed.

KSL 1.12

M-22 OUTPUT TO FACTOR LOADINGS

PURPOSE

This routine takes M-22 output and converts it to the desired number of factors where the number of factors is equal to or less than the number of factors in M-22 output. Output from 1.12 is scaled 10^{-1}, by columns, and has an N at the end of each column and a J after the last N.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 1.12	2405J
2. Parameter	24078
3. Data (M-22 Output)	24078

At stop 24078 raise black switch to read an additional data tape using the same parameters.

To read a new set of parameters at 24, move the white switch up and down.

TAPE PREPARATION

Master - Duplicate Library Master Copy KSL 1.12

Parameter - 003+

00F 00fF

00F 00vF

00F 00d_0F

24120N

where

f = number of factors to be converted (equal to or less than number of factors in M-22 output)

v = number of variables in each factor

d_0 = number of decimal digits in the output

Data - Output from M-22

ESTIMATED TIME

Master - 1 minute to read in

Data - approximately 150 characters/second

Output - approximately 50 characters/second

EXAMPLE

Data tape = 10 factors; 50 variables; 7 decimal places

Data read in = $\frac{10 \times 50 \times 7}{150} \cong 26$ seconds

Data output = 10 factors; 50 variables; 10 decimal output

Data output time = $\frac{10 \times 50 \times 10}{50}$ = 100 seconds

Total time = 26 seconds + 60 seconds + 100 seconds $\cong$ 3 minutes.

FF STOPS — None

KSL 1.20 - 251

CENTROID FACTORS WITH FIXED COMMUNALITIES

PURPOSE

This routine takes an R-matrix of v variables, (output from K-2, K-8, K-9) a set of communalities provided by the user (of v size) and reduce the R-matrix into a set of f linearly dependent factors. The communalities introduced replace the diagonals in the R (correlation) matrix.* There is no re-estimation of communalities as in KSL 1.21. The resultant factor matrix satisfies the equation $R_v = FF' + R_{(v-f)}$ where $R_{(v-f)}$ is the matrix of residuals. This program is limited to 111 variables and 99 factors. Except for a title at the front, results from this program may be fed directly into KSL 1.90, 1.80 without any modifications of the tape. The output contains a title, the factors signed, scaled 10^{-1} by column - an N at end of each column. The factors are followed by a variance table containing the factor number; the latent root or eigenvalue; the percentage of variance extracted for each factor and the cumulative percentage of variance extracted. This, in turn, is followed by a column of residuals left in the communalities and the $R_{(v-f)}$ matrix which can be fed in later for further factor extraction. If this is done, the operator tape at the end of the output is necessary.**

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 1.20	24190
2. Parameter tape	2414L
3. Communality tape	34185
4. Data tape (correlations)	24190

A new parameter tape, communality tape, and R-matrix may now be read in without rereading the master.

TAPE PREPARATION

1. Master tape - Duplicate library copy KSL 1.20
2. Parameter tape - this consists of three unsigned decimal numbers separated by a fifth-hole character (space,

carriage return, delay) they are f space d_f space d_r space.

Where f is the number of factors desired, (must be < 100)

d_f is the number of decimal places wished in each element of the factors and

d_r is the number of decimal places wished in each element of the residual matrix d_f and $d_r < 12$. (Results accurate to only 8 places).

**If d_r is set to zero no residual matrix is punched. This can be an Illiac time saver if you feel the number of factors you are extracting are sufficient. However, if you wish to extract more factors at a later date you will need the residual matrix and the operator tape that follows it. To do this - follow "OPERATOR'S INSTRUCTIONS" above - but clear machine and read in operator's tape first. Machine will stop on 240SJ. At this point read in KSL 1.20 master with a black switch up and follow steps 2, 3, and 4 making sure that your R-matrix in step 4 is the residual matrix of previous run.

3. Communality tape - this tape consists of a series of v signed elements, scaled 10^{-1} and terminated by an N. If ones are used they are entered as +1, any other number used has a leading zero, i.e., +0992.

4. Correlation tape - the correlation tape is the lower triangular matrix output of K-8, K-9 or any other triangular matrix of correlations. This tape consists of $v(v+1)/2$ signed fractions scaled by 10^{-1}. The diagonal entries must be present, but it is unimportant what these are, for they are replaced by the communalities.*

TIME ESTIMATE

1 and 1/2 minutes for input of master, parameter and communalities. Time in seconds for input of R-matrix

$$\simeq \frac{v(v+1)d_i}{300}$$

Time in seconds for output $$\simeq \left[\frac{fv}{50} + \frac{(v)(v+1)}{70}\right]d_o$$

Calculation time/factor = 1 minute if $v < 60$
2 minutes if $N > 60$

FF STOPS

FF - location 026 (1) overfactoring; (2) underestimation of communalities; (3) R-matrix is non-Gramian; (4) machine errors; and (5) errors in data tape. Raise white switch up and down. Variance table, $R_{(v-f)}$ and residual communalities will be punched out.

FF - location 04J drum transfer failure.

NOTES

* If you wish to obtain an R-matrix with diagonals some value other than one - this may be done by setting f = 0 and $d_f = 0$ in parameter, assigning a value to d_r. No factor extraction will be performed. The machine will read in the R-matrix, replace the diagonals by the communalities you read in on the communality tape, and punch out the same R-matrix with new diagonals.

If d_f and d_r are set equal to 0 in the parameter, and f is assigned some value (usually greater then the number of factors you feel are present), the machine will print out only the variance table. From this table particularly the eigenvalues - the number of factors to extract can be determined.

KSL 1.21

ESTIMATED CENTROID WITH ESTIMATED COMMUNALITIES

PURPOSE

This routine takes a correlation matrix in lower triangular form (K-2, K-8, K-9, and K-17 output) and breaks it down into a rectangular matrix of f factors (columns) and n variables (tests rows) where n is equal to the number of columns and rows in the input correlation matrix. The elements of the output matrix of 1.21 are called factor loadings. The highest valued element (+ or -) in each row of the correlation matrix is selected and replaces the diagonal element (usually +100) in the correlation matrix inputted. It will also give a residual correlation matrix after all factors are extracted. The program will accept no more than 111 variables (tests). The number of factors extracted is limited to < n-1.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 1.21 (part 1)	243F7
2. Parameter	24032
3. Correlation tape	343F7
4a. Master tape (resume at place removed for 1) black switch up	243F7 *
4. Master tape, black switch down	0F **

NOTE

Master must be re-read and above instructions followed for each new set of factors extracted.

* At this point only "f" factors signified in parameter will be extracted (no residuals).

** By reading master tape into end, residual matrix is given.

Step 4a may be eliminated on operator's instruction sheet indicating 4 instructions.

TAPE PREPARATION

Master - Duplicate library master copy KSL 1.21

Parameter -

004+
00F 00fF
00F 00d_1F
00F 00d_0F
00F 00nF
2450N

where

$\underline{f}$ = number of factors to be extracted (must be less than n below)

$\underline{d}_i$ = number of decimal digits in each correlation

$\underline{d}_0$ = number of decimal digits in output

$\underline{n}$ = number of variables (tests) or rows or columns in correlation matrix.

Correlation tape - In general this will be the triangular output from programs K-2, K-8, or K-9. It will be scaled 10^{-1}, signed (+ or -) and will have +100 in the diagonals, although any diagonal entry will be replaced by highest value in column.

For example: +1000
+0936
+1000 or +1000+0936+1000-0032+0625+1000
-0032
+0625
+1000

For this example $\underline{f}$ could be 1 or 2; $\underline{d}_i = 4$; $\underline{d}_0 = 3$ to 11; $\underline{n} = 3$.

ESTIMATED TIME IN SECONDS

Input of Master - 1 minute

Input of data - $\dfrac{3v^2 d_i + 13v^2 + 15000}{100}$

Extraction of factors - $\dfrac{[f(12.5v^2 + 17d_0 + 150v + 1000) + 20{,}000]}{100}$

Punching residuals - $\left[\dfrac{(v)(v+1)}{2}(d_0 + 4)/50\right]$ seconds

For example, if v = 10; f = 5 d_0 and $d_i = 4$

Then time

$$1 \text{ min.} \frac{+1200+1300+15000}{1000} + \frac{[5(1250+68+1500+1000) + 20{,}000]}{1000}$$

$$\frac{+440}{50} \cong 2 \text{ minutes.}$$

FF STOPS

FF location $148_{(16)}$. Error may be due to: (1) R-matrix is non-Gramian; (2) machine error, and (3) errors in data tape. By raising white switch up and down, residuals will be printed. These may help in discovering source of error.

NOTE

The output from KSL 1.21 is in the following form:

CENTROID FACTORS (title)

ESTIMATED COMMUNALITIES

+0004.170 (absolute sum of elements in square correlation matrix inputted)

+0.223) Factor I
+0.315)

+0000.960 (absolute sum after 1 factor extracted)

+001 +003 (reflections after 1st factor)

. .
. .
. .

RESIDUALS

+0000.112 (absolute sum in residuals)

+0.010)
+0.023 -0.032) residual correlations

00400+)
80F 00001F) (operator tape to be used in further
00F 00000F) extraction of residual matrix at later
24999N) date)

As may be readily seen, to use the factors from KSL 1.21 in KSL 1.91, KSL 1.80 or other routines using unrotated factors; the factors must be placed on a new tape with N's at end of each column.

DO NOT PLACE OUTPUT FROM KSL 1.21 BACK INTO ILLIAC WITHOUT MAKING NECESSARY DELECTIONS AND INSERTING AN N AFTER EACH FACTOR COLUMN.

KSL 1.52

ESTIMATION OF COMMUNALITIES BY METHOD OF MAXIMUM LIKELIHOOD

PURPOSE

This program will calculate and print out a column of communalities that may be used in KSL 1.20 (Centroid Analysis with Fixed Communalities). It is limited to a correlation matrix (R-matrix) not larger than 84 variables (84 rows or columns in K-8 output). Through an iteration process, it converges to a relatively fixed level. The user must indicate how many factors to be used and how many iterations the program is to make. He also indicates at which point he wishes the iterations to be punched out. The output also includes a variance table, giving number of factors, number of iterations and the amount of variance extracted by each iteration.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 1.52	2402S
2. Parameter	24038
3. Communality tape	2404N
4. Correlation R-matrix	2402S

(At this point, a new parameter, communality tape, and correlation R-matrix may be read in without re-reading master. Should the white switch be moved up and down, another iteration will be performed).

TAPE PREPARATION

Master - Duplicate library master copy KSL 1.52

Parameter - 4 unsigned decimal numbers; a fifth hole character (i.e., space, carriage return or delay) separating each decimal character. The characters must be in this order: f space d space I space T space

where

f is the number of factors for which communalities are to be calculated.

d is the number of decimal digits in output.

I is the number of iterations before printout occurs. (This must never be larger than T below)

T is the total number of iterations desired.

COMMUNALITY TAPE

The number of positive signed values in this tape is equal to the number of variables (rows or columns) in the correlation R-matrix. This column of numbers must be ended with an N. The numbers may be 1's (+100) if there is no idea of what the communalities may be. However, time will be saved on the machine if trial values less than one are used. These values are signed and scaled 10^{-1} (i.e., a value of .56 would be +056). Many times a trial communality tape will consists of the highest value of each row in the correlation matrix.

CORRELATION R-MATRIX

In general, this will be the triangular output from K-2, K-8 or K-9. It will be scaled 10^{-1}, signed (+ or -) and will have +100 in the diagonals.

ESTIMATED TIME

$\{TF[13v^2 + 36v + 55] + T[2v^2 + 18v + 460] + (T-I)(175v) + 1500\}$ seconds x 10^{-3}.

For example, a 10 factor; 30 variable; 20T; 10I problem would be $\{200[11700 + 1080 + 55] + 20[1800 + 540 + 460] + 52500 + 1500\}\ 10^{-3}$ or $\cong$ 45 minutes.

(See parameter description for code: v = variables)

FF STOPS

There are no FF stops listed for this program.

NOTE

Output is in this form

+0239
+0327
+0432
N

+0241
+0326
+0433
N

VARIANCE TABLE

The particular iteration to be used in KSL 1.20 can be duplicated from output tape. Use only the particular column desired plus the N following it as a communality tape for KSL 1.20.

KSL 1.53

EXTRACT ELEMENT WITH LARGEST ABSOLUTE VALUE FROM A MATRIX

PURPOSE

This routine takes the lower half of a triangular matrix (K-2, K-8, K-9 output) and extracts the largest absolute value element, printing it out in columnar form, scaled 10^{-1} and signed with +'s. An N terminates the column - the output is thus available for use as a communality tape in KSL 1.20. The size of the matrix (N) must be < 900.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape	3400S
2. Parameters	24027
3. Matrix tape	2400S

A new parameter and matrix tape may be read in without rereading master.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 1.53
2. Parameter - d_0 space n space where d_0 = number of decimal places in output ≤ 9 and n = size of matrix
3. Matrix tape - this consists of $n(n+1)/2$ signed fractions in triangular form as given in K-2, K-8, K-9.

TIME ESTIMATE

Time in seconds $\tilde{=} v^2(.002d_0 + .006) + v(.019d_0 + .053)$

FF STOPS

FF000 location 043 - sum check failure on master.

FF001 indicates that d_0 in parameter ≥ 10. A new parameter can be read in with white switch up and down.

FF002 indicates a sign digit is out of place. White switch up and down will cause matrix to continue to be read.

KSL 1.71

OBTAIN CORRELATION COSINE MATRIX AMONG PRIMARY FACTORS AND FACTOR PATTERN STRUCTURE

PURPOSE

Given a cosine matrix between reference vectors (factors) and an oblique factor matrix of reference vectors, this program will provide the cosine matrix (symmetric) among primary factors and a factor pattern matrix of the same number of factors and variables as the inputted factor matrix of reference vectors. The size of the cosine matrix fed in and received as output must be no larger than 22. The number of columns (factors) in the reference vector matrix must be equal to the number of columns in the cosine matrix. Generally, the V_{fp} matrix (primary factor pattern) obtained in this program is used for KSL 1.72. The two matrices obtained from this program should be identical (except for rounding error) to the corresponding matrices obtained from KSL 1.91.

OPERATOR'S INSTRUCTIONS

		Stops
1.	Master KSL 1.71	243F7
2.	Parameter	24035
3.	C_r tape	340F7
4.	V_{rn} tape	0F

NOTE

Master must be re-read for each problem.

TAPE PREPARATION

Master - duplicate library copy KSL 1.71

Parameter -

0010+
00F 00fF
00F 00d_iF
00F 00vF
2453N

where

$\underline{f}$ = number of factors or columns in reference vector matrix

$\underline{d_i}$ = total number of decimal places in one variable of reference vector structure (< 12) (all variables contain same number of digits)

$\underline{v}$ = number of rows or variables in reference vector matrix

$\underline{C}_r$ tape - punch a square symmetric matrix. Each element is signed and each element should contain no more than 11 digits. Terminate each row with an N. Scale each element 10^{-1}.

$\underline{V}_{rn}$ tape - Punch by rows - no terminating symbols. Each variable of element should be signed and contain no more than 11 digits.

NOTE

In the $\underline{V}_{rn}$ tape any element of +1.00 must be entered as a +099.

ESTIMATED TIME

$\underline{C}_f$:

order of C_f	time (in seconds)
5x5	40
10x10	70
15x15	90
20x20	120
22x22	140

$\underline{V}_{fp}$: time = $\frac{\text{NO. DIGITS x v x n}}{50}$ seconds

where

$\underline{v}$ = number of variables or elements in row of V_{rn}

$\underline{n}$ = number of rows in V_{rn}

FF STOPS

FF hangup in location 02J indicates a drum failure. Re-read master, try again.

Hangup on 0N3276NN3277 in location $139_{(16)}$ indicates that matrix C_r has no inverse.

KSL 1.72

ESTIMATION OF OBLIQUE FACTOR SCORES

PURPOSE

Given four matrices: $\underline{F}_0$ - arbitrary unrotated orthogonal factor matrix; $\underline{V}_{fp}$ - primary factor pattern obtained from KSL 1.71; $\underline{\Lambda}_r$ - matrix that transforms $\underline{F}_0$ to $\underline{V}_{rn}$; and $\underline{F}^2$ - a single column of $1-\underline{h}^2$ (see preparation of tapes for description) this routine gives the three matrices: $\underline{\Lambda}_f$ - a transformation matrix that takes $\underline{F}_0$ into $\underline{V}_{fp}$; $\underline{V}_{fs}$ - the primary factor structure matrix and; $\underline{B}$ - an $\underline{n}$ x $\underline{k}$ matrix of regression coefficients. The $\underline{B}$ matrix can be multiplied by the Standardized Raw Score Matrix to obtain oblique factor scores. The matrices $\underline{F}_0$, $\underline{V}_{fp}$, $\underline{V}_{fs}$ must have no more than 22 columns ($\underline{F}$) or 111 rows ($\underline{v}$).

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 1.72	2404F
2. $1-h^2$ tape (residual communalities)	34128
3. V_{fp} tape	34051
4. Λ_r tape	24154
5. F_0 tape	2404F

(A new problem may now be run by inserting a new $1-\underline{h}^2$ tape and proceeding through steps 3, 4, and 5).

PREPARATION OF TAPES

Master - duplicate library copy KSL 1.72

$1-\underline{h}^2$: This is a column of unscaled fractions. An N must be punched at the end of this column. None of these fractions can be smaller than +.01. $\underline{h}^2$ is obtained by summing the squares of each element in a row of $\underline{F}_0$.

EXAMPLE

	$\underline{h}^2$	$1-\underline{h}^2$ tape
+.90 +.30 -.20	.94	+06
+.20 + 99 - 30	1.13	+01 (see above)
+.40 + 50 + 70	.90	+10
		N

$\underline{V}_{fp}$ tape: This is the $\underline{V}_{fp}$ output tape from KSL 1.71. It may be copied directly from 1.71 output and used. It is punched row by row. Each row has an N at the end. Each element is scaled 10^{-1}.

$\underline{\Lambda}_r$: This transform matrix is punched by columns; scaled 10^{-1} and has an N at end of each column. If the transform is used from KSL 1.96, it must be remembered that it is output from KSL 1.96 by rows. Therefore, it must be transposed or punched by columns.

$\underline{F}_0$ tape: This is punched row by row. Each row has an N at end. Each element should be punched as a signed, unscaled fraction of no more than 11 digits. (This will generally be a matrix of factors obtained from either KSL 1.20 or KSL 1.21. The results from these two programs must be transposed (see KSL 5.30) and scaled 10^{+1} before using on KSL 1.72).

TIME ESTIMATE — None available

FF STOPS — FF location 02J - drum failure - re-read master

FF location 145 - wrong number variables in a row

0N3276NN3277 location 139 of $\underline{V}_{fp}$ - $\underline{\Lambda}_r$ is singular

NOTE — The rows of $\underline{\Lambda}_r$ = columns of $\underline{\Lambda}_r$ ($\underline{\Lambda}_r$ must be square)

The rows of $\underline{F}_0$ = rows of $\underline{V}_{fp}$ = the number of elements in $1-\underline{h}^2$ tape

The columns of $\underline{F}_0$ = columns of $\underline{V}_{fp}$ = rows of $\underline{\Lambda}_r$.

KSL 1.80

VARIMAX ROTATION

PURPOSE

This routine takes a set of unrotated factors (a $\underline{f}$ x $\underline{v}$) matrix (usually obtained from KSL 1.20 or KSL 1.21) and rotates them maintaining an orthogonal reference system according to the Varimax criterion. Each output contains 5 kinds of data in this order: (1) the communalities ($\underline{h}^2$ see KSL 1.72) of the $\underline{v}$ variables, scaled at 10^{-2}; (2) a set of criterion values of 12 digits each, scaled 10^{-1}. These values are used by the machine to determine when convergence is reached; (3) the variance contributed by the $\underline{f}$ varimax factors, scaled at 10^{-2}; (4) the parameters $\underline{v}$, $\underline{f}$, and $\underline{d}_0$ (these will be identical to parameter inputted (see below)); (5) the $\underline{f}$ variamx factors scaled 10^{-1}. Each column or $\underline{f}$ varimax factor is followed by an N. The number of variables must be < 127. The number of factors must satisfy this inequality: $(N+1)(f+1) < 10240$.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 1.80	2400K
2. Parameter	24043
3. Data (black switch down)	0F

(To read in new parameter and data, center black switch and white switch up and down. Repeat 2 and 3 above). (If problem must be removed before 0F is reached, center black switch; when 340SF occurs white switch up and down. Output will ensue and parts (4) and (5) of the output tape (see PURPOSE above) may be then run at a later date to continue rotation problem).

PREPARATION OF TAPE

Master - duplicate library copy KSL 1.80

Parameter - $\underline{v}$ space $\underline{f}$ space $\underline{d}_0$ space in that order, where

$\underline{v}$ = number of variables

$\underline{f}$ = number of factors, and

$\underline{d}_0$ is the number of digits desired in output (since output is scaled 10^{-1}, a d_0 of 3 will contain only 2 significant figures).

Data - This is an unrotated factor matrix punched column by column. It is scaled 10^{-1}, has a sign in front of each element and has an N at end of each column. Output from KSL 1.11, KSL 1.20, and KSL 1.30 may be used as data without changes. KSL 1.21 output may be used if correct changes are made (see KSL 1.21).

TIME ESTIMATE 200 + .0045vf 3 seconds is a conservative estimate

FF STOPS None given other than library subroutines.

KSL 1.90

OBLIMAX ROTATION OF FACTORS

PURPOSE

This routine takes an f (factor) x v (variable) unrotated matrix and rotates it to an oblique set of factors. See the library printup of this routine (pages 4-7) to obtain an idea of the mathematical procedure used, and its limitations. According to options specified on parameter tape (see below) the user may obtain varying combinations of results. The limits to this program are given under NOTES. With appropriate manipulations to the output, it can be used in KSL 1.95, KSL 1.96 or KSL 1.72. See those routines to determine what manipulations are needed.

OPERATOR'S INSTRUCTIONS

A.

	Stops
1. Master KSL 1.90	2407J
2. Parameter tape	240F0
3. Factor tape, run with black switch down	0F

If black switch is placed in center, machine will stop at 34092 after each iteration. A white switch up and down at this 34 stop will cause print out of results obtained at that point. This allows interruption of problem before it is completed. To read another parameter tape and factor tape once 0F is reached, white switch up and down.

B. Subsequent run with a transform (either from interrupted run as in A or if transform is available at beginning of run).

	Stops
1. Master KSL 1.80	2407J
2. Parameter tape	240F0
3. Transform tape (white switch up and down)	340L8
4. Factor tape, run with black switch down	0F

PREPARATION OF TAPES

Master - duplicate library copy KSL 1.90

Parameter - This tape consists of four unsigned integers, each separated by a fifth-hole character (carriage

return-space-delay). They are d_0 space v space f space and x; where d_0 is the number of digits in output (results are scaled 10^{-1} - ask for one more digit than you wish significance in); v is the number of rows or variables in one factor; f is the number of factors or columns in the factor tape, and x is a directive telling what options to print out. (See below).

OPTIONS

A directive may run from 0 to 15.

Directive	Results obtained
0	criterion values only
1	T and V_r (by columns - used for subsequent runs)
2	T and V_r by rows
3	T and V_r by both rows and columns
4	C_r, C_f, V_f, D^{-1} and D
5	T and V_r by columns C_r, C_f, V_f D^{-1} and D
6	T and V_r by rows; C_r, C_f, V_f D^{-1} and D
7	T and V_r by rows and columns; C_r, C_f, V_f, D^{-1} and D

Increase 0 thru 7 by 8 (i.e., 8 thru 15) and the oblimax rotation process will be suppressed. [If you have a T and V_r rotated and wish to get C_f and V_f a directive of 14 will do this without the rotation process being performed in the machine].

In the above directive options the following symbols are used:

T is the transformation matrix (scaled 10^{-1}) used to transform the factor matrix to a

V_r which is the rotated factor matrix (scaled 10^{-1})

C_r is the reference vector correlations. It is obtained by finding T'T, is symmetric, scaled 10^{-1} and is punched in triangular form by rows

C_f is the primary factor correlation matrix punched in triangular form by rows and scaled 10^{-1}

V_f is the primary factor pattern punched by rows, N at the end of each row, J at the end of matrix, scaled 10^{-1}

D is a diagonal matrix of the reciprocals of the square roots of the diagonal elements of C_r^{-1}. D is punched as a column is unscaled and has NJ at end.

D^{-1} is the inverse of D. Decimal points indicate the scaline. It has an NJ at end of column.

THE TRANSFORM TAPE

(For a run as described under operator's instructions, B, above). Punch by columns with an N at end of each column. They must be scaled 10^{-1} and have signs.

THE FACTOR TAPE

A set of factors (obtained from KSL 1.20, 1.21 or other factor program) punched by columns with an N at the end of each column, signed fractions and scaled 10^{-1}.

TIME ESTIMATE

1. Read master - 2 minutes
2. Read data tape - $v\ f\ (.009 + .004d_i)$ seconds
3. Approximate calculation time/iteration
   $f^2(.230 + .007f + .010v)$ seconds
   (The number of iterations varies for each problem. The larger the problem - the more iterations).
4. Printing time when x = 5 or 6
   $[5f + .354f^2 + .012f^3 + .033d_0\ [f^2 + vf]]$ seconds.

For example, a v of 50, f of 10 and d_i and d_0 of 5 might take 10 iterations. Time would be approximately 2 minutes

$+[500(.009 + .016) + 1000(.230 + .07 + .5)]$

$+ [50 + 35.4 + 12 + 19.8] = 18$ minutes

(Since the number of iterations are undeterminable for large problems, it is suggested that the user run a problem for a block of time, after which the Illiac operator is instructed to remove problem as described above in Operator's Instruction. The problem may then be finished in a subsequent run or two. To do this, both T and V_r are required by columns for subsequent machine runs. The directive x (see PREPARATION OF TAPES - parameter) should be an odd number).

FF STOPS

FF014 - sum check failure - reread master

FF00S - C_r matrix is singular (has no increase)

FF015 - limit of factors exceeded (see NOTES below)

NOTES

If the directive x (see parameter tape) is 4, 5, 6, 7, 12, 13, 14 or 15 the limit on the number of factors, f, must be less than 21. For any other x directive the limits are $d_0 \leq 12$; $v \leq 128$, $f \leq 40$.

KSL 1.94 (Procrustes)

SOLUTION OF MATRIX EQUATION At=B WHERE A AND B ARE KNOWN

PURPOSE

Given two matrices A and B, this program will give you a Transformation matrix (if one exists) which will transform matrix A into matrix B; it also punches out At. The columns of A and B $\leq$ 22. The rows of A or B $\leq$ 136. The number of columns of A need not equal the number of columns of B but the number of rows in A and B must be equal. The sum of squares of columns of t are equal to one (the columns are normalized). Because of this the value of At will not be exactly that of B.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 1.94	24046
2. Data A	3409N
3. Data B	3405l
4. Parameter	24046

A new problem may be begun by reading in a new data A and following steps 3 and 4.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 1.94
2. Data A - this is punched by columns with an N terminating each column. The last N is followed by a J. Each element begins with a sign, has less than 12 digits and is scaled 10^{-1}.
3. Data B - punch exactly as Data A
4. Parameter - d_0 space where d_0 is one character from 2 thru S to indicate the number of decimal places wished in output.

TIME ESTIMATE

No time estimate given.

FF STOPS

FF order indicates wrong number of elements in a column (as compared to the first column).

KSL 1.95

PLOTTING FACTOR PAIRS

PURPOSE

This routine films the plots of all factor pairs of a set (no. frames = $\frac{n(n+1)}{2}$). It would be used if you only wish one set of plots, i.e., you do not plan on rotating further. If you do plan to do so, it is advisable to use KSL 1.96, (one exception - this routine would be used should the number of factors you wish to film exceed the capacity of KSL 1.96).

OPERATOR'S INSTRUCTIONS

		Stops
1.	Master KSL 1.95	243F7
2.	Parameter Set output to scope	24267
3.	Data (black switch down) Clear machine - advance camera one frame	0F
4.	ID	

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 1.95
2. Parameter
   00279+
   00F 00vF
   00F 00(f-1)F
   24267N

   where v = number of variables
   f = number of factors
3. Data - this tape consists of the data you wish plots on. It is scaled 10^{-1} has signs in front of each element, an N after each column and a J after last N.
4. ID tape - this tape consists of any identification you wish placed on film to help you identify it. The first character is a 9. Your identification follows. A one-hole delay follows the identification. Do not put a one-hole delay in the identification.

TIME ESTIMATE

A very rough estimate can be obtained by this formula.

Time = $2f(f+1)$ seconds

FF STOPS

Only in library subroutines.

KSL 1.96

ROTOPLOT ROUTINE

PURPOSE

Given an unrotated (Fxv) matrix (obtained usually from KSL 1.20 or KSL 1.21); a transformation matrix (fxf) and a shift matrix (fxf) this routine will multiply the S matrix times the T matrix, normalizing columns in the resultant new T_i matrix, multiply this new T_i matrix times the unrotated matrix to obtain an oblique V_{rn} matrix. It outputs the T_i matrix, the V_{rn} matrix, a hyperplane count (see below) and the upper half of the cosine of angles matrix between the reference vectors (f). The routine is limited to 111 variables and 22 factors. The output of the λ and V_{rn} matrices is unscaled and limited to 2 decimal places. The routine then places on film scatter plots between all factors; (1 with 2, 1 with 3, 1 with 4, etc.; 2 with 3, ...) until $\frac{(n)(n-1)}{2}$ frames have been filmed. This program is used when you wish to visually rotate the output to arrive at simple structure.

OPERATOR'S INSTRUCTIONS

		Stops	
1.	Parameter (preceded by DOI)	243F7	on one tape (see preparation of tapes)
2.	Master tape KSL 1.96	2403N	
3.	Data 1 (S_i matrix)	240NK	
4.	Data 2 (λ_i matrix)	240N8	
5.	Data 3 (V_0 matrix)	2403N	

When Illiac stops on 2403N at end of Data 3 set output to scope.

6.	Black switch down	OF	

When OF is reached, advance one frame, clear Illiac and read in ID tape.

NOTE

On operator's instruction sheet to indicate the number of frames required use formula $\frac{f(f+1)}{2}+1$ where f = number of factors.

PREPARATION OF TAPES

Master tape - the master tape can be duplicated from library copy. It is preceded either on same tape or by extra smaller tape to be read in first. This extra tape contains a copy of the DOI (in library) and the parameter (see below).

Parameter tape 0010+

00F 00rF

00F 00fF

00F 00vF

24999N

where r = number of rows in transformation tape (λ_i tape)

f = number of factors in V_0 tape

00F 00vF = number of variables in V_0 tape

Data 1 (S_i matrix)

This is a shift matrix (to be multiplied times the previous transformation (λ) matrix to give a new λ matrix). It is punched by rows, with +1000 in the diagonals and the other elements signed and scaled 10^{-1}. Each row is terminated by an N. The final N is followed by a J. On the first run of this program, the shift matrix is usually an identity matrix, i.e., +1000's in the diagonals and 10000's for every other element. A + can be punched for a +0000.

Data 2 (λ_i matrix)

This matrix is punched by rows. Each row is terminated by an N. The last N is terminated by a J. Elements are signed and unscaled. The first run on this program requires a λ matrix punched by hand. It is usually obtained from any KSL routine that gives a λ matrix. On subsequent runs with the same V_0, each new λ matrix can be duplicated directly from the output of the preceding run. (Do not duplicate the heading on the output tape).

Data 3 (V_0 matrix)

This is generally the output from KSL 1.20 or KSL 1.21. Since the Data 3 is unscaled and punched by rows, before using output from KSL 1.20 and 1.21, it must be run on KSL 5.30, transposed and scaled 10^{+1}. This will automatically complete the other requirements, an N at end of each row, a J after final N.

ID tape - this tape places on film your name and any identification you wish. It is constructed by punching a 9 and then what you wish to appear as identification.

A one-hole delay comes at the end, and only the end of your identification.

TIME ESTIMATE

$$\text{Time} = \frac{f}{30}\left(f + \frac{v}{8} + 9\right) \text{ minutes}$$

where f = number of factors

v = number of variables

FF STOPS

FF indicates wrong number of variables in a row.

KSL 1.97

MAXPLANE (An Oblique Factor Rotation Program)

PURPOSE

This routine takes in a transformation matrix and a signed V_0 centroid matrix obtained from KSL 1.20 or 1.21 performs a set of oblique rotations in an attempt to arrive at simple structure through use of criteria introduced by the user and included within the program. When these criteria are met, the machine punches out the rotated matrix, the new transformation matrix and the correlation matrix of angles between the factors. The rotated factors are then plotted on film (1 vs 2, 1 vs 3, ..., 2 vs 3, 2 vs 4, ..., etc.) until $\frac{(n)(n-1)}{2}$ factor pairs have been plotted. The number of factors ≤ 20; the number of variables ≤ 111.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 1.97 black switch up	20--- 24 location 001

2. Data tape (black switch down) - read in order

A new set of data may be read in without rereading master by inserting the new data in the reader - while the previous problem is calculating. If during a run, you wish to interrupt the run, center black switch. The machine will stop on a 20 at end of a cycle. If the white switch is moved up and down - the first three elements of the output data (see below - Output Data Form) will be punched and the computer will stop on a 24 order. If the black switch is placed down - computation will begin again. If the white switch is moved up and down - a complete data output will ensue.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 1.97
2. Data tape - the data tape is rather complicated. An error on the data will cause the machine to stop on an FF stop (see FF stops). The data tape consists of the following - in order given:

d_0: number of digits after decimal point in output (+, a number 1-9, and an N)

t_1: greatest hyperplane thickness

t_2: middle hyperplane thickness

t_3: least hyperplane thickness
(+, 0, decimal point, thickness as an n place decimal fraction $\leq$ 10 for each of the three thicknesses. The third thickness is followed by an N).

λ_0: an identity matrix (+ ones in diagonal, +0's elsewhere) if a centroid V matrix from KSL 1.20 or 1.21 is used, or the output matrix from KSL 1.10 (max. 20x20) (sign (+0-), 0 (i.e., 10^{-1}) element value as an n place decimal fraction $\leq$ 11 for each element. Elements are punched by column - each column terminated by a J. Last J followed by L).

V_0: V matrix output of KSL 1.20 or KSL 1.21 always (max. 111x20). (Sign, 0 (i.e., 10^{-1}) element value as an n place decimal fraction $\leq$ 11 for each element in matrix). Elements are punched by columns. Each column terminated by a J. The last J is followed by an L.

Example of data tape: 3 factors; 4 variables

d_0 = +5N

t_1 = +0.10

t_2 = +0.075

t_3 = +0.0125N

λ_0 =

```
+1000 +0000 +0000J          +0900 +0500 +0800J
+0000 +1000 +0000J    or    +0001 +0700 +0600J
+0000 +0000 +1000JL         +0300 +0510 +0000JL
```

```
+0923 -0621 +0321 -0211J
-0326 -0921 +0032 -0022J
+0432 -0222 -0662 -0331JL
```

FORM OF OUTPUT:

1. The first integer refers to the number of rotations performed on data.
2. A 3-column table giving the factor number, number of variables in that factor that lie within a + or - value equivalent to least hyperplane thickness (in example above - those variables falling between $\pm$ .0125).
3. A correlation matrix of cosines between factors by columns.

4. The parameters d_0, t_1, t_2, t_3* (to be used for future runs with λ_n data output should problem need to be interrupted and removed from machine - see Operator's Instructions).

5. λ_n: a new transformation matrix formed by the routine. (Same form as λ_0 except with d_0 decimals).

6. V_n: this rotated matrix has same form as V_0 (with d_0 decimal places) and is obtained by multiplying $V_0\lambda_n$.

7. Scope plots for each different pair of factors, plotted on orthogonal axes. Each axis is labeled with its V_n matrix column number. The upper right hand corner contains the associated C matrix entry.

NOTE

* 5 carriage returns and 15 two-hole delays are printed between the last element of the C matrix and the d_0 on output tape to enable the user to find and read required portion of output data tape back in to Illiac should the occasion arise.

TIME ESTIMATE

1. Read master tape - 2 1/2 minutes

2. Read input data in seconds = $.07 + .004\ f^2(d_i) + .604\ fvd_i$

3. Calculation time - variable: $f < 5$; $v < 50$ run less than an hour. Larger problems should be run for a fixed time - removed as instructed in Operator's Instructions - and rerun from this point at a later time. If rerun, be sure to use outputted λ_n with V_0 - not with outputted V_n.

4. Punch and plot output in seconds:

$$\frac{5f}{2} + \frac{f^2}{6} + \frac{(4+d_0)}{60} f\ (f+v) + 3(f-1)\ (f)$$

FF STOPS

FF1: Improper terminating symbol on d_0

FF2: Improper terminating symbol on t_1, t_2, t_3

FF3: Incorrect number of t's input

FF4: Improper terminating symbol on λ_0

FF5: Incorrect number of elements as a column of λ_0

FF6: Incorrect number of columns input for λ_0

FF7: Improper terminating symbol on V_0

FF8: Incorrect number of elements in a column of V_0

FF9: Incorrect number of columns in V_0

FF10: Drum error

KSL 1.98

ROTATION MASTER

PURPOSE

Given an unrotated (Fxv) matrix (obtained usually from KSL 1.20 or KSL 1.21); a transformation matrix (fxf) and a shift matrix (fxf) this routine will multiply the S matrix times the T matrix, normalizing columns in the resultant new T_i matrix, multiply this new T_i matrix times the unrotated matrix to obtain an oblique V_{rn} matrix. It outputs the T_i matrix, the V_{rn} matrix, a hyperplane count (see below) and the upper half of the cosine of angles matrix between the reference vectors (f). The routine is limited to 128 variables and 52 factors. The output of the λ and V_{rn} matrices is unscaled and limited to 2 decimal places. The routine then places on film scatter plots between all factors; (1 with 2, 1 with 3, 1 with 4, etc.; 2 with 3, ...) until $\frac{(n)(n-1)}{2}$ frames have been filmed. This program is used when you wish to visually rotate the output to arrive at simple structure.

OPERATOR'S INSTRUCTIONS

		Stops
1.	Clear start parameter tape	2405J
2.	Master KSL 1.98 black switch up	24188
3.	Data 1 black switch up	24163
4.	Data 2 black switch up	24172
5.	Data 3 black switch up	0F000
6.	Clear start label tape	

PREPARATION OF TAPES

Master - duplicate library copy KSL 1.98

Parameter tape - 009+

00F 00eF

00F 00pF

00F 00dF

00F 00rF

00F 00fF

00F 00vF

24999N

where

e*	= number of elements on a line of the print-out	$1 \leq e \leq 70/(p+1)$
p	= number of significant places in the output	$1 \leq p \leq 12$
d	= dimensions = number of rows of λ matrix	$2 \leq d \leq 52$
r	= number of rows of S matrix	$2 \leq r \leq 52$
f	= factors = number of columns of S matrix	$2 \leq f \leq 52$
v	= variables = number of rows of V_0, V_n matrices	$2 \leq v \leq 128$

Data tape - data tapes are prepared precisely as for KSL 1.96.

Output - output is in the same order as with KSL 1.96. However, there are slight differences:

1. With large matrices, it is impossible to print an entire row of the matrix on one line of the teletype paper. This program prints e elements on a line, then executes a carriage return and line feed. (e is a program parameter). At the end of a matrix row, a carriage return and two line feeds are executed. (Exception: spacing of the C matrix is twice this e.g., two line feeds between lines, and four line feeds between rows).
2. Output is not limited to two significant figures. Instead, the number of significant figures is determined by a program parameter, p. If p=5, 1/16 would be punched as +06250; if p=2, 1/16 would be punched as +06.
3. All matrices are punched by rows. The C matrix is punched as an upper triangular matrix, with the diagonal entries (which are all one) deleted. Due to restrictions placed on the format by the length of a teletype line, the C matrix will not appear as a triangle. Nevertheless, it is an upper triangular matrix.
4. On the oscilloscope output, a "+" in the extreme upper right corner is to be interpreted as a zero, and a one to carry, i.e.,:

   +57+ means +580 +09+ means +100

   -09+ means -100

* e should be ≤ 17 if $p = 3$

A time estimate is given in terms of the program parameters f, v, and p. The number of frames is given in terms of the program parameter f.

WARNING: Any attempt to use parameters d, r, f, or v larger than the maxima shown may cause Illiac to hang up. Even if Illiac does not hang up, the results are guaranteed to be incorrect.

FF STOPS

FF000 } Both denote drum transfer failures. FF000 is a
FF010 } program stop, FF010 is the Y1 subroutine stop.

TIME ESTIMATE

$$\frac{f \cdot v(p+1)}{220} + \frac{f^2}{30} = 4 \text{ minutes}$$

where f, v, and p are defined under Parameter Tape Preparation.

NUMBER OF FRAMES

$1/2\ f(f-1) + 6$

where f is the number of factors.

KSL 2.01

K-8 REVISED TO ACCEPT SINGLE, UNSIGNED DIGITS

PURPOSE

This routine will give you the same results as obtainable in Illiac library routine K-8 (Pearson Product Moment Correlations). The number of variables is limited to 144. The sample size or number of observations is unlimited. The output contains columnar correlations followed by 2 columns, the first means, 2nd standard deviations.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 2.01	3401K
2. Parameter	2408S9
3. Data	2401K

A new parameter and data tape may now be read into Illiac.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 2.01
2. Parameter - $d_0F\ d_0L\ d_0J\ ssS$ $\begin{matrix} 0 = \text{A} \\ 1 = \square \end{matrix}$

where d_0F = decimal output in correlation (results scaled 10^{-1})
d_0L = decimal output in covariances (results scaled 10^{-1})
d_0J = decimal output in means and standard deviations
ssS = number of rows of observations; sample size
0 = triangular output (use this if you wish to put results back in Illiac. Do not get covariances if you are reading results back into Illiac, i.e., set d_0L = 0L.
1 = square output (symmetric) with N's at end of each row or column. Use only if you wish to go on further with output on Illiac.

3. Data tape - this consists of a set of single unsigned digits (from 0-9) with an N terminating each row of variables.

ESTIMATED TIME

This may be obtained from the time chart included with library writeup of K-8.

FF STOPS

FF015 = wrong number of variables in a row
FF016 = variance is negative
FF017 = sum check fails on master
FF023 = arithmetic error, possible from scaling.

KSL 2.03

MEANS, STANDARD DEVIATIONS, THIRD AND FOURTH MOMENTS ABOUT THE MEANS

PURPOSE

For each of a set of v variables, this routine will calculate the mean, standard deviation, and the third and fourth moments about the mean. The program reads the data in the form of signed multi-digit elements or in unsigned single digits 1 to 9. The number of variables is limited to < 145. If means and standard deviations are required (for example, to be used in KSL 4.10) the program can be used with as much as a 90% saving in Illiac time over K-8 or K-17.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 2.03	34084
2. Parameter	2404N
3. Data tape	24084

A new parameter and data tape may be read in without re-reading master.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 2.03
2. Parameter tape - this consists of rNd_0JnFpL

where

r = 0 if data consists of unsigned digits

r = 1 if data consists of signed digits

d_0 = number of decimal places in M - 0 output. The third and fourth moments punched to 10 places

n = number of rows - i.e., sample size

p = 0 if no third and fourth moments are desired

p = 1 if third and fourth are desired

3. Data tape - the v variables or tests are punched by rows and consists of n rows. Each row is terminated by an N.* If r = 1 (see Parameter) decimal digits for each element is ≤ 12.

FORM OF OUTPUT

Means and standard deviations are printed out in parallel columns terminated by an N (ready for use in KSL 4.10).** If moments are calculated, they follow in like parallel manner, terminated by an N.

TIME ESTIMATE

Master tape readin - 30 seconds

Data readin - .004 $(d_i + 1)$ vn seconds

Calculation - .003 vn seconds

Punch - .035v $(d_0 + 1)$ for means and standard deviations
.4v for third and fourth moments

FF STOPS

FF003 sum check failure

FF000 location 0S1 wrong number of variables as compared to row 1

FF002 location 0L5 variance is negative due to rounding errors and outside tolerance limit (10^{-10}) white switch up and down to bring about punching.

NOTES

* If an N in data tape is replaced by an F, the machine will stop on a 34 order allowing a new data tape to be read in with a black switch. This allows the user to split a large data tape in to smaller, more manageable data tapes.

** If the scaling on variable j is 10^{-P_j} then in the results the means and standard deviations will also be scaled by 10^{-P_j}. The third moment will be scaled by 10^{-3P_j} and the fourth moment will be scaled by 10^{-4P_j}.

KSL 2.40

PHI'S OR COVARIANCES FOR DICHOTOMOUS DATA

PURPOSE

This program reads in a data matrix consisting of responses coded 1 and 0 and punches out a matrix of correlations or covariances ($< .25$ for covariances). The number of responses is ≤ 157. The number of persons or sample size ≤ 2535.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 2.40	34066
2. Data tape	241K4
3. Parameter tape	241K4

To read additional parameters to operate or save data at the end of step 3, raise black switch. To begin a new problem with different data at the end of step 3, raise the white switch up and down.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 2.40
2. Data tape - this consists of a series of ones and zeroes (1 indicates a positive response, 0 a negative response). These digits are unsigned. An N follows each person's response. The final N is followed by an L.
3. The parameter tape is as follows:

   d_0 space x space where d_0 is the decimal output desired and x is a directive indicative of desired output. The meanings of the directive are as follows:

X	Coefficient	Form	Printed	Matrix
0	Phi's	Triangular	By rows	Complete
1	Phi's	Triangular	By rows	Submatrix (See below)
2	Phi's	Square	By rows	Complete
3	Phi's	Rectangular	By rows	Submatrix
4	Covariances	Triangular	By rows	Complete
5	Covariances	Triangular	By rows	Submatrix
6	Covariances	Square	By rows	Complete
7	Covariances	Rectangular	By rows	Submatrix
8	Phi's	Triangular	By columns	Complete

X	Coefficient	Form	Printed	Matrix
9	Phi's	Triangular	By columns	Submatrix
10	Phi's	Square	By columns	Complete
11	Phi's	Rectangular	By columns	Submatrix
12	Covariances	Triangular	By columns	Complete
13	Covariances	Triangular	By columns	Submatrix
14	Covariances	Square	By columns	Complete
15	Covariances	Rectangular	By columns	Submatrix

Note: If a complete matrix is desired, the directive must be an even number - and the parameter is as given above (d_0 space x space). However, if an incomplete (submatrix) matrix is desired, the directive must be followed by four more numbers; i space j space r space s space where i and j are the subscripts for the first coefficient to be printed and r and s are the subscripts for the final coefficient to be printed. For example, if the Phi's calculated could form a matrix as such

$$\begin{matrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{matrix}$$

and you are interested only in the coefficients forming this submatrix $\begin{matrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{matrix}$ then your parameter might be d_0 space s (odd) space 2 space 3 space 3 space 4 space.

FORM OF OUTPUT

If matrix printed is triangular, no terminating symbols. If the punched matrix is square or rectangular, an N will be punched at the end of each row or column. Phi coefficients are scaled 10^{-1}. Following the coefficients will be printed the means and standard deviations unscaled. A table showing the number of positive responses for each item will be printed.

TIME ESTIMATE

To read data in seconds: number of persons (.007 var. + 020) Calculate and punch in seconds: (.055 + .017 d_0 + .0005 number of persons) per coefficient.

Add 2 minutes for miscellaneous reading and punching.

FF STOPS

FF location ONl - character read not 0, 1, N, F or L. Item and person numbers printed. Raise white switch up and resume checking of data.

FF location ONJ - a person's number of items does not agree with row 1. Computer will print item and person number.

FF location 1F8 - sum check failure on master readin.

KSL 2.50

FISHER'S Z TRANSFORMATION: $(Z - 1/2\ en(1 + m)/(1 - m))$

PURPOSE

This program allows you to determine Fisher's Z which can be used for averaging coefficients of correlating and in testing the significance of differences between correlations.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 2.50	24006
2. Data tape (black switch up for each row)	24006

As many data tapes can be read in without rereading master. When machine reads an N - a 24006 stop occurs. When a J is encountered, an OF occurs. On this OF more data tapes can be read by raising white switch up and down.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 2.50
2. Data tape - the data tape consists of a series of signed fractions scaled 10^{-1} and terminated by an N or NJ (see Operator's Instructions). The typical data tape is the output of K-8 or K-9 or any other correlation routine. The data tape can be triangular output (insert an N) on rectangular output (N's already present).

TIME ESTIMATE

About .415 seconds for each number readin.

FF STOPS

FF location 1L5 - sum check failure on read in of master.

KSL 2.51

COMPUTES A TRIANGULAR CORRELATION MATRIX CONTAINING VARIANCES-COVARIANCES

PURPOSE

Given a triangular variance-covariance matrix this routine computes and outputs a triangular correlation matrix where each correlation obtained is $= \frac{\sigma_{ij}}{\sigma_i \sigma_j}$ where σ_{ij} is covariance of i and j. σ_i is standard deviation of i and σ_j is standard deviation of j. The number of columns must be < 40.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 2.51	24001
2. Parameter tape	2403K
3. Data tape (black switch down)	OF

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 2.51 (Ec 44)
2. Parameter - + c + d_0 N where c = number of columns must be < 40 d_0 is the number of decimal places in output must be < 12.
3. Data tape - a triangular variance-covariance matrix of the form $\sigma_{11}, \sigma_{12}, \sigma_{22}, \sigma_{23}, \ldots, \sigma_{nn}$N.

Samples can follow one another. The last sample should be followed by an NJ.

No time estimate or FF stops given.

KSL 2.61

CHI-SQUARE FOR M BY C FREQUENCY TABLES

PURPOSE

This routine will calculate a set of chi-square values for a set of cell frequencies for a table of M rows and C columns. If desired, expected frequencies can be obtained. These are found by multiplying the ith row sum by the jth column sum and dividing this product by the total frequency of the table. For each table a chi-square value, the degrees of freedom will be punched on tape, as well as the expected frequencies. (See Operator's Instructions). $2rc + r + c < 680$ is the capacity.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 2.61	2408K
2. Data tape	OF

To get expected values, chi-squares and degrees of freedom white switch up and down to read in data. To get only chi-square degrees of freedom read data in with black switch. Additional data tapes can be read in by raising white switch up and down and then using either white switch or black switch as indicated above.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 2.61
2. Data tape - the cell frequencies are punched on tape as positive signed integers. Each row is terminated by an N, and the last N of the table is terminated by a J. After the last table on the data tape, an L is placed.

FOR EXAMPLE

+3 +29 +4N
+4 +32 +1N
+16 +2 +9N
+111 +49 +1NJ ..... NJL

TIME ESTIMATE

Chi-square and degrees of freedom in seconds per table (.033r c + 1.2)

With expected values, chi-square and degrees of freedom in seconds per table (.210r c + 1.8).

FF STOPS

FF location 125 - sum check failure on master readin.

2009K - number of elements in row does not agree with number of elements in first row. Machine prints row number.

2408S - If frequency is negative due to data tape error - machine prints row number.

2408S - If chi-square value is undeterminate (row or column sum totals zero) machine prints error undeterminate.

To continue reading remainder of data with the three listed black stop error, raise black switch.

KSL 2.70

D STATISTIC

PURPOSE

The program computes the index $\left[\sum_{v=1}^{N} (a_{vi} - a_{vj})^2\right]^{1/2}$ for all i and j with $j < i = 2, \ldots, m$. The results are printed in a triangular form where columns are associated with index j and rows with index i. This set of indices is computed successively for each of a sequence of persons. Size of each item must be a single digit. Number of items per description ≤ 100 and number of descriptions < 12.

OPERATOR'S INSTRUCTIONS

		Stops
1.	Master tape KSL 2.70	24046
2.	Parameter tape	3404L
3.	Data (black switch down)	read in order 2404K if black switch up

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 2.70
2. Parameter tape - the number of descriptions per person (m) and the number of items per description (n) are given in this form +00M
   +00N

   For example, 10 = m; 99 = n would be +010
   +099
3. Data tape - all the item scores for one description given by a person followed by all the item scores on next description by that person, and so on until all the data for that person is given. An N is punched at the end of each person. An NJ is punched after the last person on the tape.

EXAMPLE

Consider a problem in which there are two persons, each filling in 4 items on each of five descriptions. Then the parameter tape would be:

space +005+004 spaces and the data tape would be:

	d_1	d_2	d_3	d_4	d_5	
P_1	1234	1234	6541	3256	4711	N
P_2	6666	1111	2244	4421	1244	NJ

The results would be printed as follows:

001 = person number

	m	m+1	m+2	m+3
m+1	00001			
m+2	06633	06633		
m+3	03464	03464	06633	
m+4	06856	06856	04123	08185

002

10000			
06325	04472		
07000	04359	04583	
07000	04359	01000	05099

TIME ESTIMATE

Number of seconds = $\frac{m(m-1)}{2} \times \frac{N}{2}$

where m = number of descriptions and

n = number of persons

FF STOPS

None listed

KSL 2.94

AGREEMENT PATTERNS, CODE H

PURPOSE

This program finds the highest agreement score among all the persons. An agreement score is a tally of the number of items on which two persons agree. Additional persons are selected as long as the joint scores multiplied by the number of persons in the group is increased. When the score no longer increases, the persons and joint scores (agreement score) are printed, and the joint pattern of responses is removed from the measurement matrix forming a residual measurement pattern matrix. The process continues by again finding the highest agreement score in the residual matrix, repeating the above steps resulting in a second residual matrix. Whenever the agreement scores become lower than some predetermined value, the process is at an end. The capacity can be determined by the following:

$$780 > \frac{I}{19} P + P \ (P > 25); \quad 758 > \frac{I}{19} P + P \ (P < 25)$$

where I = items and

P = persons.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 2.94 (part I)	24013
2. Data tape	243F7
3. Master tape (part II) resume where stopped on (1) above	243F7

Black switch up to begin calculating. If the amount of time required to reduce the matrix to zeros is going to be prohibitive, insert the following parameter with a black switch up and proceed by black switching to begin calculating:

0084+

00F 00SF

2685N where S is the agreement score to stop at.

PREPARATION OF TAPES

1. Master - duplicate library copy of KSL 2.94
2. Data tape - item responses can be 1, 2 or 0. No signs, a - at end of each person and an N, J, F or L at end of the measurement matrix.
3. Parameter tape - see OPERATOR'S INSTRUCTIONS.

FORM OF OUTPUT

An item count is made on each person. If data tape is correct (only 0, 1, or 2's and the same number of items for each person), the machine will print number of items and persons before beginning punching of agreement scores. If there are errors, the machine will punch the type of error and the person in which the error occurred. The machine will then stop on a 24014.

TIME ESTIMATE

To read in data tape: $\frac{PI}{150}$ seconds

Calculation: $\cong 2P + 22I$ seconds (I = number of items
P = number of persons)

Printout: $\cong \frac{PI}{50}$ seconds

FF STOPS

None

KSL 4.00

FREQUENCY DISTRIBUTIONS

PURPOSE

The routine will read a set of n signed integers. These integers will be grouped in ranked categories where the lowest boundary of all categories is set by the user. When the letter N is read at the end of a signed integer, the routine will print a frequency chart with a bar graph alongside. The width of the bar graph is automatically adjusted so as not to exceed the width of the teletype paper. The maximum number of categories is 100.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 4.00	340L7
2. Parameter tape	2411N
3. Data tape	2411N

The machine will stop on a 2411N at the end of each group to be categorized. A black switch up will read a new group using the same parameter. If a series of n integers are to be categorized using the same lower boundary and increment between categories, a black switch can be used. However, if a new parameter is to be used, white switch up and down to read in the new parameter at a 2411N stop or when the machine reads a J, at which time it will stop on an OF.

TAPE PREPARATION

1. Master - duplicate library copy KSL 4.00
2. Parameter tape - the tape consists of two signed integers and is terminated by an N. $\pm$ B + I N. $\pm$ B is the lower boundary of the lowest category. I is the increment between boundaries.

EXAMPLE

100 integers ranging -9 and +90 in value. Parameter might be -9 + 1N. This will give a frequency distribution over 99 categories. 1000 integers ranging between -100 and +200: Parameter might be -100 + 10N. This would give you frequency distributions over the intervals -100; -90; -80; -70; ..., +180; +190; +200 - a total of 31 intervals with actual

numbers grouped to fall within these respective categories. If you should fail to list the lowest number in the n integers, the numbers falling below the lowest category given in the parameter and above the range of 99I will be printed out as an ERROR and the remaining numbers will be categorized.

Data tape - the data tape consists of signed integers terminated by an N.

TIME ESTIMATE

$[(4d_i + 40)\, n + 650\, c]\, 10^{-3}$ seconds

where

d_i = digits per integer

n = number of integers

c = number of categories

FF STOPS

FF location 143 - sum check failure on master read in - read again.

KSL 4.05

RANKED NUMBERS

PURPOSE

This routine takes in a series of numbers and ranks them according to a preset parameter. Each row is ranked and on the output is designated as a set. Two columns appear on the output tape. One contains the row numbers of the variable and the positive numbers ranked in descending order. The left column contains the negative numbers in descending order. The problem is limited to 26 columns of 200 numbers each. Output is in four digit form.

OPERATOR'S INSTRUCTIONS

		Stops
1.	Master KSL 4.05	243F7
2.	Parameter	20019
3.	Data	0F if J at end of tape

A new parameter and data tape may be read in at this point by rereading master.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 4.05
2. Parameter -

0012+
00F 00S_BF
00F 00S_EF
00F 00NF
00F 00P_NF
2620N
+L_AN

where S_B = begin set (usually 1)

S_E = set end. (S_B and S_E is affect give the limits of number of columns) i.e., S_E = number of columns

N = number of signed elements in each column

P_N = limit on number of numbers wished printed ($P_n \leq N$). [If you have 200 elements (N) and wish to know highest 30 elements (both + and -) set P_n = 30].

L_A* = Lower limit (absolute value) cut off point. This is obeyed before P_n is obeyed. For example, you have a column of 200 numbers. You are interested in numbers with an absolute value of 0200 and over. When all numbers of 0200 and over are printed, regardless what P_n is, machine will go on to next column. However, if you ask for a P_N of 10 and there are 20 numbers of absolute value over 0200, it will only print out the highest 10.

*NOTE (If your data is scaled 10^{-1}, 10^{-2}, etc., take this into account when setting L_A - i.e., a L_A of 2000 with a column of numbers scaled 10^{-1} - K-8 output - will cause no numbers to be ranked - they all lay below this cutoff point).

3. Data - this is a series of signed numbers, each row or column terminated by an N. They may be scaled or not.*

TIME ESTIMATE No time estimate given. Generally this is read in time 150 characters per second and print out time 50 characters per second. At one/half minute for each set for calculation. A six digit, 100 number, 20 column problem would be approximate (printing out only top 20 numbers).

$$\frac{d_1 \text{ s } N \times S_E}{150} \text{ seconds} = \text{Input} = \frac{6 \times 100 \times 20}{150} = 80 \text{ seconds}$$

$$\frac{\text{Set}}{4} \text{ minutes} = \text{Calculation} = \frac{20 \times 1}{4} = 5 \text{ minutes}$$

$$\frac{4 \times P_n \times S_E}{50} \text{ seconds} = \text{Output} = \frac{4 \times 20 \times 20}{50} = 48 \text{ seconds}$$

Total time $\tilde{=}$ 7 minutes

FF STOPS No FF stops given.

KSL 4.10

STANDARD SCORES

PURPOSE

This routine takes in a raw score matrix by row and standardizes these scores by columns according to preset parameters provided. The standard score obtained is $\frac{\sigma'}{\sigma}(x - m) + m'$ where σ' is standard deviation of the standard scores, σ is standard deviation of the scores, x is the raw score, m the mean of raw scores and M' the mean of standard scores. M' and σ' are preset by user. The number of v (elements in a row) < 145. Sample size is unlimited, the output is signed, *** each row terminated by an N and the last N followed by a J.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 4.10	24001
2. Parameter tape	24053
3. m - σ tape	3406K
4. Raw score tape	24065

(a) one more person's raw scores may be read in by raising black switch.

(b) to read in a new parameter, m - σ tape and raw score tape, place parameter tape in reader, raise white switch up and down, then repeat steps 3 and 4.

PREPARATION OF TAPES

Master - duplicate library copy KSL 4.10

Parameter - d_0 space n space +m' +** σ' ** N or J

where d_0 is the number of decimal places in output < 11. n is the number of rows, people or sample size, m' is the mean of the standard scores. σ' is the standard deviation of the standard scores.

If you wish the output in columnar form use an N; if you wish it printed in rows use a J. (There are no carriage returns provided in output by rows except at the end of a row).

$\overline{M}$ - σ tape - this tape contains the means and standard deviations of the raw scores. These are output of K-8 or KSL 2.03 with an N added after the last σ.

If these are punched directly on tape by the user, they must be signed and follow this pattern

M_1 σ_1 of 1st variance

M_2 σ_2 of 2nd variance

M_3 σ_3 of 3rd variance

.

.

.

M_v σ_v of nth variance

An N must terminated the $\bar{M}$ - σ tape.

The number of digits in each number is < 11.

Raw score tape - this tape is punched person by person. (As for K-8 data tape). Each element (variable) must be signed and have < 12 digits. An N must terminate each row.* The same number of variables must be in each row. Scaling is unimportant by row. It is important by variable. For example, there follows a sample 3 variable 4 person matrix:

+001 -02 +0011N Not scaled by rows

+010 -21 -0101N Scaled by columns

+111 +00 +1246N

+099 -32 -0321N

ESTIMATED TIME Approximately $\frac{16}{100}$ seconds per raw score

FF STOPS FF001 means wrong number of variables in a row.

NOTES * If an F terminates a person scores (in place of an N), the computer will stop, allowing another data tape to be read in. Raise black switch to begin reading in next tape.

** Suppose M' is set equal to +50.00 and σ' = +10.00. These will be punched on the parameter tape as +5 and +1. Likewise, a M' = +.5 and σ' = .1 are punched as +5 and +1. However, if M' = +5.00 and σ' +10.00, these are punched as +15 and +10.

*** If the raw scores are scaled by 10^{-x}, then the standard scores will be scaled by 10^{-x+1}.

KSL 4.15

CUMULATIVE PROPORTION

PURPOSE

This routine reads in a matrix of raw scores, and converts each element to a cumulative proportion value lying between 0.000 and 1.000. The user has two options on output. (a) The cumulative proportions may be printed out in the form of a frequency table giving score, frequency and cumulative proportion table or (b) the score matrix may be punched out, each element of the raw score matrix being replaced by its corresponding cumulative proportion value. If B option is used, the results from this program may be fed directly into KSL 4.16 or KSL 4.17. If option A is used, no further use can be made on the computer with the output tape. The number of scores that can be ranked at one time is 650 or less. The number of groups that may be run is limited by only the amount of time allowed on Illiac for one run. The output consists of signed (+) four digit numbers, each row terminated by an N, the final row terminated by an NJ.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 4.15	2405J
2. Parameter	24013
3. Data	243F7

A new parameter may be read in with a black switch up. Then, a new data may be read in.

TAPE PREPARATION

1. Master - duplicate library copy KSL 4.15
2. Parameter - 006+
   00F 00cF
   00F 00(*)F
   00F 00rF
   2419N

where

c = number of elements in one row or columns in raw score matrix.

* = type of output; a 0 will give cumulative scores replacing raw scores. This output usable in KSL 4.16 or 4.17; or 1 will give cumulative scores in a frequency table.

KSL 4.15

CUMULATIVE PROPORTION

PURPOSE

This routine reads in a matrix of raw scores, and converts each element to a cumulative proportion value lying between 0.000 and 1.000. The user has two options on output. (a) The cumulative proportions may be printed out in the form of a frequency table giving score, frequency and cumulative proportion table or (b) the score matrix may be punched out, each element of the raw score matrix being replaced by its corresponding cumulative proportion value. If B option is used, the results from this program may be fed directly into KSL 4.16 or KSL 4.17. If option A is used, no further use can be made on the computer with the output tape. The number of scores that can be ranked at one time is 650 or less. The number of groups that may be run is limited by only the amount of time allowed on Illiac for one run. The output consists of signed (+) four digit numbers, each row terminated by an N, the final row terminated by an NJ.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 4.15	2405J
2. Parameter	24013
3. Data	243F7

A new parameter may be read in with a black switch up. Then, a new data may be read in.

TAPE PREPARATION

1. Master - duplicate library copy KSL 4.15
2. Parameter -

```
006+
00F 00cF
00F 00(*)F
00F 00rF
2419N
```

where

c = number of elements in one row or columns in raw score matrix.

* = type of output; a 0 will give cumulative scores replacing raw scores. This output usable in KSL 4.16 or 4.17; or 1 will give cumulative scores in a frequency table.

3. Data tape - this tape consists of a raw score matrix. The usual raw score matrix has c columns and r rows where the c are variables and the r are persons. IF YOUR RAW SCORE MATRIX IS IN THIS FORM YOU MUST TRANSPOSE IT BY KSL 5.30. This will give you a raw score matrix of c* columns and r* rows where the c* are now people and the r* are variables. Therefore, the cumulative scores will be given over all persons for each variable. Each element must be preceded by a plus sign. Each group of scores to be ranked must be terminated by an N. If your raw score matrix has values that are negative, add a constant to each element so that all elements are positive.

TIME ESTIMATE

$(5 \times r \times c \times \text{highest score} \times 10^{-3})$ seconds to calculate. (See note).

$(.1 \times r \times c)$ seconds

FF STOPS

FF015 - number of elements in row does not agree with number in first row.

FF06L - a negative number - white switch up and down - minus sign will be replaced by a plus and problem will continue.

NOTE

Since this program gives cumulative proportions according to the position of the score in the group, a very high value may be replaced by a smaller one - thus saving much machine time. For example, scores are +1 +3 +0 +2 +999. The 999 will receive the highest cumulative proportion value as long as it is higher in value then the other 4 scores. Therefore, a 9 maybe used to replace this value without affecting the final results, but with the advantage of saving machine time.

KSL 4.16

T-SCORE PROGRAM

PURPOSE

This program converts a set of signed (+) four-digit cumulative proportion (results of KSL 4.15) numbers to normalized T-scores. It is essentially a table-look up operation. The output tape consists of signed (+) three digit numbers, ranging between the values of 19.0 and 80.0. Groups of scores (n people over 1 variable) are terminated by an N. The last row in the matrix is terminated by an NJ. This output may be transposed by KSL 5.30 and is then ready for use in K-8. No limit to size of matrix.

OPERATOR'S INSTRUCTIONS

		Stops
1.	Master KSL 4.16	24015
2.	Data	OF

TAPE PREPARATION

1. Master - duplicate library copy KSL 4.16
2. Data - normally this will be output from KSL 4.15. However, it may be prepared on the teletype equipment. If so, it consists of rows of signed (+) four-digit numbers lying between +0.000 and +1.000. Each row is terminated by an N, the last row is terminated by an NJ.

TIME ESTIMATE

Approximately 1/3 second per T-score (r x c).

FF STOPS

FF stop - a + sign is missing.

An N on data tape causes N to be printed on output tape. A J on data tape causes machine to print a J on output tape, then come to an OF stop.

KSL 4.17

STANDARDIZED T-SCORES

PURPOSE

This routine takes signed (+) four-digit cumulative proportion numbers (results of KSL 4.15) and converts them to standardized scores - punching them out on tape. The standardized scores lie between the values of -3.00 and +3.00. The output tape consists of signed three digit numbers. Each row is terminated by an N. The final row is terminated by an NJ. There is no limit to the size of the matrix used. The output may be transposed by KSL 5.30 and is then ready for K-8.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 4.17	24015
2. Data	OF

TAPE PREPARATION

1. Master - duplicate library copy KSL 4.17
2. Data - normally this will be output from KSL 4.15. However, it may be prepared on the teletype equipment. If so, it consists of rows of signed (+) four-digit numbers lying between +0.000 and +1.000. Each row is terminated by an N, the last row is terminated by an NJ.

TIME ESTIMATE

Approximately 1/3 second per T-score (r x c).

FF STOPS

FF stop - a + sign is missing.

An N on data tape causes N to be printed on output tape. A J on data tape causes machine to print a J on output tape, then come to an OF stop.

KSL 4.50

LIMITED INFORMATION ESTIMATION, SINGLE EQUATION

PURPOSE

This routine estimates parameters in economic models by the limited information single equation method. Consider the equation: $y_1 = B_{12}\, y_2 + B_{13}\, y_3 + \cdots + c_{11}\, z_1 \cdots + u_1$. The routine estimates the B's and C's and also calculates their standard error. A parameter tape specifies which of the endogenous and exogenous variables are to be included in the equation under study. To find estimates for other structural equations on the same data, use a different parameter tape.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 4.50	343KS
2. Data tape I	24108
3a. Parameter	24108

New parameters for Data I can be read in with black switch. To read in a new data tape II, move white switch up and down then black switch in new parameters.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 4.50
2. Data - the data tape consists of one or more covariance matrices. (The order or size of these covariance matrices must be ≤ 14). Each covariance matrix in the set must have the same number of elements. The elements are punched as signed fractions, and the matrix is punched in triangular form (such as the output from K-8).

The final matrix in the set S is terminated by an NJ. The largest number of matrices in any set S, must satisfy this in equality: $S(n^2 + n) < 18{,}600$ where n is the number of rows in one matrix.

PARAMETER TAPE

Each parameter tape determines a structural equation and operates in turn on each of the covariance matrices in the set. The parameters for different equations in the same mode may follow one another on the same tape.

$+a_1 +a_2 +\ldots+a_f N$ where a's represent the row numbers of the covariance matrices which are to be the endogenous variable in the equation.

$+b_1 +b_2 +\ldots+b_g N$ where b's represent the row numbers of the covariance matrices which are to be the endogenous variables in the system.

$+c_1 +c_2 +\ldots+c_h N$ where c's represent the row numbers of the covariance matrices which are to be the exogenous variables in the equations.

$+tN$ t is the number of observations used.

Limits of f, g, h, and t are

$2 \leq f \leq 6$ $\quad 1 + f + g < t$

$2 \leq g \leq 8$ $\quad f + g \leq N$ (size of matrix)

$0 \leq h \leq 6$

TIME ESTIMATE

Read master - 90 seconds

Read set of covariance matrix $\tilde{=}$ 5 seconds/matrix

Calculate and punch results $\tilde{=}$ 15 seconds/matrix.

FF STOPS

FF location 3KK - sum check failure on master read in. Symbols may be punched in the results. The machine does not stop but reads in next matrix. The meaning of these are as follows:

0/xxx number of elements in a covariance matrix disagree with number in first matrix of the set. The matrix number, xxx, is in sexadecimal.

1/ a submatrix is singular or nearly so.

2/ overflow on an element in matrix multiplication.

3/ failure in rescaling W_1 R or $R^{-1}W$. Scale the covariance matrix down and try again.

4/ failure in adjusting estimates or standard errors to proper scaling for printing. Try a correlation matrix instead of a covariance matrix (see complete library writeup for description on how to do this).

5/ failure in forming $F_{(u)}^{-1}$. The value of λ is probably near zero.

6/ trial vector of $(R^{-1} W - \lambda I) u = 0$ has not converged after 30 iterations.

7/ failure in rescaling elements of $F_{(n)}$.

KSL 5.02

MULTIPLICATION OF A MATRIX BY ITS TRANSPOSE

PURPOSE

Given a matrix A_{ij}, this routine will form AA^T or A^TA where A^T is the transpose of the matrix. If the columns in the original matrix A_{ij} are ≤ 64, then the number of rows must be < 160; if the columns are ≤ 128, the number of rows must be < 80. The output matrix of either AA^T or A^TA will be symmetric. (Lower triangular half = upper triangular half).

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 5.02	34087
2. Parameter tape	240K7
3. Data (Matrix A) tape	24087

At stop 24087, a new parameter tape can be read by raising the black switch. Follow this by reading in a new data tape.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 5.02
2. Parameter - d_0 JtF x L $\pm$ sS

   d_0: number of decimal digits desired in output

   t : t = 0, A^TA will be formed

   t = 1, AA^T will be formed

   x : x determines the form of output matrix

   x = 0 output in triangular form by columns with carriage returns after each element.

   x = 1 output in triangular form by rows punched across the page.

   x = 2 output will be square by columns with a carriage return after each element and an N after each column.

   x = 3 output will be square by rows across the page with an N after each row.

   $\pm$s: +s or -s is the integer exponent of ten. To scale by 100 +s = +2; to scale by .01, -s = -2. Remember, matrix elements read in to machine are treated as if decimal point is after sign. (See FF stops).

+0s: If no scaling is desired, sign precedes the 0.

3. Data tape - the elements of matrix A are punched by rows as signed fractions with an N at the end of each row. The final N must be followed by a J. If an F is punched instead of an N, the computer will stop on either 300K8 or 300SS. By raising black switch another tape containing the remainder of matrix A may be read.

TIME ESTIMATE

AA^T square or triangular form in seconds:
$(\text{number of rows})^2$ $(.02d_0$ + .002 number of columns + .12) + number of rows $(.005d_0$ number of columns + .018 number of columns + .03) +40.

A^TA in square or triangular form in seconds:
substitute number of columns for number of rows and number of rows for number of columns in formula above.

FF STOPS

FF001 location 0SF - number of elements in rows are not equal

FF002 location 11F, 171 } overflow in an element of matrix c.
FF003 location 112, 175 } To read in a new scaling parameter -sS only, move white switch up and down; the white switch again to eliminate having to reread data. Output will ensue.

FF004 location 13K } sum check failure in matrix A or A^T.
FF005 location 13J } white switch three times; re-insert data, and then black switch to start.

FF006 location 3F5 sum check failure on master read in.

KSL 5.11

MATRIX INVERSION

PURPOSE

This routine takes either a triangular matrix (output from K2, K8, K9) or a square matrix and computes and punches out the diagonals of the inverse matrix and/or the inverse matrix. The inverse matrix can be punched out in the form of a triangular and/or square matrix.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 5.11	3409S
2. Parameter	200K3
3. Data	2409S

To run new data tape on same parameter, white switch up and down.

TAPE PREPARATION

1. Master - duplicate library copy KSL 5.11
2. Parameter d_0 space N space X space

where d_0 = number of digits in each element of output

N = order (number of rows or columns of matrix)

X = 0 - diagonals only punched

1 - diagonals and triangular output

2 - diagonals and square output

3 - diagonals and triangular and square output

(The above are used if a triangular matrix is data matrix).

X = 10 - diagonals only punched

11 - diagonals and triangular output

12 - diagonals and square output

13 - diagonals and triangular and square output

(The above are used if a square matrix is data matrix).

3. Data tape - the data tape can be either a triangular or square matrix. If it is triangular the termination symbol at end of entire matrix must be an N, J, F, L + or -. If it is a square matrix, each row must be terminated by an N, J, F, or L. In either case, each element must have a sign and not more than 12 digits in an element.

TIME ESTIMATE	Not given.	
FF STOPS	FF018 - diagonal entry negative FF01N - diagonal is zero	} punch error
	FF01+ - singular matrix	
	FF019 - any diagonal is negative - to be used as pivit.	

KSL 5.16

RESCALE M13 OUTPUT

PURPOSE

This routine takes the output of M13, scales all elements by the same power of 10 and punches the data, column by column, as a square matrix with N's after each column and a J at the end of matrix.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 5.16	243F7
2. Parameter tape	2406F
3. Data tape	243F7

A new parameter and data tape may be read in with a black switch.

PREPARATION OF TAPES

1. Master tape - duplicate library copy KSL 5.16
2. Parameter -

004+
00F 00(d_i)F
00F 00(d_o)F
00F 00(c)F
00F 00(r)F
00F 00(p)F
26100N
+(s)

where d_1 = number of digits in each element input
d_o = number of digits desired in output
c = number of columns of each matrix on problem tape (if same parameter is used for more than one problem - the number of columns and rows must be identical for each matrix).
r = number of rows of matrix
p = number of matrices on problem tape
s = the scaling of M13 output (given a 10 column matrix from M13, the scalers at the end of each column may vary between +1000 and +00000 ... 1. If for the matrix the smallest scaler is +00001, then (s) would be this number).

3. Data tape - this is simply the output from M13.

TIME ESTIMATE

Essentially input-output time.

$$\text{Input time in seconds} = \frac{c \times r \times d_i}{150}$$

$$\text{Output time in seconds} = \frac{c \times r \times d_o}{50}$$

where c, r, d, and d_o are defined under parameter's above.

FF STOPS

None except in library subroutines.

NOTE

The scaling factor of this output can be determined by the formula 10^{K-y-1} where K is determined by the scaling of input into M13 - (10^{-K}) and y is the scale factor of output of M13.

For example: input of M13 scaled: 10^{-1}

scale factor (+001): 10^{-3}

KSL 5.16 output scaled $10^{1-3-1} = 10^{-3}$.

KSL 5.20

MATRIX ADDITION OR SUBTRACTION

PURPOSE

Given two matrices A_{ij} and B_{ij}, equal in number of rows and columns, this routine will add the elements of A_{ij} to B_{ij} and print out the new matrix C_{ij}. The routine will also subtract the elements of B_{ij} from the elements of A_{ij} and print out the new matrix D_{ij}. It is possible to sum a series of B_{ij} matrices before subtracting or adding the elements of B_{ij} from or to A_{ij}. The A_{ij} and B_{ij} matrices can be rescaled by any power of 10 up to 10 to prevent overflow of the elements of C. The size of the matrices must satisfy this inequality $i(j+1) \leq 10{,}240$ where i = number of rows and j = number of columns in one matrix which must be less than 400.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 5.20	34022
2. Parameter tape	2403N
3. Data tape A	34057
4a. Data tape B_1	24057
4b. Data tape B_2	24057
Data tape B_n	0F000

To begin a new problem at stop 0F, move the white switch up and down. The computer stops on 24022. Insert the new parameter tape and raise the black switch.

TAPE PREPARATION

1. Master - duplicate library copy KSL 5.20
2. Parameter tape - this tape consists of three signed integers terminated by an N. They are in the following order: $\pm d_0 \pm n \pm sN$ where

d_0 = number of decimal places in output

n = number of B matrices. A positive sign indicates addition. A negative sign indicates subtraction.

s = the scaling integer. The A and B matrices are multiplied by $10^{\pm s}$ during input. If no rescaling is desired (s=0), this parameter may be omitted. If both n and s are omitted, two matrices, A_{ij} and B_{ij}, will be added without rescaling.

3. Data tape - the elements of A_{ij} and B_{ij} matrices are punched by rows as signed fractions with 12 or less digits in each element. Each row is terminated by an N and each matrix is terminated by a J. If no F is punched in place of an N, the computer stops, allowing a new tape to be read in.

TIME ESTIMATE

$.005n(d_0 + 2)ij + .017(d_0 + 1)ij$ seconds.

Where n = number of B matrices

d_0 = number of digits per element in decimal output

i = number of rows in matrix and

j = number of columns in matrix

FF STOPS

FF000 location 046 - elements of A vector overflows into B vector; white switch to read another row of A.

FF001 location 056 - number of elements of rows of A not equal.

FF002 location 065 - number of elements of columns of A and B not equal.

FF003 location 071 - number of elements of rows of A and B not equal.

FF004 location 137 - master tape sum check failure, reread master tape.

KSL 5.30 - 259

MATRIX TRANSPOSITION AND RESCALING

PURPOSE

This routine reads in a rectangular matrix and punches it out transposed and/or rescaled according to user's designation. The elements may be rescaled 10^{-2}, 10^{-1}, 10^{+1} or 10^{+2}. The form of output on teletype page printout may be columnar, row or in a row with a space separating elements. It does not provide carriage returns at the end of a row of teletype print except after a complete row. That is - if the total number of characters (sign included) on a line or row of teletype print exceeds 70, the resulting 2nd, 3rd etc., lines left-hand margins will be ragged in printing. Neither R (rows) nor C (columns) can exceed 775 elements. $R \times C \leq 10240$. This routine is especially helpful in going from the output of one routine to input for another. (For example, KSL 1.20 to KSL 1.96).

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 5.30	340FN
2. Parameter tape	240F8
3. Data tape	240F8

An additional data tape may be read in with a black switch at this point. It uses the same parameter. To change parameters at stop on step 3, insert parameter, raise white switch up and down and repeat step 3.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 5.30
2. Parameter - consists of d_0 space X S P

d_0: decimal places wished in output. (If you are reading in elements with 6 decimal places and scaling 10^{+2} you would ask for 4 decimal places; if you are scaling 10^{-2} ask for 8 decimal places; if you are just transposing, ask for 6 decimal places).

x = 0 to transpose matrix; x = 1 to rescale only and x = 2 if the matrix is to be transposed and rescaled.

S: If x = 0, omit S

If s = +2 (sign important) elements will be scaled 10^{+2}

s = +1 elements will be scaled 10^{+1}

s = -1 elements will be scaled 10^{-1}

s = -2 elements will be scaled 10^{-2}

P: If P = N, output appears printed by columns

P = J, output appears printed by rows

P = F, output appears printed by rows with a space separating elements.

Data tape - the data tape must have an N terminating each column or row. A J follows the final N. Each element is a signed fraction < 12 digits. For transposing, the number of elements in each row must be equal. For scaling alone, this is not important. A large tape may be broken into smaller ones at the end of a row or column and an F substituted for the N. The computer stops and a second tape may be read in with a black switch.

TIME ESTIMATE

In seconds $\tilde{=}$ .25 R x C where R = number of rows and C is the number of columns.

FF STOPS

4F location 033 - number of elements in row do not agree with number in first row.

FF location 100 - sum check failure master read in.

NOTE

If you wish to scale a triangular matrix (K-8 output), place an NJ at end of matrix and use KSL 5.30. Do not try to transpose anything but a rectangular matrix.

EXAMPLE

52-1J Parameter - transpose and scale 10^{-1}; print by rows to 5 places. [i.e., one leading zero plus 4 digits in column 10].

DATA INPUT

```
+3-4+06+100-29+30+1+2-239+0632N
+1+3+10+099-30+31+2-1-225+1111N
+2+2+11+098-32+29+1+2+321-2103N
-3+0+07-001-43+32+2-2-112+1246N
-2-2-21+056+01+31+3+3+099-0029N
+4-1+42+113-18+32+1+4-021+0632N
+9-4-21+210-19+25+1+2+125+2567N
-3-9-32+098-32+42+1-2+625-3248N
+9-9+61+217-41+32-1+6-235+1112N
+8+2+07-304+29+33-1+1+325+1113N
+7+4+09-002+42+24-2+2-249-2468N
J
```

OUTPUT RESULTS

```
+03000+01000+02000-03000-02000+04000+09000-03000+09000+08000+07000N
-04000+03000+02000+00000-02000-01000-04000-09000-09000+02000+04000N
+00600+01000+01100+00700-02100+04200-02100-03200+06100+00700+00900N
+01000+00990+00980-00010+00560+01130+02100+00980+02710-03040-00020N
-02900-03000-03200-04300+00100-01800-01900-03200-04100+02900+04200N
+03000+03100+02900+03200+03100+03200+02500+04200+03200+03300+02400N
+01000+02000+01000+02000+03000+01000+01000+01000-01000-01000-02000N
+02000-01000+02000-02000+03000+04000+02000-02000+06000+01000+02000N
-02390-02250+03210-01120+00990-00210+01250+06250-02350+03250-02490N
+00632+01111-02103+01246-00029+00632+02567-03248+01112+01113-02468N
```

J

KSL 5.50 - 261

DELETE ROWS AND COLUMNS FROM A MATRIX

PURPOSE

Given a matrix of R (rows) by C (columns) size, this routine extracts and prints out a submatrix r (rows) and c (columns) deleting those rows and/or columns designated by the user. The input matrix is a rectangular matrix of signed elements as is the output submatrix. The size of matrices involved must satisfy the inequality $(R-M) + (C-c) + C < 822$.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 5.50	34070
2. Specification tape	24087
3. Data tape	24087

Step 3 may be repeated, if you wish to delete the same rows and columns, with a new data tape by raising black switch. To read in a new specification tape, insert the new tape in the reader and raise the white switch up and down. The computer then stops at 24087 for a data tape.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 5.50
2. Specification tape - this tape consists of three parts:
   a) a set of signed integers to indicate the <u>rows</u> R to be deleted terminated by an N. (A row is a group of C elements terminated by an N). If no rows are to be deleted, only an N is punched on tape.
   b) a set of signed integers to indicate the <u>columns</u> C to be deleted, terminated by an N. If no rows are to be deleted, only an N is punched on tape.
   c) an unsigned number followed by a fifth-hole character to indicate the number of decimal places desired in the results.
3. Data tape - the data tape consists of rows (R) of signed fractions (c); each row terminated by an N and the final N terminated by a J. If an F is used instead of an N, the computer stops on a 34090, allowing another

tape to be inserted into the reader and read with a black switch. This allows the user to break a large tape into smaller, easier handled tapes.

TIME ESTIMATE

Time in seconds: $(.006)\ RC[d_0 + 1] + (.03)\ rc[d_0 + 1]$

FF STOPS

FF location 057 - sum check failure on master readin.

FF location 092 - number of elements in a row do not agree with number in first row.

KSL 5.52

MATRIX INTERLEAFER (Augment a Matrix)

PURPOSE

This routine takes a matrix A into the machine, stores it and upon reading in a matrix B punches a new matrix C out. Matrix C is composed of row 1 of A + row 1 of B terminated by an N, row 2 of A + row 2 of B terminated by an N; etc., until all rows of A + B have been punched out. The number of digits in one element of the elements of A need not be the same as those in an element of B. No terminating symbol need be employed at the end of rows A or B. There must be a terminating symbol at the end of each of the two matrices. The number of columns of A or B must not exceed 810. The number of digits in one element of A or B must not exceed 12. Matrix A must satisfy this inequality $(c_a + 1)\, r_a \leq 10240$. To augment a matrix for K-17, K-8, etc., the output will be $A + B_i$ with an N at end of row. To prepare an augmented matrix for M-13, M-24, etc., the user can direct that the augmented matrix have an N after row A_i and on N after row B_i.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 5.52	2400F
2. Parameter	24025
3. Matrix A	20026
4. Matrix B	2400F

To read in new parameter, insert parameter and black switch up. Then, read in matrix A and B with black switch. At the 2400F stop a white switch up and down will copy the tape from the reader until a fifth-hole delay is reached.

TAPE PREPARATION

1. Master - duplicate library copy KSL 5.52
2. Parameter - consists of C_a space R_a space d_a space c_b space d_b space x

where

C_a = number of elements (columns) in a row of matrix A.

R_a = number of rows in matrix A (will be same as number of rows in matrix B).

d_a = number of digits to be punched in one element of matrix A.

C_b = number of elements (columns) in a row of matrix B.

d_b = number of digits to be punched in one element of matrix B.

x = 1 if it is desired to punch an N after each row of matrix A and B in augmented form.

x = 0 if matrix output is desired as an augmented matrix with no N at end of rows of matrix A.

3. Data A - this is a matrix (subject to capacity limitations - see PURPOSE) of signed fractions. Each row of the matrix may or may not be terminated by an N, J, F or L. However the last row of the matrix must be terminated by an N, J, F or L (or + or -).

4. Data B - prepare same as Data A.

TIME ESTIMATE

$$\simeq \frac{4d_A R_A c_A + 4(d_B + 1) v_B c_B}{150} \text{ seconds}$$ (see parameter for explanation of symbols).

FF STOPS

FF000 - sum check failure on master read in - reread.

FF0L0 - drum transfer failure or input could be incorrectly prepared.

KSL 5.54

PRODUCE DATA FOR USE IN M-22

PURPOSE

This routine reads in a lower triangular matrix, signed, scaled 10^{-1} with an N terminating symbol at end of matrix. It also reads in parameters needed in M-22 and punches output to be used directly in M-22 library routine - namely on upper triangular signed matrix, scaled 10^{-2} and having N's after each row. The matrix is followed by the parameters needed for M-22 (supplied by user). See writeup of M-22 to determine capacity limitations of this program.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master KSL 5.54	340NN
2. Parameter	24JK
3. Data (to end of Δ_N)	240F9
4. M-22 parameters	240NN

Master must be reread to run new problem.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 5.54
2. Parameter - d_0 space n space

where d_0 = number of decimal places in output

n = size of triangular input matrix (number of rows)

3. Data - this is a lower triangular correlation matrix, signed, scaled 10^{-1} with an N at the end of the matrix. Generally this will be K-2, K-8, or K-9 output, but it may be a matrix punched on tape by the user.

M-22 parameters - these are the parameters as needed for running the output on M-22 (see that writeup for more detailed description). It will consist of n numbers of 0, 1, or 2 plus 2 sexadecimal characters indicating the decimal output of eigenvalues and eigenvectors. All of this is followed by an L.

TIME ESTIMATE

Essentially readin-readout

Read in time in seconds = $\frac{(n)(n+1) \times d_i}{300}$ d_i = number of decimal places in one element

Read out time in seconds = $\frac{(n)(n+1) \times d_0}{50}$ d_0 = number of decimal places in one element

FF STOPS

FF stops on master read in - sum check failure

FF001 - No N at end of triangular matrix.

KSL 5.55

TRIANGULAR MATRIX TO SQUARE (or Rectangular)

PURPOSE

This routine takes a matrix in triangular form (K-8, K-17, K-9 output) and punches it out in square form with an N terminating each row and a J at the end of the matrix. The user may also delete designated rows or columns from the triangular matrix before it is punched out. The number of decimal places in each element of output must be less than or equal to 8. The size of the symmetric matrix cannot exceed 111 rows or columns.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 5.55	3409N
2. Parameter tape	240NF
3. Data tape	2409N

A new parameter may be read in by raising the black switch.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 5.55
2. Parameter - if a square matrix is desired (no deletions) the parameter is d_0 space n space 0 space

where d_0* = number of digits in one element (must be ≤ 8)

n = number of rows or columns in triangular input matrix (≤ 111)

If a rectangular matrix is desired, (i.e., columns or rows are to be deleted) the parameter is d_0 space n space x space s_1 space s_2 space ... s_x space

where d_0 = number of digits in one element (≤ 8)

n = number of rows or columns in triangular input matrix (≤ 111)

x = number of rows to be deleted ($0 < x \leq 111$)

$s_1, s_2, \ldots, s_x$ = selected row numbers to be deleted (NOTE the number of s_i's = x).

3. Data tape - the data tape consists of a triangular matrix in the following order:

$r_{11}\ r_{21}\ r_{22}\ r_{31}\ r_{32}\ r_{33} \cdots r_{n1}\ r_{n2} \cdots r_{nn}$

Each number is punched as a sign followed by d_0 digits.

TIME ESTIMATE

To read matrix and punch out a square matrix

$$n^2(.020d_0 + .037)$$

To delete rows and punch out a rectangular matrix

$$n^2(.0025d_0 + .0035) + nx(.0167d_0 + .0332) \text{ seconds.}$$

See parameter for definition of n_1 d_0 and x)

FF STOPS

FF123 $d_0 > 8$; $s_i = 0$; or $s_i > n$ (computer prints out which of three has occurred). To correct read in correct parameter tape with white switch up and down.

FF000 sum check failure on master - reread master.

* A triangular matrix of unsigned single digits may be read in if d_0 is set = 0.

KSL 5.57

PAGE OUTPUT CORRELATIONS

PURPOSE

This routine reads in a triangular correlation matrix (+1000's in diagonals), punches these back onto tape, unscaled, and in page form, with the columns and rows correctly labeled with column and row numbers running from 1 through v where v is the number of variables in the correlation matrix. v must lie between 2 and 148. The number of digits in one element of the correlation matrix read in must lie between 3 and 9. The output from this routine cannot be read back into Illiac.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 5.57	2401K
2. Data tape	24056

At this point raise the black switch (if complete data has been read in). If 24056 occurs and there is more data to be read in, raise white switch up and down to complete reading of data, then raise black switch once all data has been read in. After the results have been punched, computer will stop on 2401K. To begin a new problem, insert next data tape, and raise black switch, repeating steps described at beginning at this paragraph.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 5.57
2. Data tape - the data tape consists of the output from routines K-8, K-9, KSL 2.40 or any other triangular matrix of correlations.

The machine records the number of fifth-hole characters after the first diagonal entry (first +1000 above). Whenever the number of fifth-hole characters are following diagonal entries (+1000's) exceeds the number after the first, the machine will stop on 24056. (An extra carriage return after a +1000 will cause this stop to occur). If the complete matrix has not been read in, raising the white switch will cause the machine to continue reading

until the next time an extra fifth-hole character is encountered. (In general, if your data tape is output from any of the above routines and is one tape, and has not been tampered with), the above stop will not occur until all the correlations have been read in. However, there will generally be a short part of data left - the means and standard deviations. Be sure to note to the operator that "all the data will not read in."

TIME ESTIMATE — In seconds $.025\ n^2(d_i+1)$

FF STOPS

FF location 0J7 sum check failure on master readin - try again.

FF location 01F no + for first diagonal +1000.

FF location 021 no 1 for first diagonal.

FF location 049 no + for subsequent diagonal +1000.

FF location 04N no 1 for subsequent diagonal +1000.

Raising the white switch for any of the last four FF stops causes the computer to continue reading; your output will not be affected.

NOTES

In the special case when $d_i = 3$ and $n > 99$, the column heading will be out of line for column heading > 99. Otherwise, the results will be punched correctly.

KSL 5.60

MATRIX NORMALIZATION

PURPOSE

This routine will normalize (convert into a unit vector) a matrix by rows or by columns. If r is the number of rows and c is the number of columns, and you normalize by rows, the capacity is $1 < c < 701$ and r is not limited. When normalizing by columns $1 \leq c < 351$ and $r(c+1) \leq 10240$. The elements of the normalized matrix will be printed by rows.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 5.60	34016
2. Parameter tape	2001K
3. Data tape	0F000

To begin a new problem at 0F, move white switch up and down. Computer comes to a 24016 stop. Raise black switch to read new parameter and new data.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 5.60
2. Parameter tape - $+d_0$ N or J. Where $+d_0$ is a signed number < 12 indicating the number of decimal places desired in one element of the output. If an N is used, normalization will be done by rows. If a J is used, normalization will be done by columns.
3. Data tape - the elements of the matrix are punched by rows as signed fractions, with an N after each row, and a J after the final N. If, instead of an N, an F or L is punched at the end of a row, the computer will stop. This enables you to read in another tape containing the remainder of the matrix by raising the black switch.

TIME ESTIMATE

Read in of master - 20 seconds

Read in, calculation and punch out of normalized matrix in seconds - $.022\ rc(d_0+1)$

FF STOPS

FF001 wrong number of elements in a column

FF002 wrong number of elements in a row

FF003 sum check failure on master read in, try again.

KSL 5.70

SUMS OF ROWS AND COLUMNS OF A MATRIX

PURPOSE

This routine reads in a matrix, punches out the sum of rows and columns. Capacity, 2 times number of rows + number of columns < 920.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 5.70	2405J
2. Parameter tape	24048
3. Data tape	24048

Data tapes of identical size may be read in with black switch. To change parameter, white switch parameter in, read in data tape with black switch.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 5.70
2. Parameter* -

```
005+
00cF 00cF      c is number of columns
007+
00rF 00rF      r is number of rows
2472N
```

3. Data - the data tape is punched with N's at end of each row. Scale all data at least 10^{-1} to keep overflow from occurring. No J at end of data.

TIME ESTIMATE

$$\text{Input time in seconds} = \frac{r \times c \times d_i}{150} \qquad d_i = \text{decimal places in input}$$

$$\text{Output time in seconds} = \frac{rd_o + cd_o}{50} \qquad d_o = \text{decimal places in output}$$

FF STOPS

None listed.

NOTE*

The number of decimal places in output will be six (6) unless a change is added to beginning of parameter to indicate a desire for more than 6 places. This change will be in the form

```
003+
00F 00(d_o)F
```

followed by regular parameter.

KSL 5.71

SUMS OF SQUARES OF ROWS AND COLUMNS OF A MATRIX

PURPOSE

This routine forms the sums of squares across rows and down columns of a matrix and in addition it calculates the total sums of squares (2R+C). Twice the rows plus the number of columns < 920.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master tape KSL 5.71	24014
2. Parameter tape	2401S
3. Data tape	2401S

A black switch up will cause new data tapes to be read in on same parameter. For a new parameter, white switch up and down - read in new data with black switch.

PREPARATION OF TAPES

1. Master - duplicate library copy KSL 5.71
2. Parameter - d_0 space - where d_0 = number of decimal places wished - 6 or above and < 12 is usually wise to take care of scaling.
3. Data - rows of a signed matrix terminated by N's with final N followed by a J. Preferably scaled 10^{-2}. In most cases 10^{-1} is O.K.

TIME ESTIMATE

$$\text{Input time in seconds} = \frac{r \times c \times d_i}{150}$$

$$\text{Output time in seconds} = \frac{rd_0 + cd_0}{50}$$

FF STOPS

FF stops occur for overflow.

FF stop location 02J - wrong number of elements in a row.

ITERATIVE ESTIMATION OF COMMUNALITIES: PRINCIPAL AXES METHOD

PURPOSE

Given a lower triangular correlation matrix, this routine makes a principal axes factoring the matrix, re-estimates the communalities, substitutes the new estimates for the old, and prepares for another cycle. The latent roots and new communality estimates are punched after each cycle. The size of the correlation matrix is 18 or less.

OPERATOR'S INSTRUCTIONS

	Stops
1. K7 master (part I)	24014
2. Data	343F7
3. K7 master (part II from where removed in 1) black switch down	OF

If it is desired to interrupt run before OF is reached, center black switch. When Illiac stops on 240F9, white switch up and down. The output will be used in preparing the data tape for further iterations.

TAPE PREPARATION

1. K7 master (duplicate Illiac library copy)
2. Data tape - this tape consists of the parameter followed on the same tape by two copies of the lower triangular half of the correlation matrix. The parameter is d_0JvFfL where d_0 = decimal places desired in output (scaled 10^{-1}); v = number of variables or rows in correlation matrix and f is the number of factors you desire to have communalities estimated for. The correlation matrix must be scaled so the sum of squares of elements over the entire matrix is less than 1/2. The ones in the diagonal of the correlation matrix may be replaced by communality estimate < one to speed time of calculation. The correlation matrix is punched by rows (lower left triangle). Each element is signed and the last element is followed by an N. This matrix is duplicated - that is, two copies are needed on the data tape.

TIME ESTIMATE	Dependent on number of iterations and size of matrix.
	11 variables - 1 iteration - 1 minute
	18 variables - 1 iteration - 5 minutes
FF STOPS	None listed.

PRODUCT MOMENT CORRELATIONS, VARIANCE, COVARIANCES, MEANS AND STANDARD DEVIATIONS

PURPOSE

Given a raw score matrix of v variables or measurements and s samples or observations this routine gives product moment correlations and/or variance-covariances and/or standard deviations. The maximum number of variables allowed is 144. There is no limit on the number of observations. However, if, upon consulting the time chart enclosed you find your problem will run longer than 2 hours it is best to use K-17 with the interrupted run option. If the output asked for in this routine is correlations in triangular form, the output will be usable directly in KSL 1.20, 1.21, 1.52 as well as other routines.

OPERATOR'S INSTRUCTIONS

	Stops
1. K8 master (operator's copy)	3401K
2. Parameter	240J9
3. Data	2401K

A new parameter may be read in with a black switch. This is followed by a new data tape.

TAPE PREPARATION

1. K8 master (operator has copy - no need to duplicate)
2. Parameter d_cFd$_{vc}$Ld$_n$JsSx

where d_c = decimal places in correlation output (results are scaled 10^{-1}).

d_{vc} = decimal places in variance-covariances output (do not get this if correlations are to be read back into Illiac - set $d_{vc} = 0$).

d_m = decimal places in means - standard deviations.

s = sample size or number of observations.

x = 0 if triangular output is desired.

= 1 if square symmetric output is desired. This output will be scaled 10^{-1} and printed by columns, each column being terminated by an N.

3. Data - each observation is punched as a signed fraction of up to 12 decimal places. An N must be punched at the end of each row. If an F follows a row, the machine will stop and another part of the data tape can be placed in the reader and the problem continued. No scaling is required on the data other than as described below: If your data consists of 3 rows of 3 variables

```
+1   +21 -321
+16  - 2 -1192
-111 + 0 +  26
```

it would be punched as follows:

```
+001 +21 -0321N
+016 -02 -1992N
-111 +00 +0026N
```

TIME ESTIMATE — See accompanying chart.

FF STOPS

FF017 - sum check failure on master read in - reread master

FF015 - number of variables in a row does not agree with number in first row - return problem

FF016 - variance is negative - return problem

FF023 - arithmetic error, possible from scaling.

TIME ESTIMATES FOR K-8 PROGRAM

The following times are estimates only. They are for use as a guide to help provide closer estimates in the time required to run K-8 programs through Illiac. In general, estimates based on these times should not be more than ± 10% in error.

The chart below gives the time for fixed intervals of variables in seconds per sample, i.e., for a 100 sample size problem, multiply the time in column two by 100 to obtain the time in seconds the problem is to run before output occurs. The remainder of the chart gives the time, in minutes, that a square or diagonal matrix would require to output. For convenience sake, these times are given for 3, 4, 5, and 6, decimal places of output.

NO. OF VARIABLES	READ-IN TIME SEC/SAMPLE	PRINT-OUT TIME IN MINUTES TRIANGULAR MATRIX				SQUARE MATRIX			
		3 dec.	4 dec.	5 dec.	6 dec.	3 dec.	4 dec.	5 dec.	6 dec.
5	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1
15	2	1	1	1	1	1	1	1	1
20	2	1	1	1	1	1	1	1	2
25	3	1	1	1	1	1	2	2	2
30	4	1	1.5	1.5	2	2	3	3	4
35	5	1.5	2	2	2.5	3	4	4	5
40	6	2	2.5	2.5	3	4	5	5	6
45	7	2.5	3	3	3.5	5	6	6	7
50	7.5	3	4	4	4.5	6	8	8	9
55	9	4	4.5	5	5.5	8	9	10	11
50	11	4.5	5.5	6	6.5	9	11	12	13
65	13	5.5	6.5	7	7.5	11	13	14	15
70	14	6.5	7.5	8	9	13	15	16	18
75	16	7.5	8.5	9.5	10.5	15	17	19	21
80	18	8.5	9.5	10.5	11.5	17	19	21	23
85	20	9.5	11	11.5	13	19	22	23	26
90	22	10.5	12	13	14.5	21	24	25	29
95	24.5	11.5	13.5	15	16	23	26	29	32
100	27	13	15	17	18	25	29	33	35
105	31	14	16.5	18	19.5	27	32	35	38
110	34	15.5	18	20	21.5	30	35	39	42
115	37	17	19.5	21.5	23.5	33	37	42	46
120	41	18.5	21	23.5	25.5	36	41	46	50
125	44	19.5	23	25.5	27.5	38	45	50	54
130	48	21	25	27.5	30.5	41	49	54	60
135	52	22.5	26.5	29.5	33	44	52	58	65
140	56	25	28.5	31.5	35	49	56	62	69
144	61	27.5	31	34	38	54	61	67	75

(M. Foster)

QUARTIMAX ORTHOGONAL ROTATION OF FACTORS

PURPOSE

The routine finds the orthogonal rotation which maximizes the variance of the squared factor loadings. This can be shown to be equivalent to maximizing the fourth powers of the loadings. Two factors are operated one at a time. The capacity is $f \cdot v \leq 745$ where f = factors; v = variables with an N at the end of each column. For a more complete description of the mathematical method use refer to the library routine writeup.

OPERATOR'S INSTRUCTIONS

	Stops
1. K11 Master	243F7
2. Parameter	2407N
3. Data (black switch down)	0F

If continuous, punching has not begun when estimated time has elapsed, raise the black switch to OBEY (center). When Illiac stops on 24090, raise white switch up and down. Punching will ensue.

TAPE PREPARATION

1. Master - (duplicate Illiac library copy)
2. Parameter - 006+
   00vF 00vF
   0027+
   00fF 00fF
   22278N

where v is the number of variables and f is the number of factors desired.

3. Data - a matrix tape of f columns of v variables. Each element is signed and scaled 10^{-1}. Each column of factor is terminated by an N.

TIME ESTIMATE

$.36fv + cf$ seconds where c and f can be approximately determined from the following table:

No. of factors	4	6	8	12	16
c	4	7	12	21	36
f (in seconds)	17	30	50	120	200

(Centroid factors will usually require a larger c than will principal axes factors).

FF STOPS

None listed.

MULTIPLE REGRESSION ANALYSIS WITH TRANSFORMATION

PURPOSE

Given a score matrix this routine gives first order correlations on subsets chosen from the score matrix after performing indicated transformations on the scores. It also gives a multiple correlation between a dependent and independent variables in the subset. Also obtainable are covariances, means, standard deviations, standardized and unstandardized regression weights and their respective standard errors. If it is not desired to receive this information, the proper decimal output parameter may be set to 0, suppressing the output for that particular information. The total number of variables on the data tape must be ≤ 50. The largest number of variables dealt with in any parameter cannot exceed 22. The number of observations is unlimited. [If a large number of analyses are to be done on one data tape, please use K16 - which allows the data tape to be stored on the drum. K14 requires the data to be read into the machine after each parameter, K16 does not].

OPERATOR'S INSTRUCTIONS

	Stops
1. K16 master (Operator's copy)	24012
2. Parameter	200NL
3. Data tape	24012

A new parameter and data tape may be now read in with a black switch for each.

TAPE PREPARATION

1. K14 - operator has copy - no need to provide one.
2. Parameter - consists of four parts.

sS = where s is sample size - number of rows in matrix.

nN = where n is the number of variables in subset. Must be 22 or less.

d_cF = where d_c is the number of places to print correlations.

I $d_{cov}L$ = where d_{cov} is the number of places to print covariances

d_{BS}+ = where d_{BS} is the number of places to print standardized regression weights, their standard errors and multiple correlations.

d_{Bu}J = where d_{Bu} is the number of places to print unstandardized regression weights, their standard errors and standard errors of estimates.

(Any of the four decimal output numbers may be set to zero).

II Following these six pairs of sexadecimal characters are a series of ones and zeros. The ones refer to the position of the variables in the subset you wish to hold successively independent. For example, in a subset of 20 variables from a data tape of 50 variables you wish to hold the 1st, 7th and 10th variables in the subset (not over the entire set of variables) successively indpendent. The series of ones and zeros would appear on tape as 1001000001. No delays, spaces, line feeds or other characters with fifth-hole punches must appear in this series. The first character after the last one in this series must be a delay, space or line feed.

III From this point on until Illiac reads a single fifth-hole delay, whatever is on the parameter in the way of identification will be exactly duplicated on the output tape. If no identification is desired follow the fifth-hole character at the end of part II above with a single-hole delay.

IV Next follows a sequence of signed integers terminated by an N. These integers provide the clue to the transformations, if any, wished on the entire group of variables (not the subset) in the data tape. Therefore, if you have a data tape of 50 variables and are using a subset of 22, there will be 50 signed integers -- 28 of which will be +0. The transformations available are listed below.

+0 = eliminate variable

+1 = use variables as is unchanged

+2 = variable is squared - x^2

+3 = variable is cubed - x^3

+4 = take square root of variable - x - $x \geq 0$

+5 = take log variable over 10, $\frac{\log x}{10}$, $x \geq 10^{-10}$

+6 = take $\frac{1}{\sigma\pi}$ arc sin x

EXAMPLE - You have a 36 variable, 121 sample matrix. You wish correlations, multiple correlations and standardized regression weights over a subset of 22 variables, holding variables (in the subset) 1, 3, and 5 successively independent. You wish to transform variable 10 into x^2 and 20 into $\sqrt{x}$. Your parameter would appear as,

121S22N5F0L8+0J10101sCORN 1961° +0+0+1+1+1+0+0+1+1+2+0+0+1

p
a
c
e

+1+1+1+1+1+1+1+4+1+0+0+0+0+1+1+0+0+1+1+0+0+1+1+0N

The computer reads in the 36 variables, 121 row matrix and stores it. When it is ready to calculate, it selects columns (variables) 3, 4, 5, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 27, 30, 31, 34, 35 to work with, squaring variable 10 and taking the square root of variable 20 before setting up the matrix to work with. From this subset of 22 variables, the first one is held indendent, then the third one, then fifth one.

3. Data tape - the data tape consists of s rows of v variables. These are signed numbers. Each of the s rows are terminated by an N. Scaling over variables is advisable. If you have four values for variable one of +1-21+321+6213, column one would appear on the tape as

+0001 ..... N

-0021 ..... N

+0321 ..... N

+6213 ..... N

If an F terminates a row instead of an N, the machine will stop on a 3012K. This allows another tape containing more data to be read in with black switch up.

TIME ESTIMATE

Read in of master = 35 seconds.

To calculate and printout $\tilde{=}$ $[2v - 10 + \frac{d_i s v_i}{80} + \frac{v\, d_o}{7}]$ seconds

where $\underline{v}$ = number of variables in subset, $\underline{v}_i$ = number of variables on one row of data tape, $\underline{d}_i$ = number of digits in one element on data tape, $\underline{d}_o$ = decimal places in output (one element) and $\underline{s}$ = number of rows or samples in data tape.

FF STOPS

FF011 location 378 - sum check failure on master readin - reread master.

FF012 location 0J4 - row of variables does not agree in number with first row - return problem.

FF013 location 06N - drum transfer failure - try again.

NOTE

[If 2 variables are uncorrelated with each other - have 0 correlation - routine will hangup on division hangup or 00000 order].

ANALYSIS OF VARIANCE BY METHOD OF FITTING OF CONSTANTS

PURPOSE

Given a score matrix of v variables and s samples, this routine calculates and punches out RAW SUMS OF SQUARES, Fitted Constants, the inverse of the cross products matrix and "Accounted for Sum of Squares". If it is desired, the cross product matrix may be obtained in place of the inverse matrix. The total number of variables must satisfy the inequalities:

$\frac{V-K}{2}[3V - K+3] + V \leq 792$ and $V < 30$ where V = total number of variables and K = number of dependent variables. There is no limit on sample size unless the accumulated sums of squares exceeds 3^{39}. See the Illiac library writeup for a more complete description of use of results from K15.

OPERATOR'S INSTRUCTIONS

	Stops
1. K15 master (Operator's copy)	34042
2. Parameter tape	24084
3. Data	24042

A new parameter and data tape may now be read in with a black switch up for both.

TAPE PREPARATION

1. K15 master - operator has copy - no need to provide one.
2. Parameter - sS vN k + 2 carriage returns identification - a fifth-hole delay.

where s = number of rows in data tape - sample size.
v = number of variables: must be < 30.
and k = number of dependent variables. The identification can be any thing for your own convenience in identifying the output. If no identification is desired, punch after the + 2 carriage returns and a single fifth-hole delay.

If k is set to v then the inverse matrix will be replaced by the matrix of cross-products. However, this cross-product matrix will be in reverse order, i.e., variable 1 will be last, 2 next to last, etc.

3. Data tape - each observation or sample is represented by a row of signed coefficients indicating the relationship of the observation to the constants to be fitted. The dependent variable/variables is the last one to be punched in the row. These signed coefficients describe the complete classification of each observation; use a zero if the observation did not occur in a class and a 1 if it did. Each row is terminated by an N. If weights other than 1 are desired for any row of this observation matrix, precede the row by the numerical weight j punched as +jJ. The machine in effect treats these rows as if they occurred j times. If a terminating N is replaced by an F, the machine will stop so that more data can be read in.

EXAMPLE - 10 observations 4 variables, subgroups of observations = 3, 3, and 4.

Parameter tape - 10S7N1 + (2 carriage returns) IIA*

Data tape - +1+0+0+0392-0426+1123-23N
+4J+1+0+0-0263+1023-0024+19N [Treat as though four observations]
+1+0+0+1235-0032+1926+03N
+0+1+0+0032+0092-0326+09N
+0+1+0-0069-1023+0462-10N
+0+1+0+0132+6245+1111-11N
+2J+0+1+0-0294-0236+0232-62N [Treat as though two observations]
+0+0+1-2369+0326+0436-03N
+0+0+1+2469-0012+0626+05N
+0+0+1-0235+0024+0727-03N

TIME ESTIMATE

Read in $10s(v^2 + v)\ 10^{-3}$

Compute $2[v-k]^3$

Output $136[v^2 + 3v + k^2 - k]$

FF STOPS

FF010 location 03K = drum transfer failure try again

FF201 location 099 = accumulated number exceeds 39 - return problem

FF200 location 08F - number of variables in a row does not agree with number in first row. Return problem.

NOTE

If k > 1, then the covariance model is in effect and the constants printed out for the k variables are in reverse order; i.e., the last variable first, next to the last 2nd, etc.

MODIFIED MULTIPLE REGRESSION ANALYSIS

PURPOSE

See writeup of K14 for complete instructions on running this problem. The only difference is in (1) the data tape need only be read in after parameter 1 and (2) the parameters have one additional terminating symbol (see below). In addition the sample size times [the total number of variables on tape +1] must be less than 8137.

OPERATOR'S INSTRUCTIONS

	Stops
1. K16 master (operator's copy)	24012
2. Parameter (type N)	24113
3. Data	24012
4. Parameter (Type J)	24012

Any number of parameter (Type J) may be read in with a black switch. These all will work on the data read in in step 3. To read in a new data tape, black switch in a parameter (Type N) and then read in the new data tape. This may then be followed by parameters (Type J).

TAPE PREPARATION

1. K16 master (Operator has copy - no need to provide one).
2. Parameter (Type N) - identical to one described in K14 except an <u>additional</u> N is added after the final N.
3. Data - as described in K14.
4. Parameter (Type J) - identical to one described in K14 except one <u>additional</u> J is added after the final N.

[NOTE: transformations for K16 are only +0, +1, +2, +3, +4, and +5. No +6].

TIME ESTIMATE

Read in of master = 35 seconds

To calculate and print out $\tilde{=}$ $[6v + v\, d_0]$ seconds where v = number of variables in subset and d_0 = number of places in output.

FF STOPS

FF011 location 378 - sum check failure on master readin - reread master.

FF012 location 12J - number of variables does not agree with number in first row - return problem.

FF013 location 06N - drum transfer failure - try again.

NOTE

If two variables are uncorrelated - have zero correlation - with each other - machine will hangup on a division hangup or a 00000 order.

See K14 EXAMPLE. The only change would be an additional N at end of Parameter A - an additional J at end of Parameter B.

PRODUCT MOMENT CORRELATIONS, VARIANCES COVARIANCES, MEANS AND STANDARD DEVIATIONS

PURPOSE

This routine takes a raw score matrix, row by row providing center correlations between the columns in the matrix. It differs from K-8 in that a problem can be completed in two or more distinct machine runs. (See "INTERRUPTION OF PROBLEM"). The number of variables must be ≤ 144. The number of rows or sample size is unlimited. The output, if obtained in triangular form, is ready to be used directly as input for KSL 1.20, 1.21, 1.52.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master K17	2401K
2. Parameter	240J9
3. Data	2401K

A new parameter may be read in with a black switch. A new data is then read in with a black switch. If it is desired to run the problem with more than one machine run, the following is used as OPERATOR'S INSTRUCTIONS.

FIRST RUN	Stops
1. Master K17	2401K
2. Parameter	240J9
3. Data tape, part 1 (2N's at end of tape)	FF015
4. Move white switch up and down (Do not print output)	0F

SUBSEQUENT RUNS	Stops
1. Master K17	2401K
2. Tape (Output from 1st run)	24047
3. Data tape, next part	2401K

(If total problem is finished. If there is still a third data tape, instructions for steps 3 and 4 are as in FIRST RUN instructions).

TAPE PREPARATION

1. K17 master - (duplicate library copy)

2. Parameter - d_0F d_{c0} L d_m J s S x

where d_0 = number of places in correlations.

d_{c0} = number of places in covariances.

d_m = number of places in means and standard deviations (the above results are scaled 10^{-1} if 3 significant places desired set $d_c = 4$).

s = sample size or number of rows.

x = 0 if symmetric matrix of correlations and/or covariances is to be punched in triangular form with an N at end of the matrix. Use 0 if you desire to use output in KSL 1.20, 1.21, 1.52.

= 1 if a square matrix is desired. If 1 is used correlations and/or covariances are punched out by columns with an N at the end of each column.

3. Data tape - the data tape consists of s rows of n elements each. The n variables are punched as signed fractions. Each row is terminated by an N. If an F terminates a row instead of an N, the computer will stop on 30049. To continue reading the remainder of the data tape, raise the black switch, being sure the new tape is in the reader before doing so.

If you wish to run the problem in interrupted machine runs (see OPERATOR'S INSTRUCTIONS), at the end of the first part, punch an extra N terminating symbol.

TIME ESTIMATE

See accompanying chart.

FF STOPS

FF001 location 152 - sum check on reading sexadecimal tape has failed. Reread master, try sexadecimal tape again.

FF015 location 04N - an extra N has been punched following a normal N. White switch up and down to get sexadecimal tape for next run - or the number of variables in

one row does not agree with the number in the first row. Raise the white switch up and down to get partial correlations to this point - correct row in error - run rest of data as instructed for an interrupted run.

FF016 location 088 - the variance is negative and outside the tolerance limits. Remove problem.

FF017 location 3F4 - sum check on reading master tape has failed. Reread master.

FF023 location 081 - arithmetic error possibly due to scaling on data tape.

DATA OUTPUT

If both correlations and covariances are specified by the parameter tape, the correlations appear in the first column and the covariances in the second column. The correlations are scaled 10^{-1}. The scaling of the covariances depends upon the scaling of the variables on the data tape. If x_i is scaled by 10^{-1} and x_j is scaled by 10^{-j}, then the covariance ij is scaled by $10^{-(i+j)}$.

TIME ESTIMATES FOR K-8 PROGRAM

The following times are estimates only. They are for use as a guide to help provide closer estimates in the time required to run K-8 programs through Illiac. In general, estimates based on these times should not be more than ± 10% in error.

The chart below gives the time for fixed intervals of variables in seconds per sample, i.e., for a 100 sample size problem, multiply the time in column two by 100 to obtain the time in seconds the problem is to run before output occurs. The remainder of the chart gives the time, in minutes, that a square or diagonal matrix would require to output. For convenience sake, these times are given for 3, 4, 5, and 6, decimal places of output.

NO. OF VARIABLES	READ-IN TIME SEC/SAMPLE	PRINT-OUT TIME IN MINUTES TRIANGULAR MATRIX				SQUARE MATRIX			
		3 dec.	4 dec.	5 dec.	6 dec.	3 dec.	4 dec.	5 dec.	6 dec.
5	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1
15	2	1	1	1	1	1	1	1	1
20	2	1	1	1	1	1	1	1	2
25	3	1	1	1	1	1	2	2	2
30	4	1	1.5	1.5	2	2	3	3	4
35	5	1.5	2	2	2.5	3	4	4	5
40	6	2	2.5	2.5	3	4	5	5	6
45	7	2.5	3	3	3.5	5	6	6	7
50	7.5	3	4	4	4.5	6	8	8	9
55	9	4	4.5	5	5.5	8	9	10	11
50	11	4.5	5.5	6	6.5	9	11	12	13
65	13	5.5	6.5	7	7.5	11	13	14	15
70	14	6.5	7.5	8	9	13	15	16	18
75	16	7.5	8.5	9.5	10.5	15	17	19	21
80	18	8.5	9.5	10.5	11.5	17	19	21	23
85	20	9.5	11	11.5	13	19	22	23	26
90	22	10.5	12	13	14.5	21	24	25	29
95	24.5	11.5	13.5	15	16	23	26	29	32
100	27	13	15	17	18	25	29	33	35
105	31	14	16.5	18	19.5	27	32	35	38
110	34	15.5	18	20	21.5	30	35	39	42
115	37	17	19.5	21.5	23.5	33	37	42	46
120	41	18.5	21	23.5	25.5	36	41	46	50
125	44	19.5	23	25.5	27.5	38	45	50	54
130	48	21	25	27.5	30.5	41	49	54	60
135	52	22.5	26.5	29.5	33	44	52	58	65
140	56	25	28.5	31.5	35	49	56	62	69
144	61	27.5	31	34	38	54	61	67	75

(M. Foster)

L7 - 230

AUTOMATIC LINEAR EQUATION SOLVER

PURPOSE

The routine takes a set of simultaneous linear equation coefficients set up in matrix form (A_{nn} and solves the unknowns $x_1, x_2, x_3, \ldots, x_{n-1}$. If desired it will also calculate and provide the residuals. The values for the unknowns are punched out in a single column with a decimal point properly placed in each result. The capacity is 143 equations or less.

OPERATOR'S INSTRUCTIONS

	Stops
1. L7 master (operator's copy)	3401N
2. Data	24009

If residuals are wished at this point instruct the operator to re-insert data into reader and white switch up and down. If no residuals are wished, read a new data tape in with a black switch up.

TAPE PREPARATION

1. L7 master - operator has copy - he will supply it. If request is made on OPERATOR'S INSTRUCTION sheet.
2. Data tape - the elements in the data tape are signed fractions, scaled so that each coefficient or element is $< 1/2$. Each element must have 12 or less digits. Each row of coefficients must be terminated by an N.

The number of rows in the data matrix will be one less than the number of coefficients. At the end of the data matrix, after the final N, place a number d_0 from 2 thru N to indicate the number of digits in the elements to be punched out. K = 10 places, S = 11, N = 12.

An example follows:

$56\ x_1 + 23\ x_2 + 9\ x_3 = 10$

$-2\ x_1 + 3\ x_2 + 2\ x_3 = -5$

$31\ x_1 - 32\ x_2 + 8\ x_3 = 999$

The data tape asking that solution contains 10 decimal places in each result would be as follows:

+056 +023 +009 -010N

-2 +3 -2 +5N

+0031 -0032 +0008 -0999NK

TIME ESTIMATE

a) 15 seconds to input program

b) .004K seconds to input coefficient where K = (n)(n-1)

c) $.0017n^3$ seconds to solve equations

d) $.017(d_0+3)n$ seconds to punch results

where n = number of elements in one row.

FF STOPS

FF02K - master sum check failure (read master again)

FF026 - drum failure during triangularization (try again)

FF027 - drum failure during black substitution (try again)

FF028 - wrong number of coefficients (compared to first row)

FF029 - trying to solve for more than 143 equations.

AUXILIARY M7 - 150

PRINCIPAL AXES FACTOR ANALYSIS

PURPOSE

This routine takes a lower triangular matrix of correlations, scaled 10^{-2} and prints out up to 23 factors obtained by scaling the eigenvectors by the square root of the corresponding eigenvalue (latent root). Each factor is printed as a column terminated by an N. The eigenvalues are punched as a separate column after the last factor.

OPERATOR'S INSTRUCTIONS

	Stops
1. M7 master (part I)	2400K
2. Data	243F7
3. M7 master (Part II - begin where removed in 1)	0F

The master must be reread for a new problem.

TAPE PREPARATION

1. M7 master - (duplicate Illiac Auxiliary library copy)

2. Data - the first part of the tape consists of the correlation matrix to be factored. It must be scaled 10^{-2} signed and the entire matrix terminated by an N. The correlation matrix will be the lower left triangular half punched by rows. The second part of the tape consists of two numbers each followed by a sexadecimal character - they are d_0JvF where d_0 is the number of decimal places desired in the output and v is the order or number of rows in the correlation matrix.

An example of data input follows:

+1000)
+0438 +1000) This could be output from K2, K8, K9, K17
+0072 -0309 +1000)

The data tape would be

+01000
+00438
+01000
+00072
-00309
+01000N

5J results will have a sign, zero and four significant figures (i.e., scaled 10^{-1})

3F order of matrix (number of rows) is three.

THIS MUST BE ON ONE TAPE

TIME ESTIMATE (Same as for M3-117)

FF STOPS None listed.

MATRIX TRIANGULARIZATION AND COMPUTATION OF ITS DETERMINANT

PURPOSE

Given any number of square matrices up to 40 x 40 elements in size, this routine will upper triangularize and compute the determinant value - punching either or both out.

OPERATOR'S INSTRUCTIONS

	Stops
1. M12 master	20095
2. Data	20095

Additional data tapes may be read with a black switch.

PREPARATION OF TAPES

1. M12 master - duplicate library copy
2. Data tape - a square matrix, punched row by row, each element signed and scaled so its value is less than 1/2 (see Scaling). Each row of elements must be terminated by an N. If it is desired to print out triangular matrix end last row of matrix with a J instead of an N.

DATA OUTPUT

To interpret from results the value of the determinant use the following:

$(-1)^n b \times 10^2, \quad -1 \leq b < 1$

Example -.013 -16123456789 represents

if n even $-.16123456789 \times 10^{-13}$

n odd $.16123456789 \times 10^{-13}$

If an upper triangular matrix is to be punched out, it will be punched out by columns with eleven digits.

SCALING

All elements should be less than 1/2. In order to determine value multiplied by determinant result to get correct results follow example below:

if original matrix is

1	2	3
5	4	1

data tape would

.01	.004	.0091

be +.1 +.2 +.3N x 10^1 multiply value of

+.05 +.04 +.01N x 10^2 determinant obtained by

+.1 +.04 +.091N x 10^{-1} not on tape 10^2 to get correct result

10^2

TIME ESTIMATE

a) Read in matrix $[(d_i+1)n^2 + n]\ 10^{-3}$ seconds

b) Computing time and punching determinant

$$\simeq \frac{5}{4} n^3\ 10^{-3} \text{ seconds}$$

c) Punch time of triangularized matrix

$$336(n^2 + n)\ 10^{-3} \text{ seconds}$$

FF STOPS

Only in library subroutines.

COMPLETE LINEAR MATRIX EQUATION SOLVER AND GENERAL MATRIX INVERSION ($N \leq 22$)

PURPOSE

This routine solves the linear matrix equation $Ax = B$ for matrices A and B when A is non-singular and a square matrix of order n x n and B is of size n x m. The size of n and m must be such that the inequality $nm + \frac{n}{2}(n + 1) + (n + m) \leq 842$ is satisfied.
The solution x of size n x m is punched out by columns (an N at the end of each column). The last element in each column is a scaling factor (see example). If a matrix A_{nn} is to be inverted, the capacity is a matrix A of size 22 x 22.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master M13	2403F
2. Data	2403F

A new data tape may now be read in with a black switch.

TAPE PREPARATION

1. Master M13 - duplicate library copy
2. Data - the data tape for solving $Ax = B$ is prepared as follows: Punch row 1 (N) of matrix A followed by row 1 (N) of matrix B. Then row 2 (N) of matrix A followed by row 2 (N) of matrix B. Continue until row n (N) of matrix A and row n (N) of matrix B are on tape. Each row of A and each row of B is followed by an N. The final N on the tape is followed by a sexadecimal character 2 thru S indicating the number of digits desired in the output. All rows must be scaled so that no element is $\geq 1/2$. Each element must be preceded by a sign and must have digits ≤ 12 in number.
The data tape for inversion of matrix A consists of n elements and n rows, each element signed and ≤ 12 in number of digits. The elements in the matrix must be scaled so no element is $\geq 1/2$. Each row must be terminated by a J. Following the nth J will be a sexadecimal character 2 thru S indicating the number of digits desired in the output (inverted matrix).

TIME ESTIMATE

Read in a matrix A and B = $n \frac{[n+m]}{150} d_i$ seconds

Computing time for solving Ax = B when B is in columns

$$\frac{[n^3]}{500} m \text{ seconds}$$

Computing time for inversion of A = $4n - .3n^2 + .01n^3$ seconds

Punching of output = $\frac{n^2 d_o}{50}$ seconds.

FF STOPS

No FF stops. However, if an F is punched on the output tape with no other output, it indicates either overflow of elements or a singular matrix.

EXAMPLE (Input)

An example would be

$$A = \begin{matrix} 1 & 2 & 0 \\ 3 & -4 & 1 \\ -2 & 5 & 6 \end{matrix}$$

$$B = \begin{matrix} 1 & 2 \\ 0 & -1 \\ -2 & 0 \end{matrix}$$

The data tape for solving Ax = B would be

+1 +2 +0N +1 +2N
+3 -4 +1N +0 -1N
-02 +05 +06N -02 +00NK (last row scaled so that elements = 1/2)

Output will be to 10 places rounded off.

The data tape for inversion of A would be

+01 +02 +00J
+03 -04 +01J
-02 +05 +06JK (all elements scaled alike so no element $\geq$ 1/2)

EXAMPLE (Output)

A tripical output for a 3 x 3 matrix may look like the following:

+1.000
-0.500
+2.500
+1.000N → scaling factor - decimal point after 1

-20.00
+00.30
-25.00
+01.00N

+030.0
+010.0
+005.0
+001.0N

M 21 - 240

MATRIX MULTIPLICATION

PURPOSE

The routine allows two matrices A_{ij} and B_{ik} to be multiplied. $j \leq 412$ and $jk \leq 10240$. Each element of the matrix must be less than one. If overflow occurs, the program records it and punches out the proper element of the product matrix C_{ik} accordingly. This would be punched as signed integer, a decimal point and an n place fraction otherwise (i.e., elements < 1) the elements of the product matrix C_{ik} is punched as an n place fraction. The elements of C_{ik} are preceded by three constants i; the number of rows of C_{ik}; k, the number of columns, and n the number of decimal places printed. Each row is terminated by an N. The matrix C_{ik} can be read back into the computer if no elements are greater than one. If you wish to take C_{ik} output and place it directly back into the computer in the same production problem, it is wise to be sure that all elements of A_{ij} and/or B_{jk} are originally scaled 10^{-1}, that is, each element has a leading zero.

OPERATOR'S INSTRUCTIONS

	Stops
1. Master M21	30026
2. B matrix (parameter on beginning of tape)	30030
3. A matrix (parameter on beginning)	30019

A new A matrix may be read in with a black switch. If you wish to read in a new B matrix and A matrix, white switch then black switch in the new B matrix and A matrix.

TAPE PREPARATION

1. Master M21 - duplicate library copy
2. B matrix - this matrix is preceded by a parameter on the same tape. This parameter is j space k space d_0 space, where j is the number of rows of matrix B; k is the number of columns and n the number of decimal digits to be punched in each element of C_{ik}. The elements of B are punched by rows, the elements being signed fractions.

Each row is terminated by an N (replacing an N by a J causes the computer to stop, allowing a new tape with the remainder of the matrix to be inserted in the reader).

3. A matrix - punched as B matrix, except the parameters are i space j space d_0; i is the number of rows in A, j is the number of columns and d_0 is the same as in the B matrix parameter.

TIME ESTIMATE

Input of B matrix if d_i is the number of digits in one element in seconds $(4jk\ d_i + 18jk + 80k + 500)\ 10^{-3}$

Input of A matrix, calculation and output in seconds $(3.3ij\ d_i + 4ij\ d_i + 6.6ik\ d_0 + 200ik)\ 10^{-3}$

FF STOPS

FF03N - sum check failure on master readin.

FF03J - drum transfer failure. (This will occur if the j (number of columns) in matrix A does not agree with the j (number of rows) in matrix B).

NOTE

For instructions on performing a series of A · B multiplications with just new A matrices or with both new A matrices and new B matrices without stopping see the the more complete library writeup.

COMPLETE LINEAR MATRIX EQUATION SOLVER AND GENERAL MATRIX INVERSION ROUTINE

PURPOSE

Given two matrices $\mathbf{A}_{ii}$ and B_{ij} this routine will solve the matrix equation $A_{ii}\ x = B_{ij}$ for x where A is non-singular (its determinant value $\neq 0$). It also will take a matrix A_{ii} and give its inverse A_{ii}^{-1}. The limits of B_{ij} are set by $i + j \leq 164$ and $i^2 + 2ij + 3i \leq 20,420$. To get the inverse $i \leq 82$.

OPERATOR'S INSTRUCTIONS

	Stops
1. M24 master	2406K
2. Data tape	2406K

A new data tape may be read in with a black switch up.

TAPE PREPARATION

1. M24 master - duplicate library copy
2. Data tape - when matrix A and/or B are to be read from tape, they appear as a matrix in this form

$$\pm A_{11} \pm A_{12} \pm A_{13} \cdots \pm A_{1i}\ C$$
$$\pm B_{11} \pm B_{12} \cdots \pm B_{1i}\ C$$
$$\pm A_{21} \pm A_{22} \pm A_{23} \cdots \pm A_{2i}\ C$$
$$\pm B_{21} \pm B_{22} \cdots \pm B_{2i}\ C$$
$$\cdots\cdots\cdots\cdots\cdots$$
$$\pm A_{i1} \pm A_{i2} \pm A_{i3} \cdots \pm A_{ii}\ C$$
$$\pm B_{i1} \pm B_{i2} \cdots \pm B_{ij}\ C\ P$$

where $\pm A_{ii}$ and $\pm B_{ij}$ are the elements of the matrices. C is a control character for calculations. P is a control character for output.

If the problem is to solve a linear matrix equation, then the C after each row is an N. If the problem is the inversion of A_{ii}, C is a J and the elements B_{ij} are not punched. P determines the form of output. If $1 \leq P \leq S$ output will be on tape by columns where p is the number of digits to be punched for each element. Each column will be followed by a scaling factor or an N (be sure to remove the scaling factor before reading the output from M24 back into the computer).

If P = L the results of the computation will be placed consecutively on the drum by row, starting at location 2560. If the user wishes to utilize the drum for series of computations, he is referred to the more complete writeup of this in the regular Illiac library routines book.

TIME ESTIMATE

Inversion: $i = 20 \quad \tilde{=} 90$ seconds

$i = 40 \quad \tilde{=} 600$ seconds

Linear equation: $i = 10, j = 1 \sim 8$ seconds

$i = 10, j = 4 \sim 15$ seconds

For higher orders, the time increases approximately as n^3.

FF STOPS

FF010 - drum transfer failure on y - 1

FF011 - sum check error in reading program master. Read again.

MATRIX MULTIPLICATION WITH OR WITHOUT RESCALING

PURPOSE

The routine will form the product of two matrices, $(A_{ij}\ B_{ij})$. The product matrix (C_{ij}) can be punched on tape or stored on the drum. Matrix B_{ij} can be rescaled by any power of ten prior to multiplication. A set of products of various sizes can be produced without rereading parameters, and these products can be stored on the drum for use in subsequent operations. Multiple products such as $(A)^p\ (A)^{p-1}$ ... AB = C can be formed without punching intermediate results on tape. Capacity limitations are found under the heading "STORAGE OF B AND C MATRICES."

OPERATOR'S INSTRUCTIONS

4 cases are cited. The first is for a simple $(A_{ij}\ B_{ij} = C_{ij})$ multiplication.

CASE I	Stops
1. M28 (Operator's copy)	3402L
2. Parameter tape, type T	24060
3. Data B	240J7
4. Data A	2402L

To begin a new problem at 2402L read in a new parameter with a black switch.

TAPE PREPARATION

CASE I

1. M28 master - operator has a copy at Illiac reader console. Request as above. No master needed if requested as above.
2. Parameter tape, type T - this tape consists of three elements and is the same for CASES I, II and III. They are d_0 space $\pm$ s and x where

d_0 = decimal output in matrix C_{ij} when $d_0 < 13$. If d_0 is ≥ 13, d_0 will be interpreted as a drum address at which to store matrix C_{ij}. See section called STORAGE OF MATRICES B AND C.

$\pm$s = scaling exponent. Omit this if no rescaling is desired on B_{ij}. If +1s is given each element B_{ij} will be multiplied by 10 before the matrix is stored on the drum.

If -ls is given, each element of B_{ij} will be multiplied by .1. If your B_{ij} matrix has a leading zero before each element, (i.e., +092 -032) a + ls will eliminate the zero. However, if there is a chance that the product matrix C_{ij} will have elements over +1.00, rescaling matrix B_{ij} would be reverse. A scaler of -12 would cause the elements in matrix B_{ij} to all become zero - a thing to avoid.

x = a directive, either N, J, F or L. When x = J or L and d_0 is < 13, the rows of C_{ij} will be punched in columnar form with a carriage return after each element. An N will be punched after each row and a J at the end of the matrix. When x = N or F and d_0 is less than 13, the rows of the matrix C_{ij} will be printed across the teletype page. The rows will be evenly punched so that a neat printout will result. An N will follow each row of C_{ij}, and a J will be punched at the end of the matrix.

Whenever d_0 is 13 or greater, matrix C_{ij} will be stored on the drum, and no distinction is made between the directives. For example, a parameter of 10 space n means print C_{ij} with 10 decimal places by rows across the teletype page.

DATA TAPES A_{ij} and B_{ij}

The elements of both tapes (all tapes in later cases) are punched by rows as signed fractions with an N at the end of each row. If any row is terminated by an F instead of an N, the computer will stop. By raising the black switch, a second part of the matrix will be read. A terminating symbol is required at the end of each matrix. Matrix B_{ij} can have a J or L. If matrix A_{ij} has a J terminating symbol - the next tape read in (if any) must be a parameter tape. If an L terminates an A_{ij} tape, the previous parameters is retained and a new B_{ij} may be read in on this parameter.

OPERATOR'S INSTRUCTIONS (CASE II) $(A^p)(B^p) = C^p$

		Stops
1.	M28 master (operator's copy)	3402L
2.	Parameter tape, type T	24060
3.	Data tape $B^{(1)}$	240J7
4.	Data tape $A^{(1)}$ (with L terminating symbol)	24060
5.	Data tape $B^{(2)}$	240J7
6.	Data tape $A^{(2)}$ (with L terminating symbol)	24060

etc.

TAPE PREPARATION (CASE II)

1. Master (see CASE I)
2. Parameter tape, type T (see CASE I)
3. Data tapes (see CASE I)
   That only difference is that data tapes $A^{(p)}$ must contain an L as a terminating symbol.

OPERATOR'S INSTRUCTIONS (CASE III) $A^{(p)} A^{(p-1)} \ldots A^{(2)} A^{(1)} B = C$

		Stops
1.	M28 master (Operator's copy)	3402L
2.	Parameter tape, type T	24060
3.	Data tape B	240J7
4.	Data tape $A^{(1)}$ (J terminating symbol)	2402L
5.	Parameter tape, type D	240SJ
6a.	Data tape $A^{(2)}$ (L terminating symbol)	240SJ
6b.	Data tape $A^{(3)}$ (L terminating symbol)	240SJ

Continue reading Data tapes $A^{(4)} \ldots A^{(p-1)} A^{(p)}$ with a black switch. At the end of $A^{(p)}$ a J terminating symbol will cause the product matrix C to be printed.

TAPE PREPARATION (CASE III)

1. Master (see CASE I)
2. Parameter tape - type T, since the first product $A^{(1)}$ B must be stored on the drum at a location, the d_0 directive will be greater than 13. It can be determined by referring to the section below titled STORAGE OF MATRICES B AND C. An N or J must terminate this parameter. For example, a parameter of "4558 space +2N" would cause $A^{(1)}$

B to be stored by rows in successive locations at drum location 4558 after having multiplied each element of matrix B by 100 before forming the $A^{(1)}$ B product. The type D parameter tape must be terminated by either an F or an L. (When this is read in and the black switch lifted the product matrix $A^{(1)}$ B is read from the drum to form $A^{(2)}$ $A^{(1)}$ B is $A^{(2)}$ is read into the computer. This product matrix $A^{(1)}$ B will be read in from drum location D, rescaled if so indicated, and restored as a new B matrix at location 2560 on the drum). For instance a type D parameter might be "6 space -1F". This means read rows of previously formed product matrix from the drum, multiply each element by 1/10 and store as a B matrix; form the new product matrix of $A^{(2)}$ (this matrix is read in at this time) $A^{(1)}$ B and print it out to 6 places as rows across the page.

DATA TAPES

(See CASE I) Data tape B and $A^{(1)}$ will have a J terminating symbol at end of matrix. All other data tapes will have an L terminating symbol until the final product is wished printed. The last data tape read in will then have a J terminating symbol.

OPERATOR'S INSTRUCTIONS

(CASE IV) A_{ir} M_{rc} = C_{ic} where M_{rc} is at drum location D and Williams Memory has been cleared.

	Stops
1. M28 master (Operator's copy)	3402L
2a. Parameter tape, type D	240SJ
2b. Parameter B - (white switch up and down)	240SF
3. Data tape A_{ir} (with J terminating symbol)	2402L

This case would be used in the instance where a product matrix M_{rc} had been formed and stored on the drum - and was still present on the drum (i.e., the drum had not been clear). It would be possible to read in a Data tape A_{ir} and print out the new product C_{ic}.

TAPE PREPARATION

1. Master (see CASE I)

2a. Parameter type d - see CASE III

2b. Parameter B.- Consists of D space r space c space where D is the drum location of M_{rc} (see STORAGE OF B AND C MATRICES) r is the number of rows in the stored matrix M_{rc} and c is the number of columns in M_{rc}.

STORAGE OF B AND C MATRICES

The first element in each row of B is stored at 2560+ (j-1)I where j is the number of rows in matrix B and I is determined by the number of columns in B. The capacity for B is shown in the chart below:

Columns in B	I	max. rows
$0 < k \leq 65$	65	157
$65 < k \leq 129$	129	79
$129 < k \leq 193$	193	53
$193 < k \leq 257$	257	39
$257 < k \leq 311$	321	31

If matrix C is not stored on the drum, the number of rows of matrix A is not limited. The maximum number of columns in matrix B is 311.

The first available location for the storage of matrix C on the drum will be after the final element of matrix B. This can be determined by the following formula: D = 2560 + (j-1) I+k (see above paragraph for explanation of j, I and k). The number of locations required for storage of matrix C will be i (k+1) and D+i(k+1) < 12,800. For CASE III, when multiple products are formed D must be chosen large enough to accomodate the largest B matrix. As an example: you wish to form a product matrix of the following size matrices $A^{(3)}$ = 12 rows 30 columns; $A^{(2)}$ = 30 rows by 40 columns; $A^{(1)}$ = 40 rows by 15 columns and B = 15 rows by 90 columns. This will give a product matrix C of 12 rows by 120 columns. The sizes of the matrices stored are:

$A^{(1)}{}_B$ = 40 rows x 90 columns = 3600

$A^{(2)}[A^{(1)}{}_B]$ = 30 rows x 90 columns = 2900

$A^{(3)}[A^{(2)}A^{(1)}B]$ = 12 rows x 90 columns print out.
$A^{(1)}B$ provides largest matrix to be stored. Even though B read in must be stored, its size (15 x 90) is smaller than $A^{(1)}B$ to be later stored from drum location 2560 on. So matrix $A^{(1)}B$ will be termining matrix as to size of drum area to be used. In this product matrix j = 90 and k = 40.
From the chart if k = 40, I = 65 and maximum j allowed is 157. Therefore the drum location for storing C matrix will be 2560 + (90-1) 65 or 8345 = D.
As $A^{(1)}B$ will also be the largest C matrix stored on the drum it will require i (k+1) locations for storage. Since i = 40 and k = 90 this becomes 3640 locations. The number of locations for storage of B and C must fulfill the inequality $D + i(k+1) < 12800$.
8345 + 3640 = 11985 < 12800. Therefore in the parameter needing a D drum location D = 8345 and enough space is left for storage of matrices C as they are formed.

TIME ESTIMATE

To read master and parameters - 30 seconds
To read matrix B from tape and store on drum -
$jk(.005d_i + .013)$ seconds
To read matrix C from drum and restore as B matrix -
$i(.005k + .018)$ seconds
To read matrix A and calculate -
$ij(.005d_i + .010) + ik(.004j + .009)$ seconds
To punch matrix C -
$ik(.0167d_0 + .034) + .067i$ seconds
To store matrix C on drum -
$i(.002k + .008)$ seconds

FF STOPS

FF001 location 06K J or L at end of lst row of B; white switch to continue.
FF002 location 084 Number of elements of B are not equal.
FF003 location 0F8 Row vector (size does not conform to column vector of B.

FF004 location 0S2	Improper drum address for store of C; a white switch up and down will calculate first available address and store C.
FF005 location 10K	Overflow on element of C.
FF006 location 109	B sum check failure; white switch to try again.
FF007 location 18F	Master sum check failure; reread.
FF008 location 0J0	Row of C read incorrectly from drum; white switch to try again.

NOTE

A type T parameter tape can be converted to a type D tape merely by raising the white switch up and down at stop 24060.